The impact of uncertainty in risk preferences and risk capacities on lifecycle investment

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Introduction

- Rich literature on how to elicit risk preferences and capacity
- Our focus on next step
  - From (possibly noisy) preference measurements to investment strategies
- Risk attitudes and risk capacities may change over time

Motivation

There are several reasons why a discrepancy between an implemented strategy and a theoretical optimum could arise

- Young people don't pay attention to pensions.
- Risk preferences can change over the investment horizon. Ideally, pension investment would be based upon risk preferences during retirement...
- There can be unexpected changes in pension contributions or in overall retirement wealth, e.g. due to divorce, disability, longevity...
- There can be difficulties in the precise measurement of risk preferences.
- Pension funds or insurers may want to group similar agents within risk classes.

Research Question

- Consider an agent saving money towards retirement over a period of $T = 40$ years, regularly making pension contributions and investing the money in a risky and a risk-free asset.
- Suppose that in the first t years, the agent's investment strategy is based on wrong assessments of future pension contributions (risk capacity) or a wrong assessment of the agent's risk preferences.
- How much welfare does the agent lose compared to an optimal strategy?

The Setting I

- Risky asset S modeled as geometric Brownian motion with drift $\mu = 0.04$ and volatility $\sigma = 0.2$, risk-free interest rate $r = 0.01$ so $(\mu - r)/\sigma^2 = 0.75$.
- Evolution of financial wealth F from period t_i to $t_{i+1} = t_i + \Delta$ given by

$$F_{t_{i+1}} = (1 - m_{t_i})e^{r\Delta}F_{t_i} + m_{t_i}\frac{S_{t_{i+1}}}{S_{t_i}}F_{t_i} + h_{t_i}\Delta e^{r\Delta}.$$

where h_{t_i} is the annualized pension contribution.

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- We assume $F_0 = 1,\ \Delta = 1/3,\ T = 40$.
- For contributions, we consider a relatively extreme baseline setting, $h_0 = 1$ until $t = 20$ and $h_1 = 2$ after $t = 20$.

The Setting II

- The agent has power utility

$$u_\gamma(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

with risk aversion parameter $\gamma = 3$ and cares about expected utility from financial wealth at retirement,

$$\max_{(m_{t_i})_i} E[u_\gamma(F_T)].$$

- Denote by H_{t_i} the present value of the agent's outstanding pension contributions (human capital)

$$H_{t_i} = \sum_{t_j : t_i \leq t_j < T} h_{t_j} e^{-r(t_j - t_i)} \Delta.$$

The Setting III

- As (a proxy for) the agent's optimal investment strategy we consider the Merton fraction at the level of total wealth $F_{t_i} + H_{t_i}$, capped by the leverage constraint $m_{\max} = 1.5$

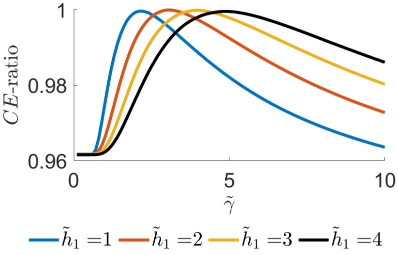
$$m_{t_i}^* = \min\left(m_{\max}, \frac{\mu - r}{\gamma\sigma^2}\frac{F_{t_i} + H_{t_i}}{F_{t_i}}\right)$$

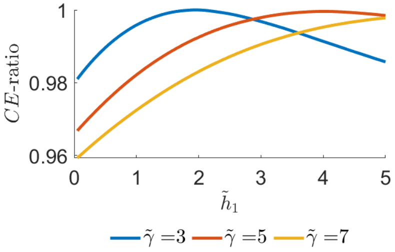
Moment of truth model

- Until the "moment of truth" $t = 20$, the agent's investment strategy is based on a wrong value $\tilde{\gamma}$ of the risk aversion parameter and a wrong value $\tilde{h}_1$ of the pension contributions after time t.
- By $\tilde{m}$, we denote the resulting investment strategy and by $F_T(\tilde{m})$ the terminal wealth from following $\tilde{m}$ until t and m^* afterwards.
- Our welfare criterion is the ratio between the certainty equivalent from following $\tilde{m}$ rather than m^*.

