The Lee-Carter Model: an update and some extensions

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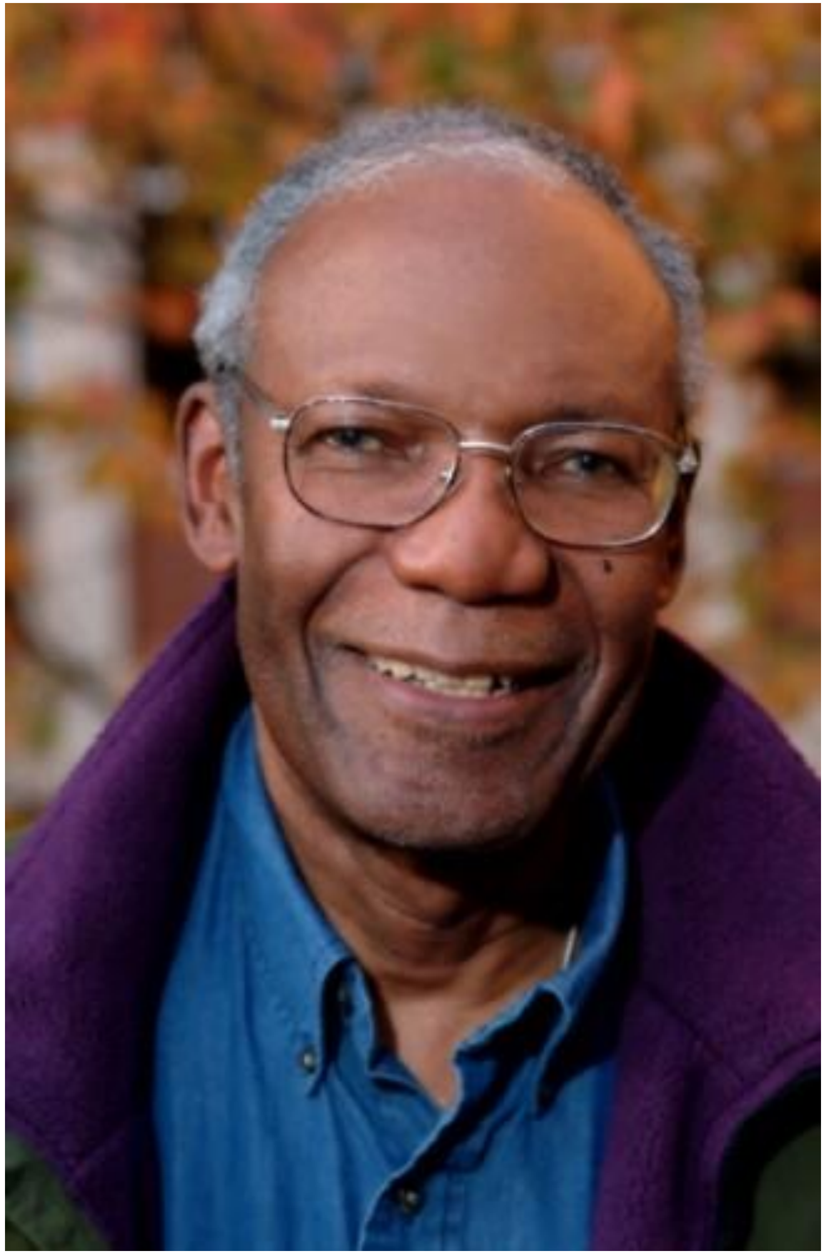
My collaborator, Larry Carter: 1943-2011



Forecasting length of life – who would dare attempt it? *Quelle folie*!

- Any of us might die at any instant; who can pretend to know or understand mortality, or be so foolish as to try to predict it?
- If we do try, surely the attempt must be rooted in biology or medicine, or bio-technology?
- A demographer or statistician's efforts might be dismissed as laughable or brash arrogance.
- And yet demographers and statisticians do try, and have even been fairly successful.
- Larry Carter and I are among those.

- We and others have had some success forecasting mortality because
  - It has a highly distinctive and regular age pattern observed over centuries of data,
  - aside from transitory shocks it moves slowly and steadily.
- It is easy to imagine reasons why the patterns might change and the trends might break, but in practice this seldom happens.
  - Trends persisted through revolutionary discoveries like antibiotics and hypertension drugs
  - Through obesity and smoking
- There have been some exceptions where trends have broken:
  - HIV/AIDS, particularly in parts of SubSaharan Africa
  - Countries of the USSR
- Nonetheless, in combination, the stable features make long term forecasts possible.
- Challenge for extrapolative forecasting is to capture these temporal and age regularities in a parsimonious way.

I will focus on my own work, with collaborators.

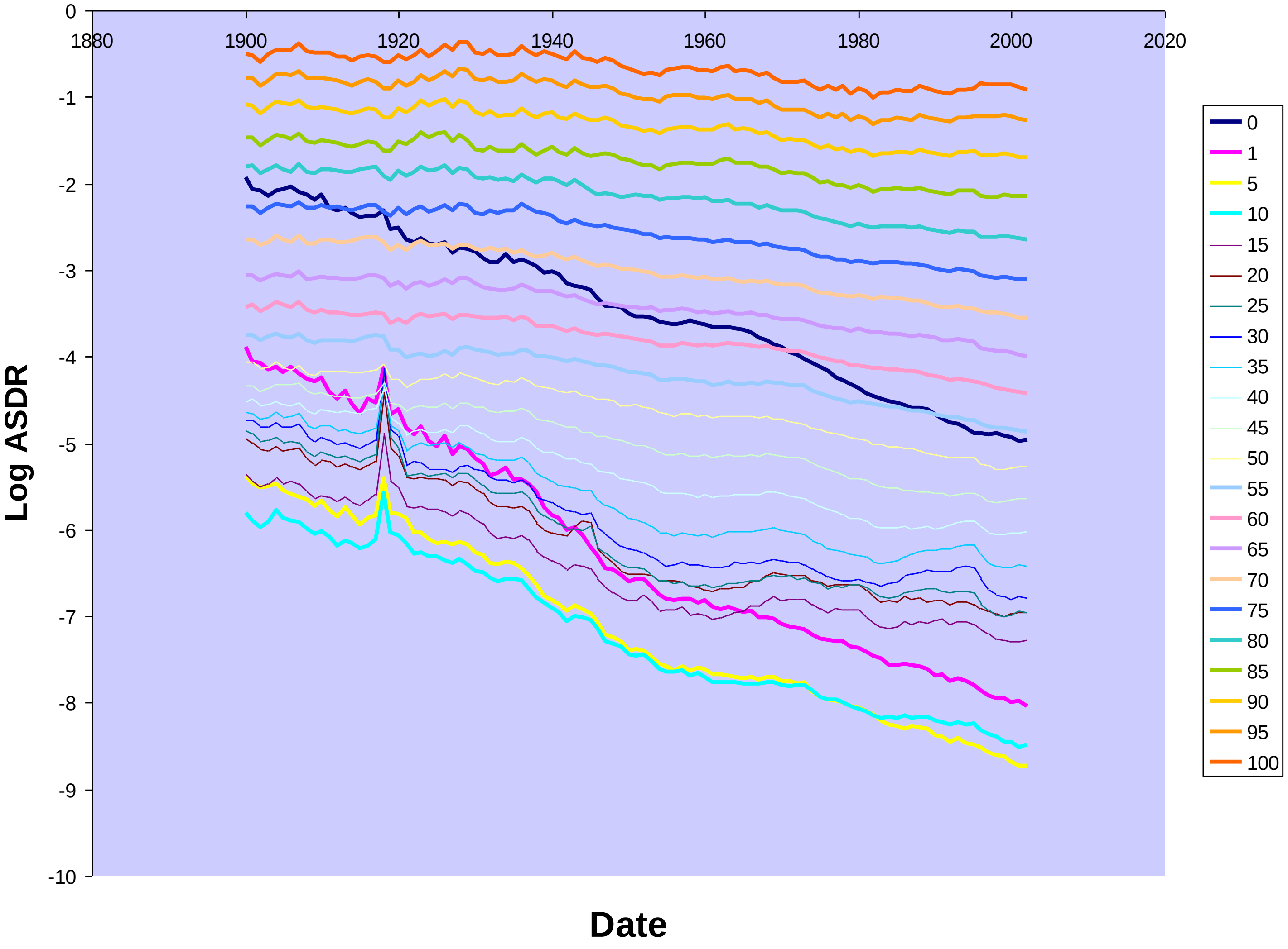
- There is a large and rapidly growing literature on new models and methods for statistical forecasting of mortality.
    - I have not kept up with it
    - I apologize to those here whose work I am overlooking.
    - Often it is more technical than I can follow.

Plan for talk

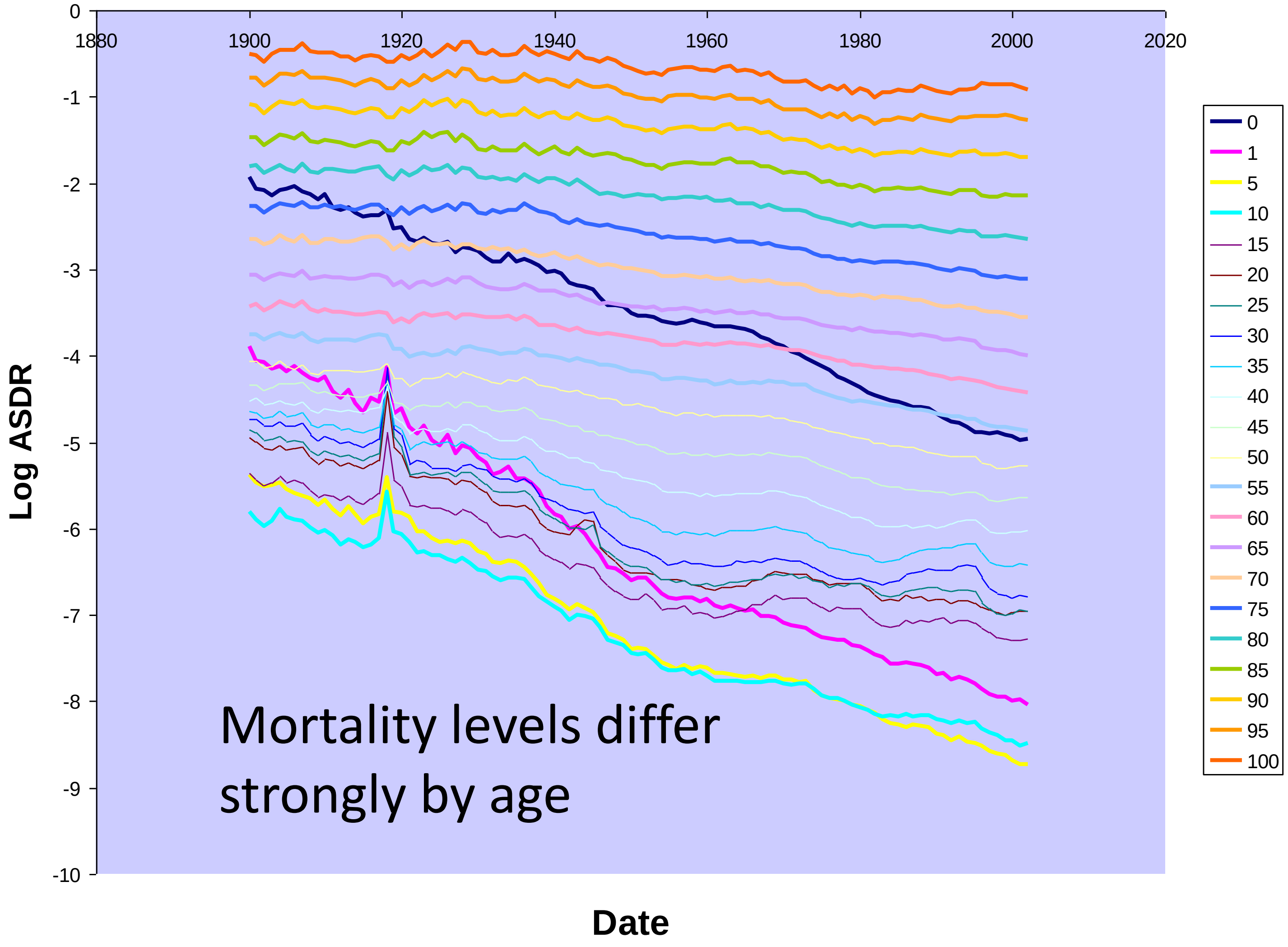
1. **The basic Lee-Carter model**
2. **Exploitation common patterns in groups of populations – “coherent forecasting”**
3. **In long run, the age pattern of mortality decline changes: b(x) is not constant**
4. **Dealing with uncertainty**
5. **The future of mortality – deeper issues**

1. The basic Lee-Carter model

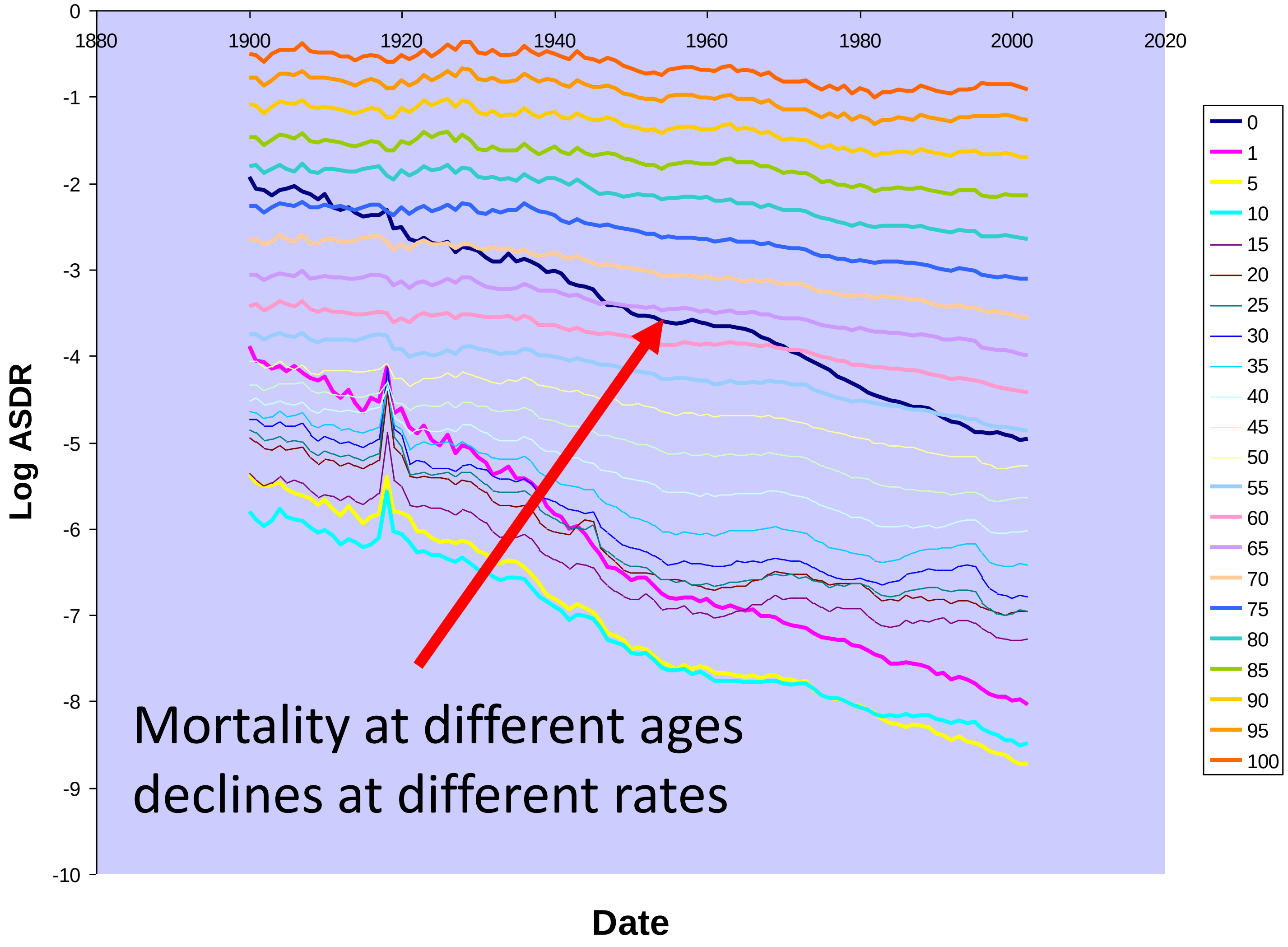
The log of US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