$$CE\text{-ratio} = \frac{\widetilde{CE}}{CE^*} = \frac{u_\gamma^{-1}(E[u_\gamma(F_T(\tilde{m}))])}{u_\gamma^{-1}(E[u_\gamma(F_T(m^*))])}$$

Baseline Results



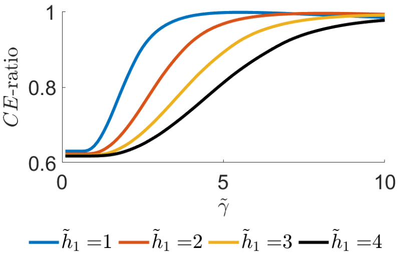


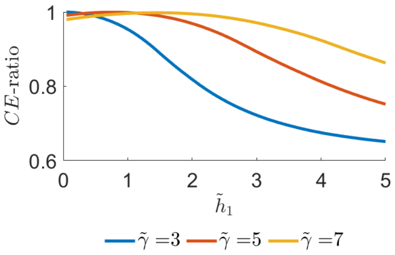
- In this example, all curves stay above 0.96 for quite extreme ranges of $\tilde{\gamma}$ and $\tilde{h}_1$.
- Leverage constraint offers protection for small $\tilde{\gamma}$.
- Mistakes in different dimensions can cancel each other out – or amplify each other.

Further Scenarios in the paper

	$Z_i,\ i =$	t	γ	h_0	h_1	m_{max}	CE^*
Base	1	20	3	1	2	1.5	84.33
Risk aversion up	2	20	**5**	1	2	1.5	79.96
Human capital up	3	20	3	1	**3**	1.5	109.86
Income drop	4	20	3	1	**1**	1.5	58.56
Disability	5	20	3	1	**0**	1.5	32.61
Early t	6	**10**	3	1	2	1.5	99.01
Late t	7	**30**	3	1	2	1.5	71.03
No leverage	8	20	3	1	2	**1**	83.92
Leverage up	9	20	3	1	2	**2**	84.49

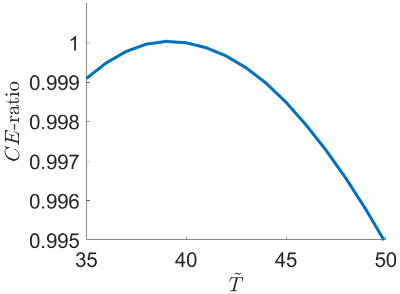
Disability Scenario





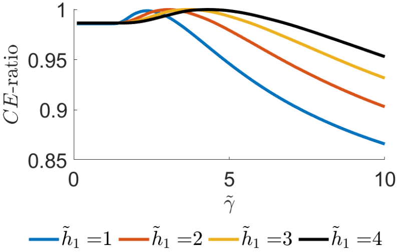
- In the disability scenario, $h_1 = 0$, welfare losses are substantial.
- This is only the *additional* loss from having invested too riskily at young ages due to anticipation of future premiums

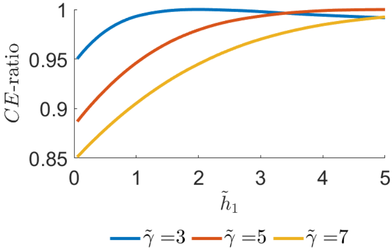
Longevity Scenario



- Under the correct beliefs agent thinks he retires at $T = 40$, the age of 67, but now we look at the investment strategies based on the expectation that he retires at $\tilde{T}$, at the age of $27 + \tilde{T}$.
  - At time $t = 35$ (five years before the real retirement at $T = 40$) agent realizes the true retirement age.