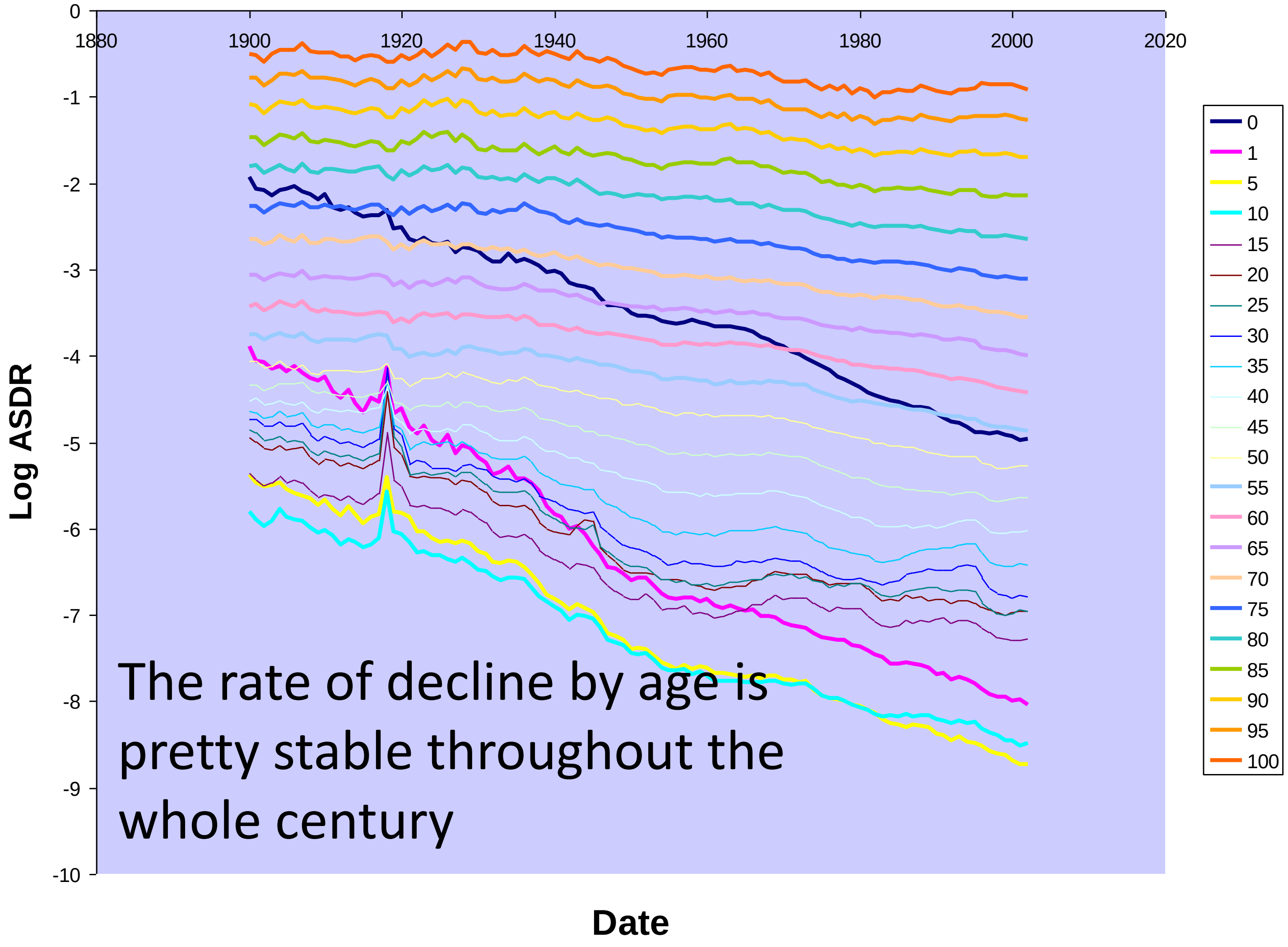
US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



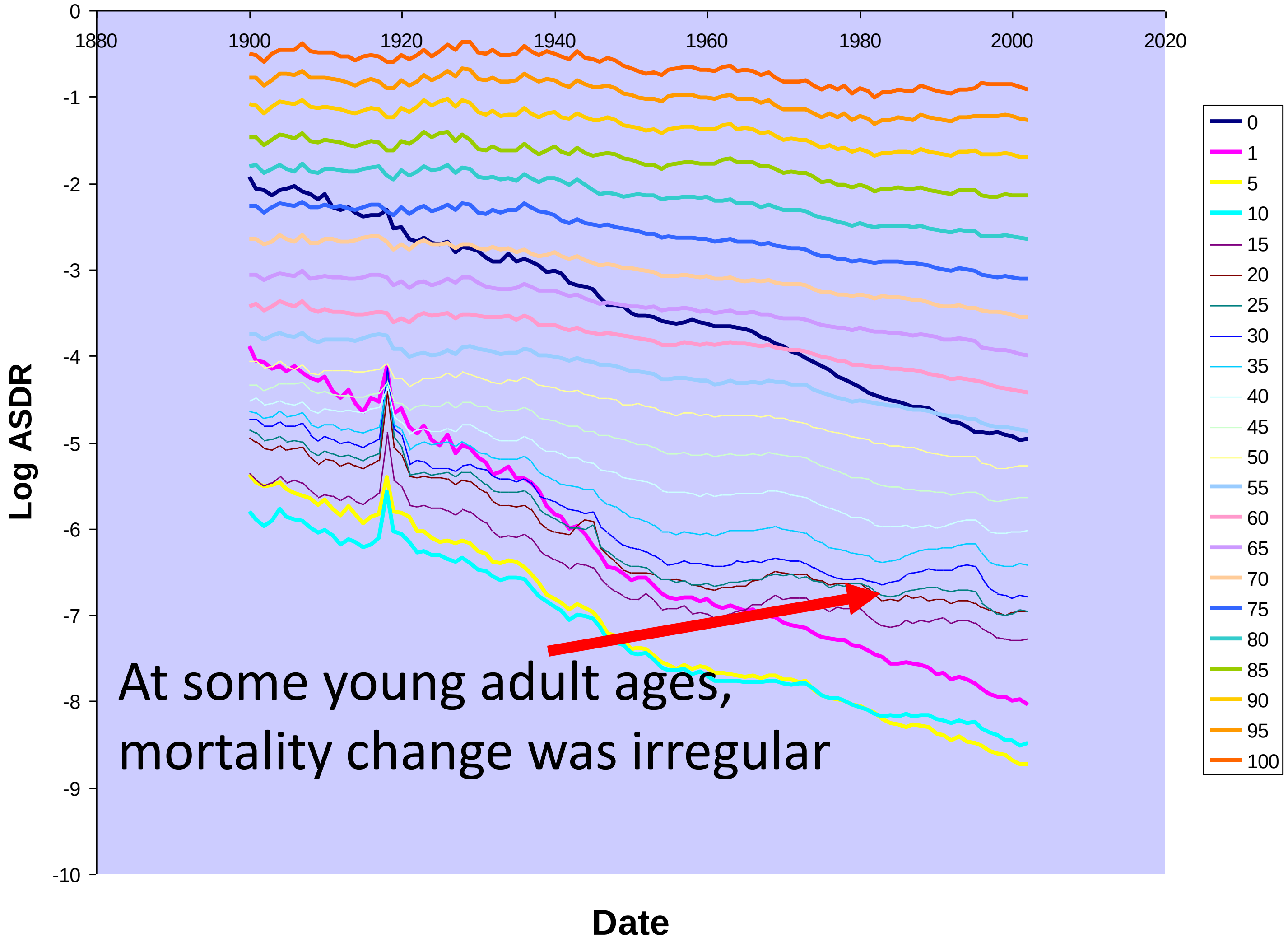
US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



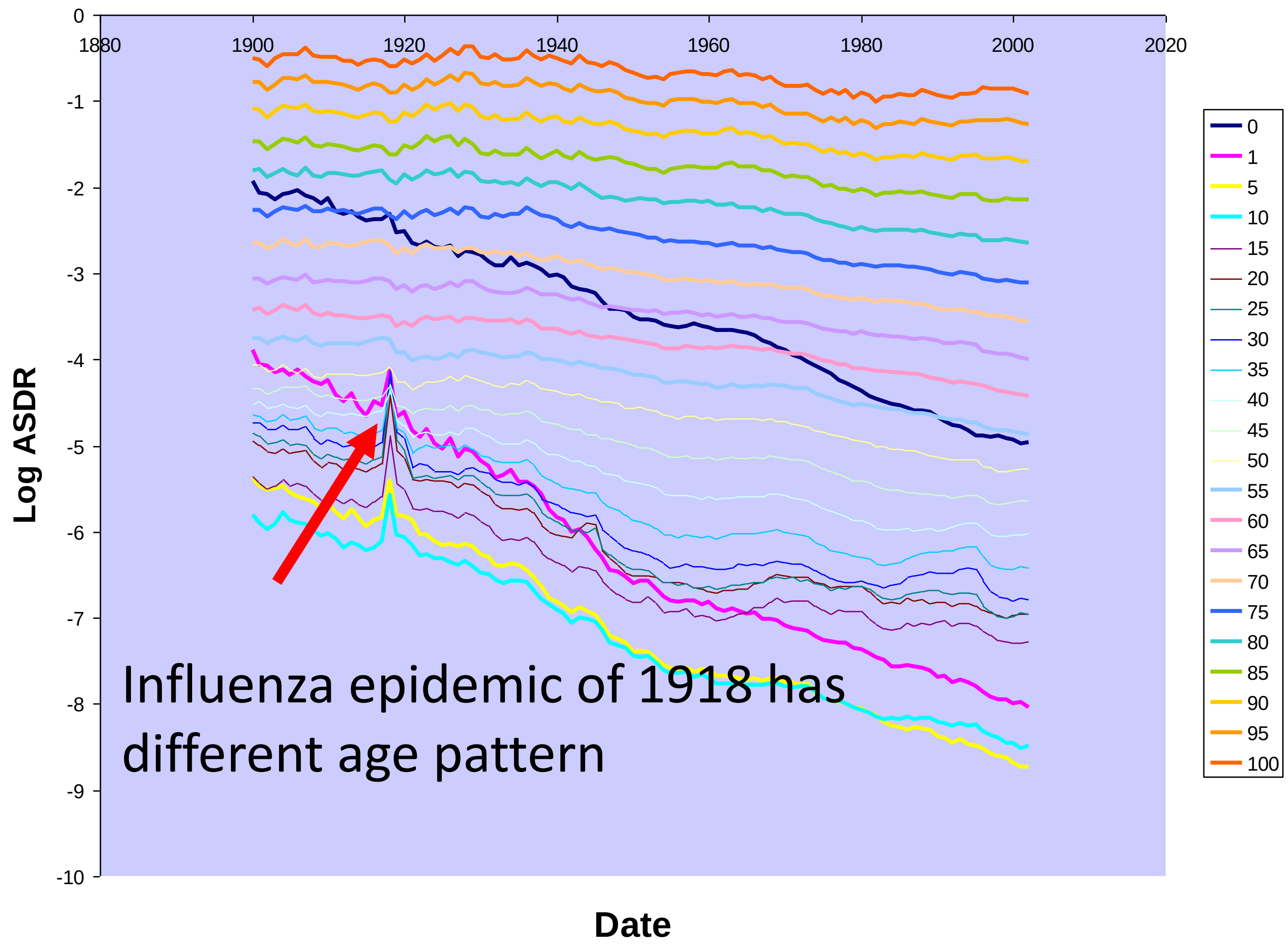
US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



US Age Specific Death Rates, Sexes Combined, 1900-2002 (logs)



Lee-Carter model captures most but not all these features with a simple expression

$m(x,t) = \text{death rate for age x in year t}$

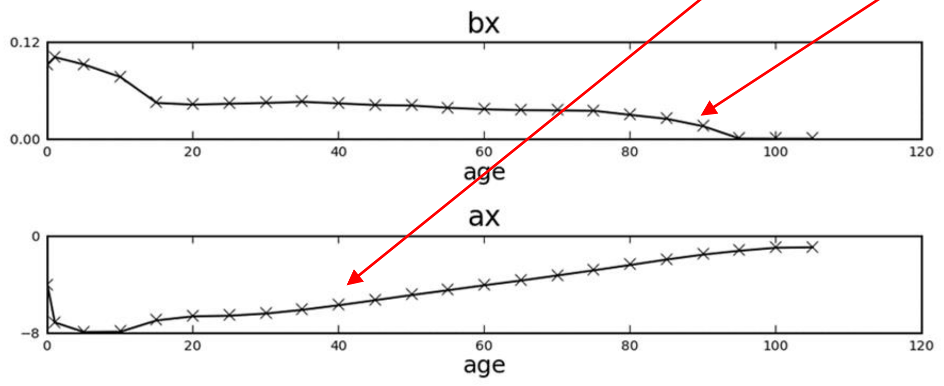
$$\ln\left(m(x,t)\right) = a(x) + b(x)k(t) + \varepsilon(x,t)$$

- Differences in mortality by age are captured by a(x)
- Differences in relative rates of change by age are captured by b(x)
- Tendency of age-specific death rates to move together is captured by b(x)k(t), where k(t) reflects year-to-year changes in the general level of mortality
- Some things it gets wrong or ignores—
  - Correlation structure of errors across ages
  - Long term trends or “rotation” of relative rate of decline by age [b(x) factor]
  - Some shocks like HIV/AIDS or 1918 Influenza Pandemic that affect ages differently
  - Any cohort-specific factors

Here are estimated values of a(x) and b(x) for the US for 1933-2013 (data from Human Mortality Database)

$m(x,t) = \text{death rate for age x in year t}$

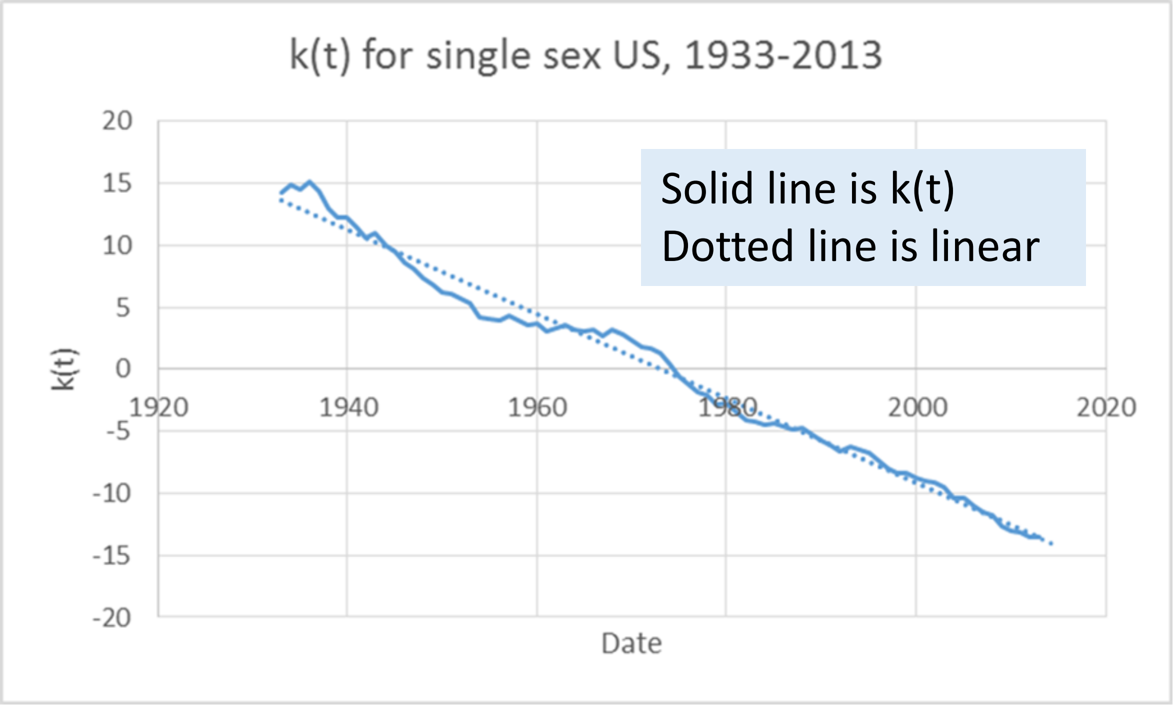
$\ln(m(x,t)) = a(x) + b(x)k(t) + \varepsilon(x,t)$



- Differences in mortality by age are captured by a(x)
- Differences in relative rates of change by age are captured by b(x).

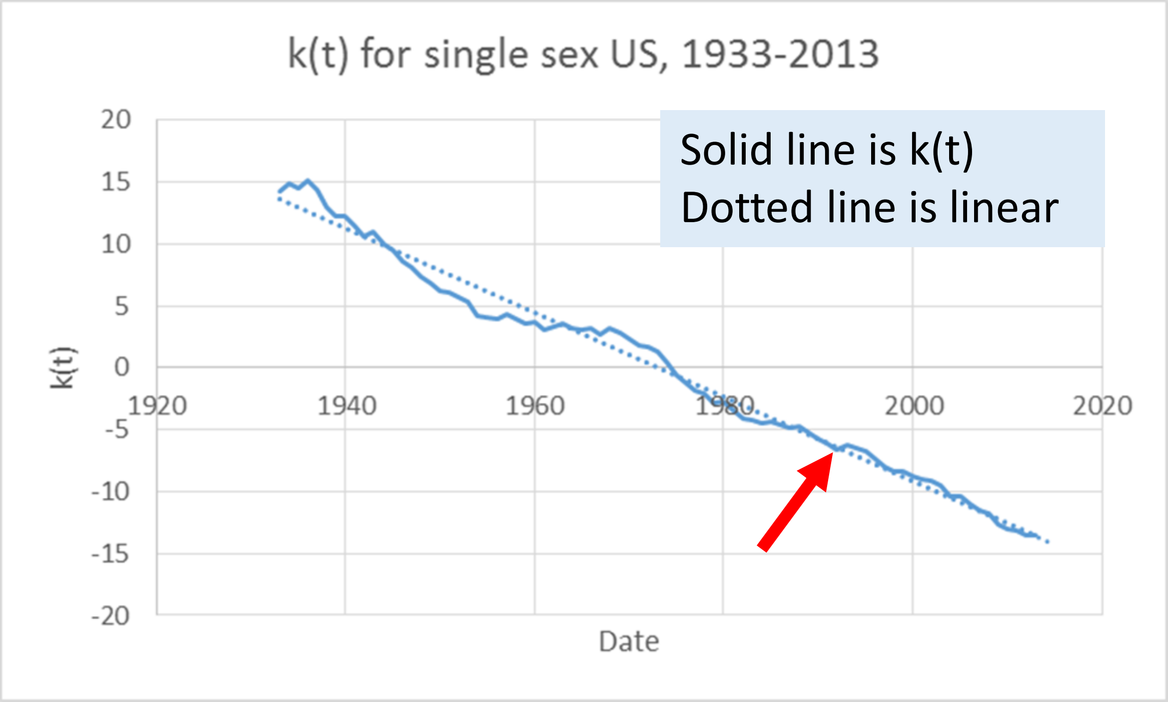
Only the k(t) factor changes over time, driving changes in age specific mortality.

- k(t) can be modeled as some time series process.
- We were surprised it was a random walk with drift in the US
- More surprised this was true in many other countries.