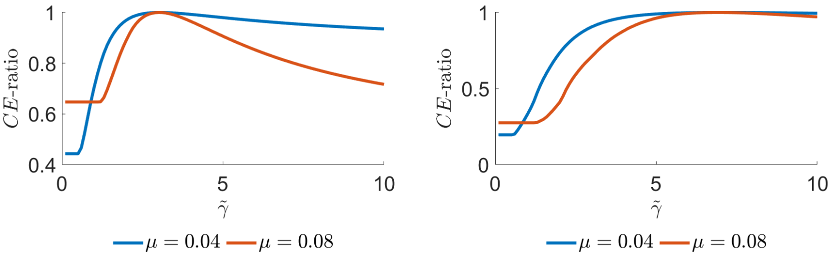
More favorable stock market





- With $\mu = 0.08$, welfare losses from overestimating risk aversion begin to play a bigger role.
- The same is true for losses from underestimating human capital.

What if the moment of truth never comes?



- In the case $t = T$, we consider only discrepancies in γ for $\gamma = 3$ (left) and $\gamma = 7$ (right).
- One motivation for $t = T$ is aggregation of agents with similar risk preferences.
- Figures suggest that a moderate number of strategies can cover interval $[1, 10]$ with minimal welfare losses.

Could we offer everyone their personal optimum?

- In order to compute

$$m^* = \frac{1}{\gamma} \times \frac{F+H}{F} \times \frac{\mu - r}{\sigma^2}$$

without error, we would need to know not only risk preferences (as captured by γ) and risk capacity (as captured by F and H) exactly but also financial market conditions as captured by μ, r and σ.

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- Accurate estimation of the drift μ is famous for being almost impossible under realistic conditions.
- However, our previous results on stability under misspecified m^* apply in similar form.

Estimates from 10,000 scenarios of 30 years of daily data

	min	$q_{0.05}$	$q_{0.25}$	mean	$q_{0.75}$	$q_{0.95}$	max
$\widehat{\mu}$	-0.129	-0.019	0.015	0.0397	0.064	0.099	0.170
$\widehat{\sigma}$	0.194	0.197	0.199	0.200	0.201	0.203	0.206
$\frac{\widehat{\mu}-r}{\widehat{\sigma}^2}$	-3.533	-0.728	0.133	0.744	1.358	2.226	4.062

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$\widehat{\mu}$	-0.129	-0.019	0.015	0.0397	0.064	0.099	0.170
$\widehat{\sigma}$	0.194	0.197	0.199	0.200	0.201	0.203	0.206
$\frac{\widehat{\mu}-r}{\widehat{\sigma}^2}$	-3.533	-0.728	0.133	0.744	1.358	2.226	4.062

- The majority of estimated investment fractions are far away the theoretical value of 0.75.
- Risk capacity and risk preferences have to be quite uncertain to become a major source of uncertainty...

What else is (not) in the paper?

- Comparison of different leverage constraints m_{max}.
- More realistic wage profiles
- Earlier and later moment of truth

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- Comparison of different leverage constraints $m_{\max}$.
- More realistic wage profiles
- Earlier and later moment of truth
- Not in the paper: Stochastic human capital, inflation, other sources of wealth like housing, inheritance etc.
- Balter, and Schweizer (2024). Robust decisions for heterogeneous agents via certainty equivalents, EJOR: has some complimentary theoretical results; like bounds on welfare loss due to grouping of agents.
- Balter, Garcia, and Schweizer (2024). Welfare effects of collective investment for heterogeneous agents, Netspar Design Paper: further research on clustering agents and computing collective investment glide paths

Some conclusions

- Investment success is remarkably stable under moderate discrepancies between true and implemented risk preferences, risk capacities and market conditions.
- Leverage constraints play an important role in diminishing the impact of underestimating risk aversion or overestimating risk capacity.
- Agents facing an unforeseen adverse event like disability face an *additional welfare loss* because their earlier financial planning was targeted at a more optimistic scenario.

Thank you!