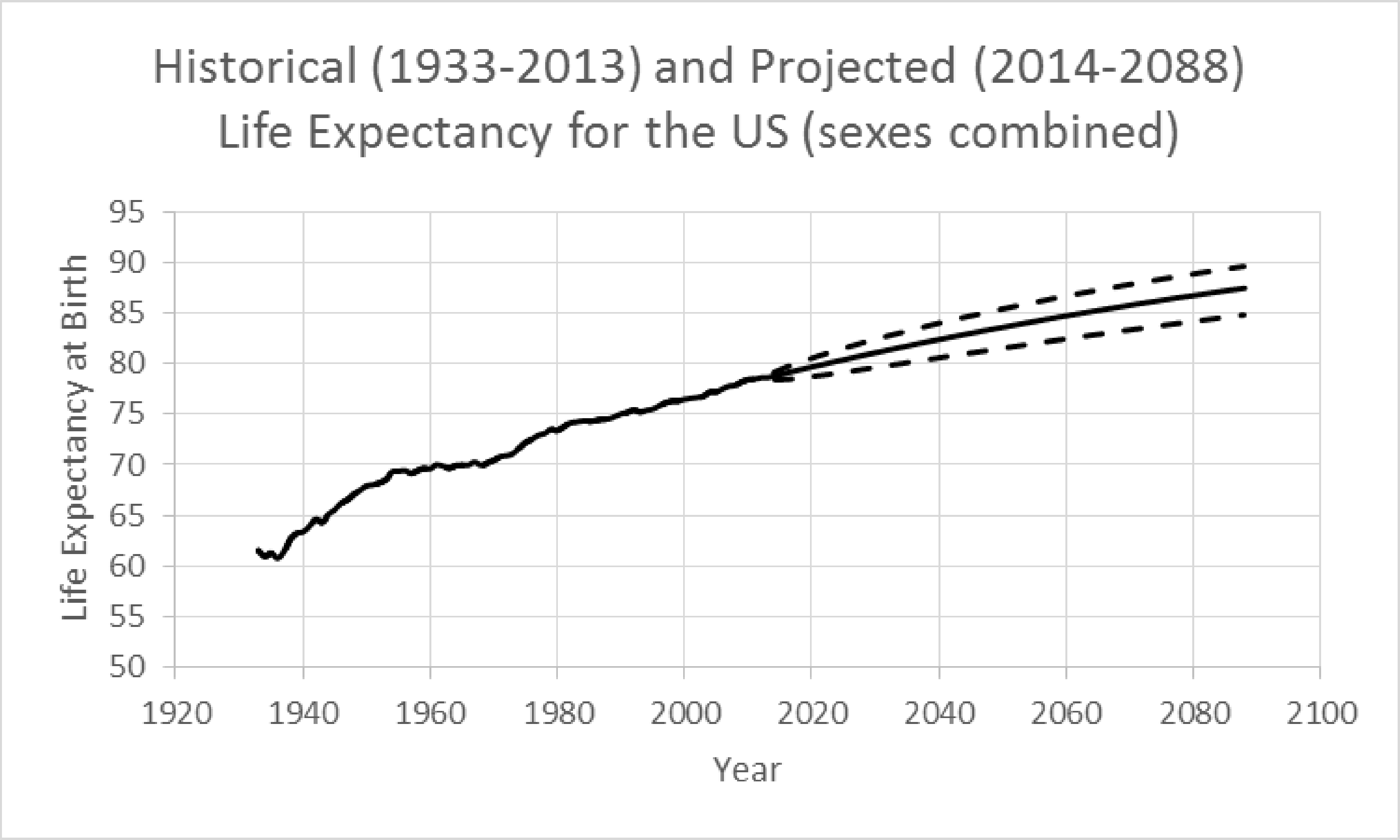


Only k(t), the index of mortality level, changes over time, driving changes in mortality

- Fit statistical time series model to k(t).
- Larry and I were surprised to find that a random walk with drift, with low innovation variation, fit k(t) for the US.
- We were more surprised when this fit in other countries as well.
- Article was published in 1992, shown by arrow; k(t) continued on linear trend until present.



From k(t) forecast with probability distribution comes Life Expectancy forecast, with probability distribution (95% interval shown here)



2. Exploiting common patterns in groups of populations – “coherent forecasting”

- Forecasts for a single population can draw strength from forecasts of similar populations (“Coherent” forecasts, see Li and Lee, 2005)
- Examples of similar populations
  - Males and Females in same country
  - Areas within a single country like French Departments or Canadian Provinces
  - The group of all low mortality nations
- Oeppen-Vaupel (2002), White (2002), Lee (2006) and Li and Lee (2005) all suggest drawing on the international context to improve individual country mortality projections.
  - White (2002) and Lee (2006) develop specific regression-based approaches
  - Li and Lee (2005) develop a method based on Lee-Carter.

The “Coherent” approach for forecasting national mortality

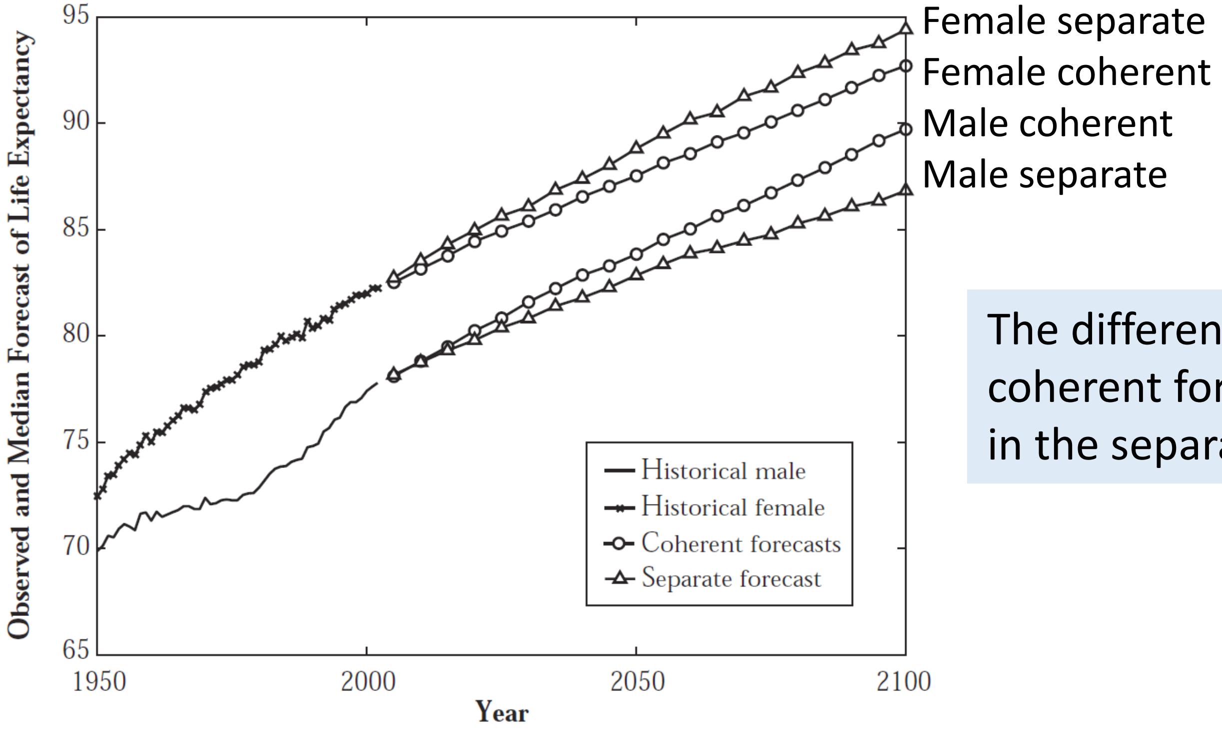
- Use judgment to form a group of countries that are believed to be similar in important ways, such that mortality trends are inter-related.
- Construct a super population by summing the population-weighted individual country mortality data.
- Fit a “Common Factor” Lee-Carter model to the group
  - Each country gets its own a(x) schedule
  - All countries get the bsame B(x) and K(t)   (capital letters indicate these common schedules)
- Initial differences in shape of age schedule (a(x)) are preserved forever in this Common Factor model, but eventually declines at the same rate.

Coherent approach (continued)

- Now use a Lee-Carter type model for the residuals for each country from this common factor model, using SVD to get a secondary b(x,i) and k(t,i) for each country i.
- k(t,i) is modeled as a first order autoregressive process.
- The model works for country i only if the estimated AR(1) coefficient is <1 so that over time, the forecast converges to the Common Factor rate of change at each age, while preserving differences in shape and level.

An example for forecasts by sex in Sweden

Figure 1. Coherent and Separate Forecasts of Life Expectancy, Sweden



The differences shrink in the coherent forecast but expand in the separate forecasts.

Li, Nan and Ronald Lee "Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method," *Demography*. 42:3, August 2005, pp 575-594. PMID: 16235614; PMCID: PMC1356525; NIHMSID: NIHMS3338; http://www.pubmedcentral.gov/articlerender.fcgi?artid=1356525

Coherent results for 15 low mortality countries
Base data 1952-1996 (so now old, out-of-date)

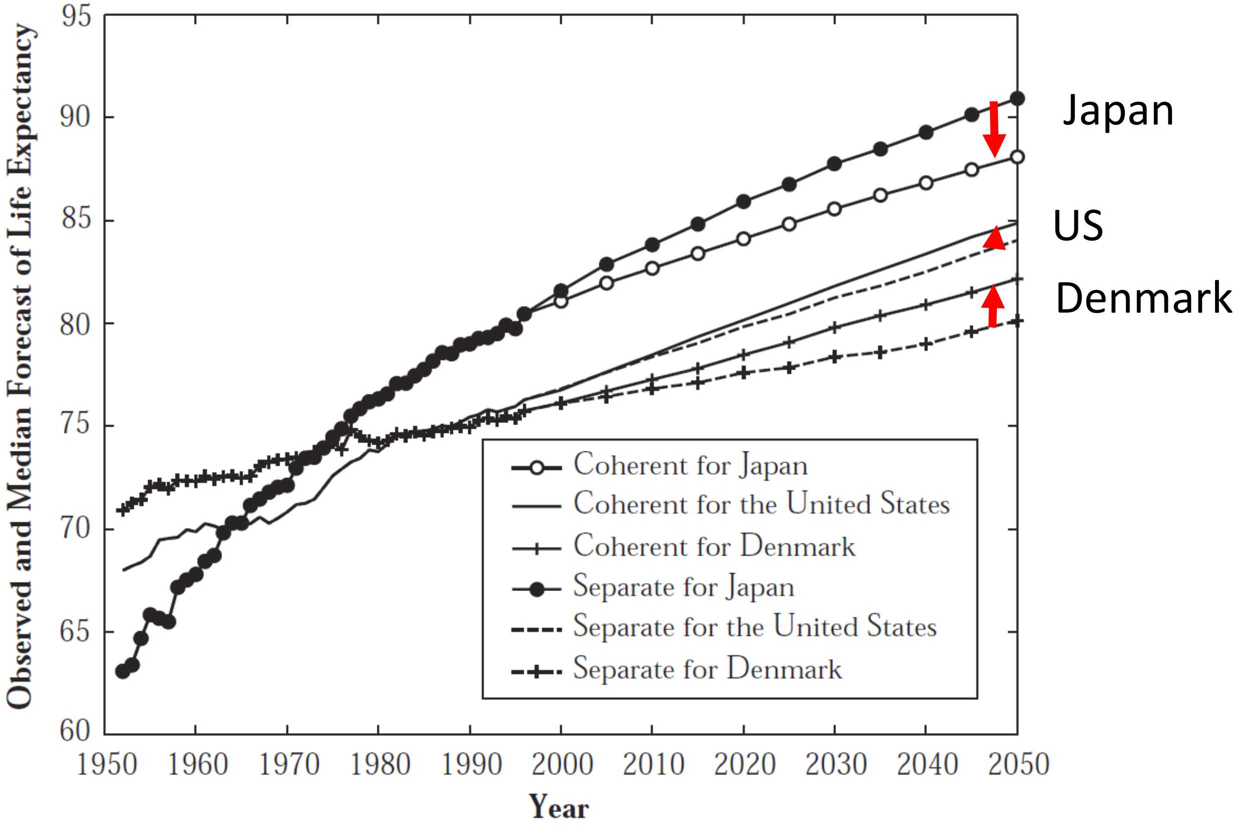
Table 4. Life Expectancies at Time *t*, *e*(*t*), of Low-Mortality Countries

Country	*e*(1996)	Median of *e*(2050) From the Coherent Forecast	Median of *e*(2050) From the Separate Forecast	95% Confidence Interval of *e*(2050) From the Coherent Forecast	95% Confidence Interval of *e*(2050) From the Separate Forecast
Austria	77.4	84.8	84.9	3.8	4.8
Canada	78.4	86.3	85.4	4.2	3.5
Denmark	75.8	82.2	80.1	10.6	9.1
England	77.1	84.9	83.7	4.8	5.9
Finland	76.9	84.7	85.8	4.1	7.9
France	78.0	85.8	86.7	4.2	6.5
Germany (West)	77.1	84.8	84.6	3.8	4.8
Italy	78.5	86.1	87.1	4.1	6.7
Japan	80.5	88.1	90.9	4.3	4.7
The Netherlands	77.6	85.4	83.0	4.5	6.5
Norway	78.2	85.2	83.0	6.1	6.5
Spain	78.2	85.9	86.2	4.0	6.8
Sweden	79.0	86.1	85.6	4.5	6.0
Switzerland	79.1	86.5	87.5	4.5	6.1
United States	76.3	84.9	84.0	5.4	5.7
Mean				4.9	6.1
SD	1.2	1.3	2.5		

Li, Nan and Ronald Lee "Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method," *Demography*. 42:3, August 2005, pp 575-594. PMID: 16235614; PMCID: PMC1356525; NIHMSID: NIHMS3338; http://www.pubmedcentral.gov/articlerender.fcgi?artid=1356525

Illustrative differences between coherent and separate

Figure 6. Coherent and Separate Forecasts of Life Expectancy



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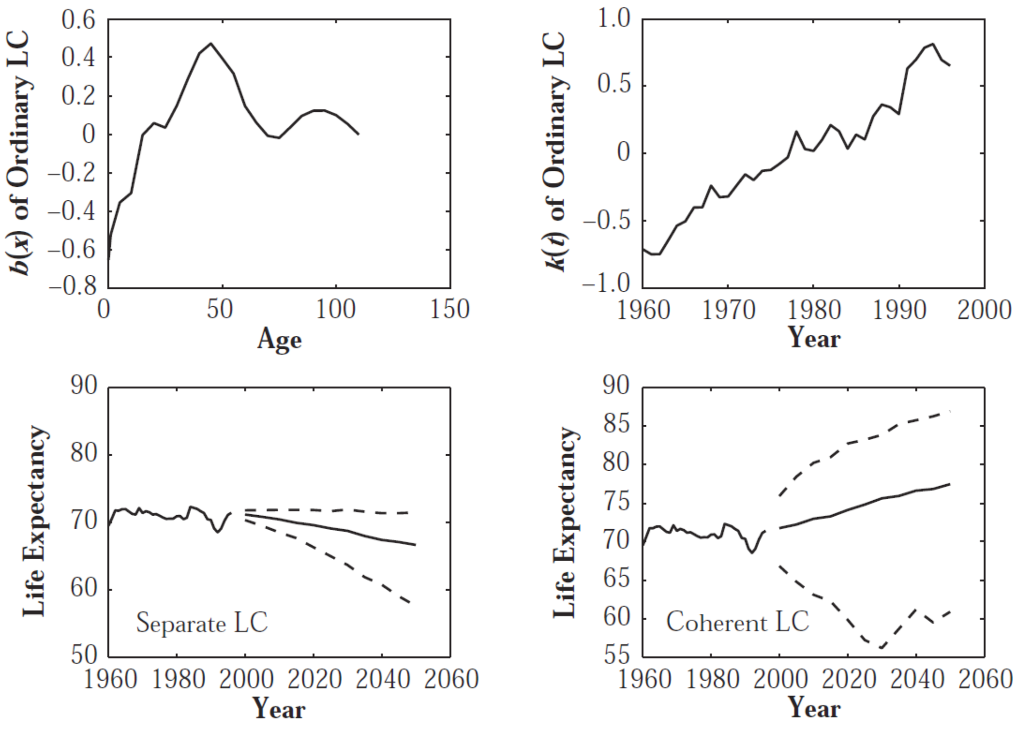
Russia/Eastern Europe – special situation

Common Factors for low mortality countries may better describe their futures

Here assume so and use the Common Factors just estimated for 15 countries.

Figure 8. Separate and Coherent Forecasts of Lithuania



Countries initially considered together:
Bulgaria, Czech, E. Germany,
Hungary, Lithuania, Russia
After diagnostics, only three remained:
Czech, E. Germany, Lithuania

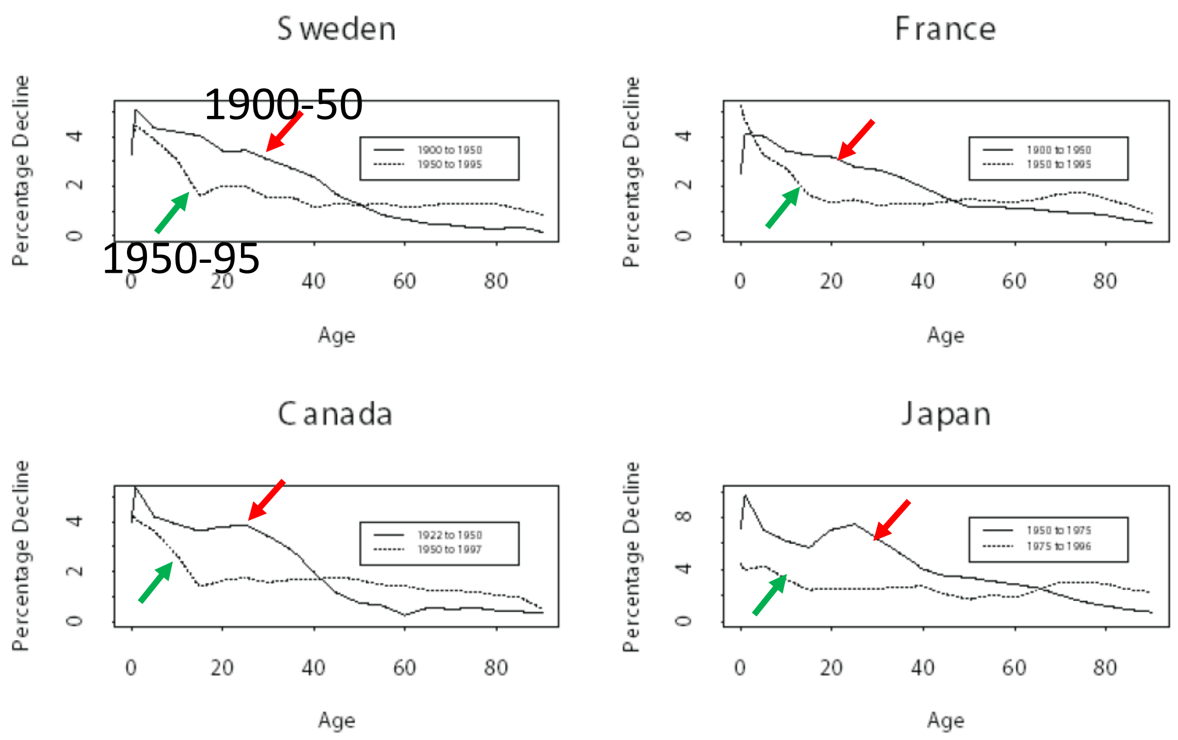
- Coherent approach changes forecast from decline to increase
- Also greatly increases uncertainty

3. In long run, the relative rates of decline by age are not constant: b(x) changes

- Lee-Carter projects a constant rate of decline for mortality at each age.
  - Rapid at young ages
  - Slower at old ages
- This pattern has weakened in low mortality countries

As mortality declines, the b(x) curve (pace of decline by age) rotates, slowing at younger ages and rising at older ages (Lee, 2006).

Average rate of decline by age, 1900-1950 and 1950-1995.



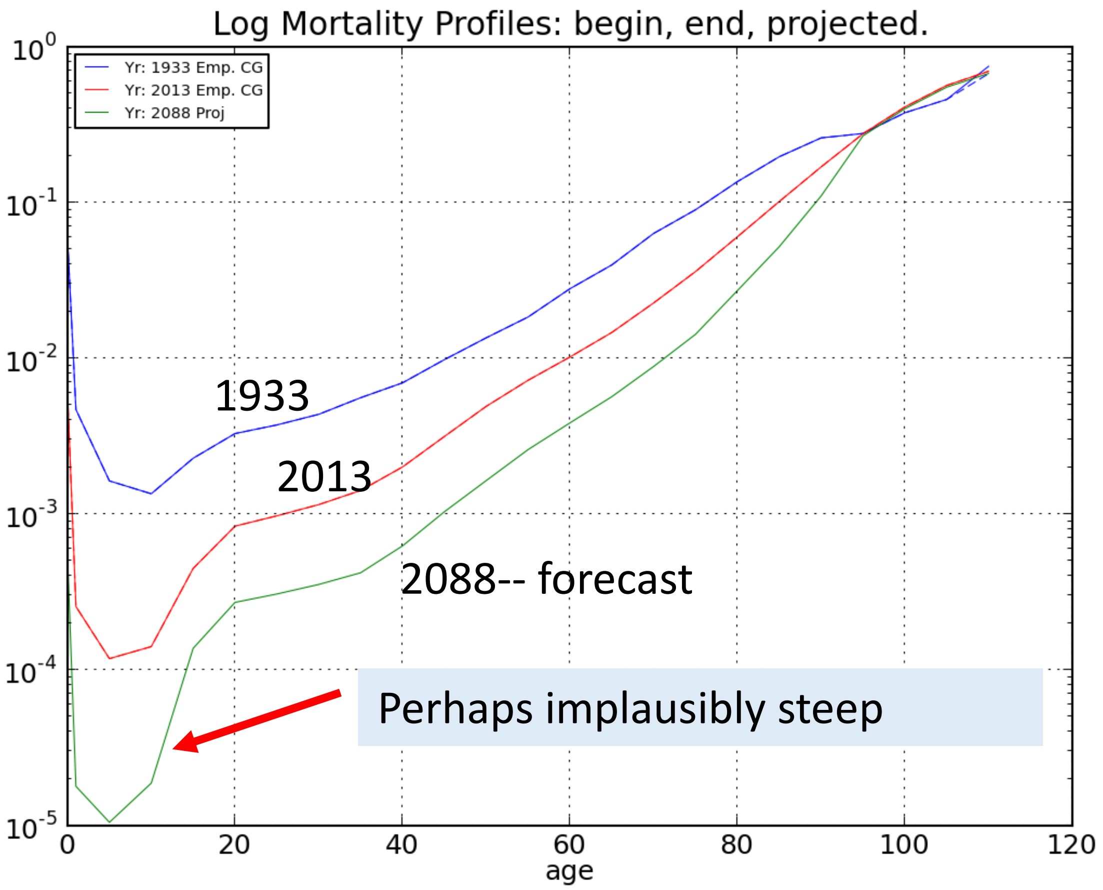
Many other countries look similar, including US.

Note that b(x) curve is becoming flat across all ages.
Clearest in Japan with highest e_0.

Lee, Ronald (2006) “Mortality Forecasts and Linear Life Expectancy Trends” in Bengtsson, Tommy (ed.) *Perspectives on Mortality Forecasting. III. The Linear Rise in Life Expectancy: History and Prospects. Social Insurance Studies. No. 3.* Försäkringskassan, Swedish Social Insurancd Agency, Stockholm, 2006, pp. 19-39. forsakringskassan.se/filer/publikationer/pdf/sis_3.pdf

Actual and 75-year forecast of mortality by age in US

- Is the age pattern in 2088 believable?
- Gerosi&King (2008) suggest that long term Lee-Carter forecasts result in implausibly low mortality in childhood, here 1/100,000.



Ratio of Infant Mortality rate to death rate at 15-19 in the 20 lowest mortality countries

- Infant mortality declines faster than m(15-19), with bigger b(x), so the ratio falls as life expectancy rises.
- After age 80 the curve flattens out, so they decline at the same rate.
- After age 80, it appears that all death rates below age 65

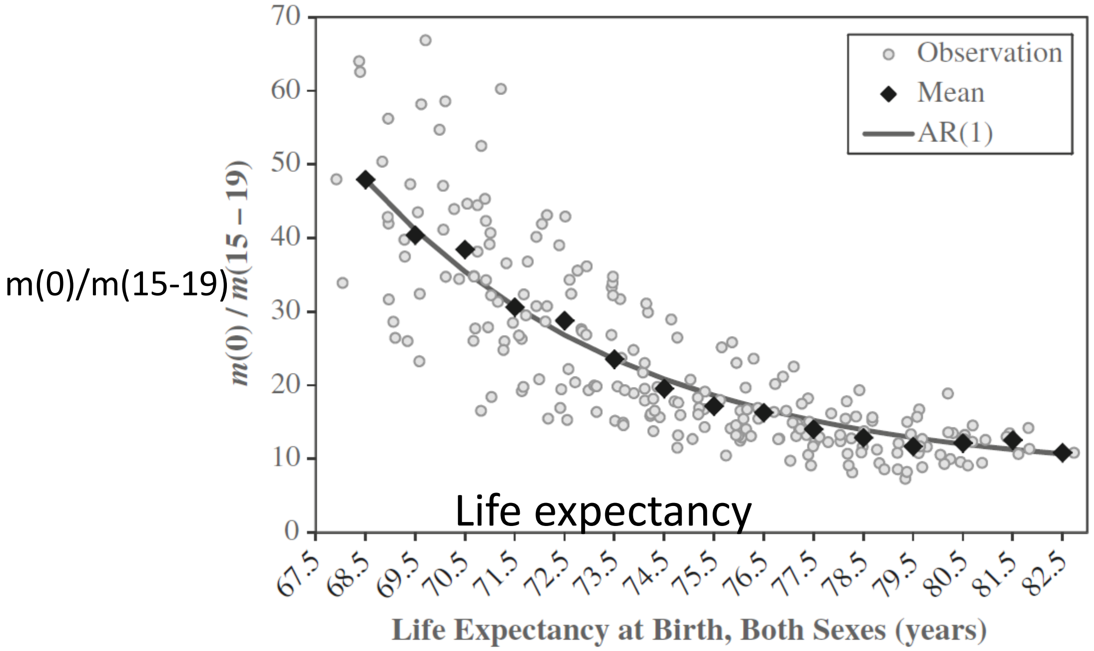


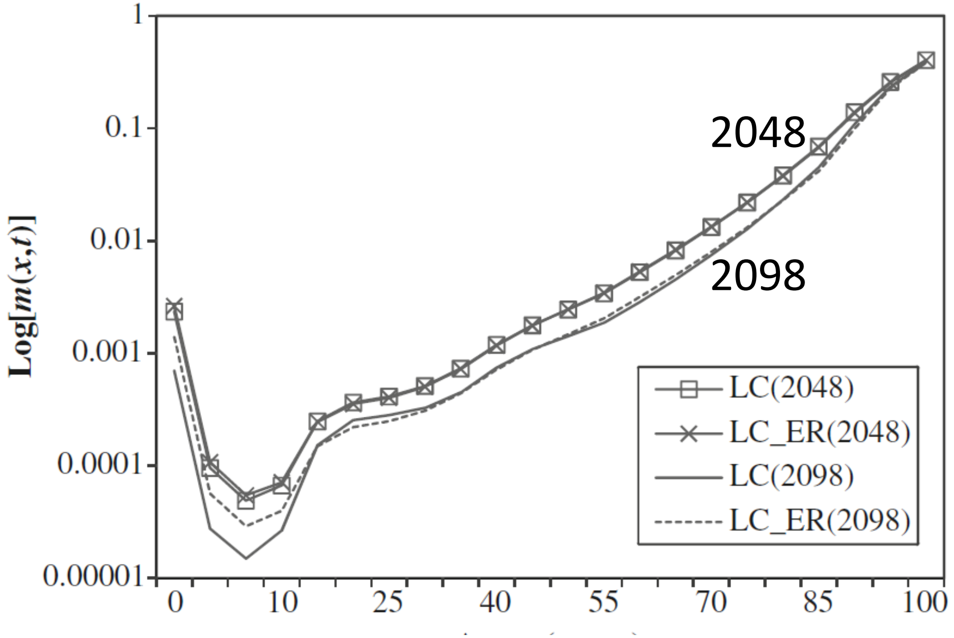
Fig. 4 Observed values by five-year periods between 1950 and 2010 of the ratio of infant mortality to the death rate for ages 15–19 ($m(0)$ / $m(15–19)$) plotted against the level of life expectancy at birth in the 20 lowest-mortality populations. The mean of the observed values in each integer interval of life expectancy are indicated by black diamonds, with fitted trend using an AR(1) model as solid line

Li, Nan, Ronald Lee, and Patrick Gerland (2013) "Extending the Lee-Carter Method to Model the Rotation of Age Patterns of Mortality Decline for Long-Term Projection. *Demography.* DOI: 10.1007/s13524-013-0232-2. Published online August 1, 2013.

Li-Lee-Gerland (2013) develops a modification to incorporate the b_x rotation in Lee-Carter, for use at United Nations.

- Up to life expectancy 80, no change in Lee-Carter method.
- After e0 = 80, the b(x) curve gradually rotates and flattens until it is constant from age 0 to age 85; then it declines.
  - K(t) is adjusted so the standard Lee-Carter e_0 projection is not changed.
- Can be combined with Coherent forecasting.

Illustration for the US



- At 2048, no difference.
- At 2098, mortality higher for children, lower for elderly

Figure contrasts standard and modified forecasts to 2048 and 2098 for US.

Li, Nan, Ronald Lee, and Patrick Gerland (2013) "Extending the Lee-Carter Method to Model the Rotation of Age Patterns of Mortality Decline for Long-Term Projection. *Demography.* DOI: 10.1007/s13524-013-0232-2. Published online August 1, 2013.

4. Dealing with uncertainty

- Population forecasters have responsibility to indicate the uncertainty of their forecasts.
- There are many approaches.
- Lee-Carter method generates probability intervals as well as central forecasts.

Official agencies have often under-predicted life expectancy gains and therefore the elderly population

- I am not sure whether this is still true.
- Studies have shown it was true in the 20^{th} century.

US National Academy of Sciences study calculated the mean error in UN projections of life expectancy carried out before 2005.

Beyond Six Billion: Forecasting the World's Population
http://www.nap.edu/catalog/9828.html

Life expectancy projections by the United Nations for rich industrial nations tended to be too low.

The mean error after 25 years was
-2 years for Europe/FSU
-4.5 years for US/Australia/Japan.

132 BEYOND SIX BILLION

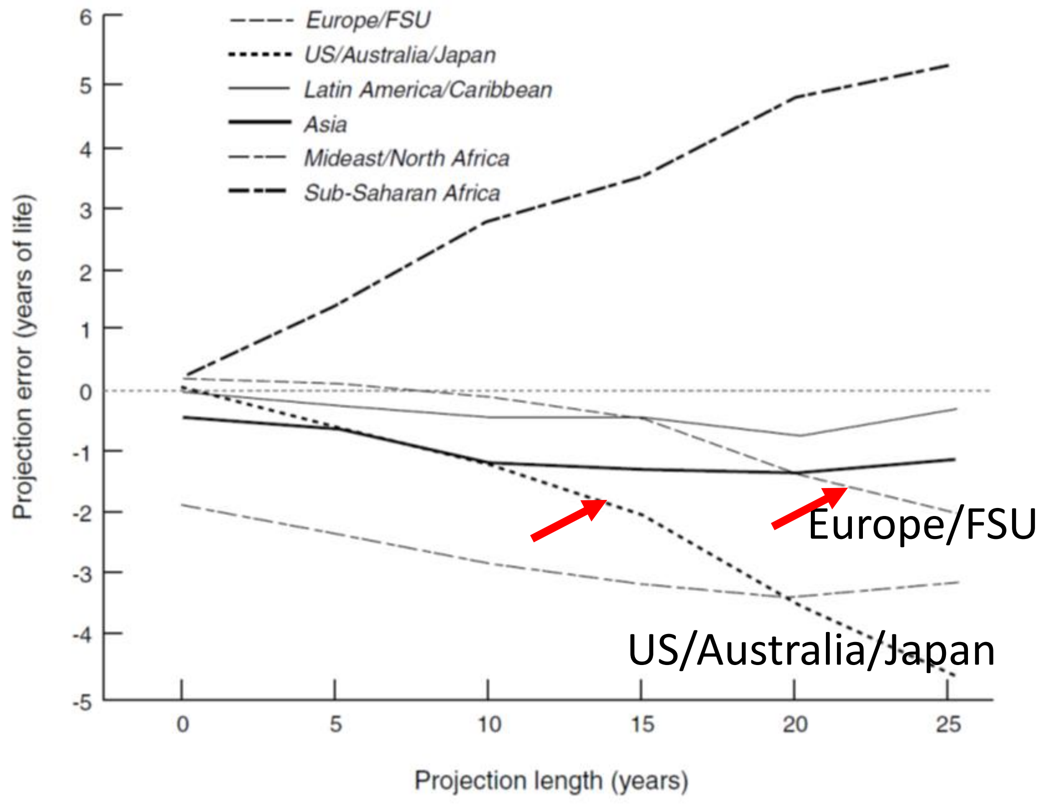
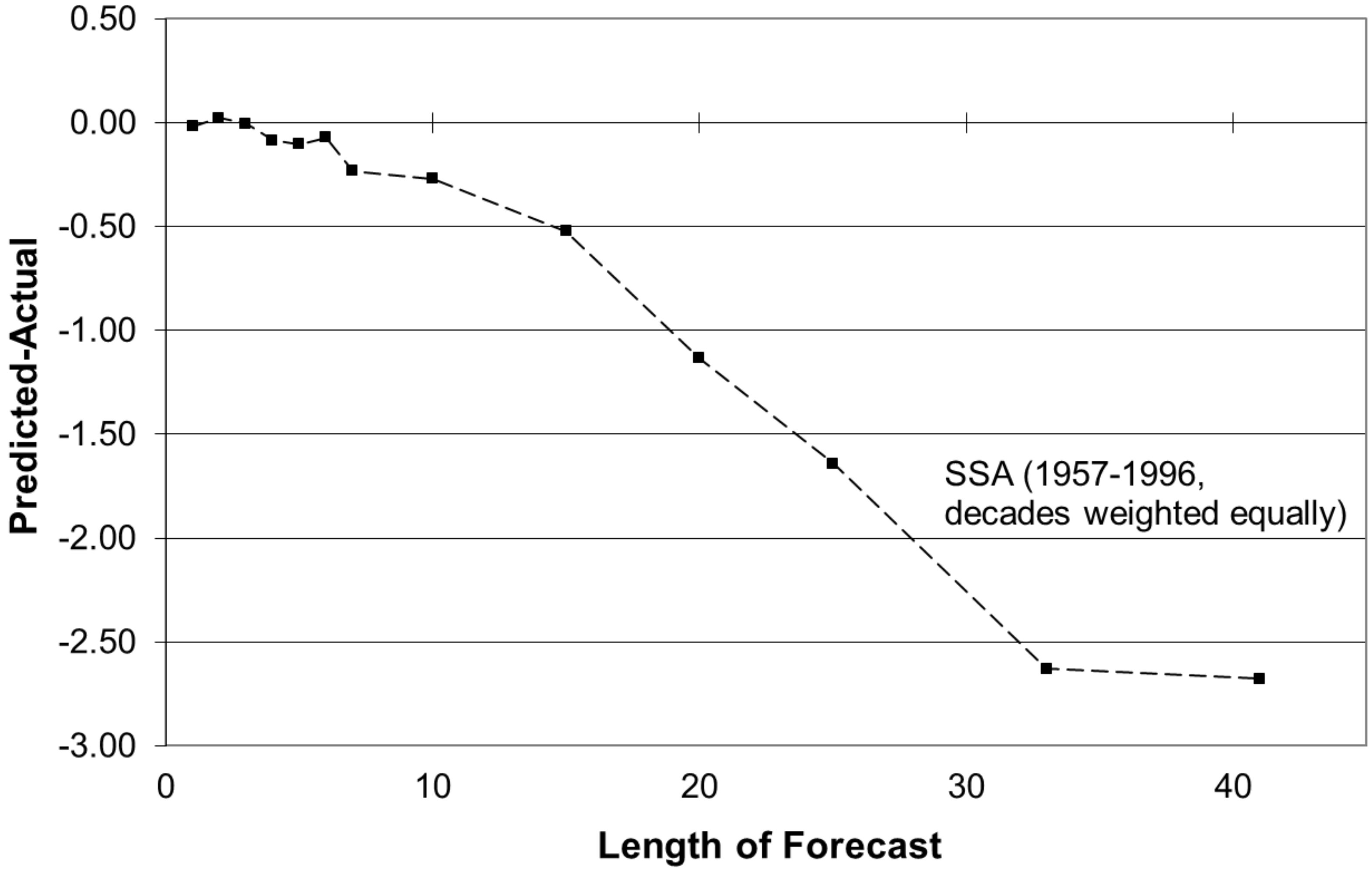
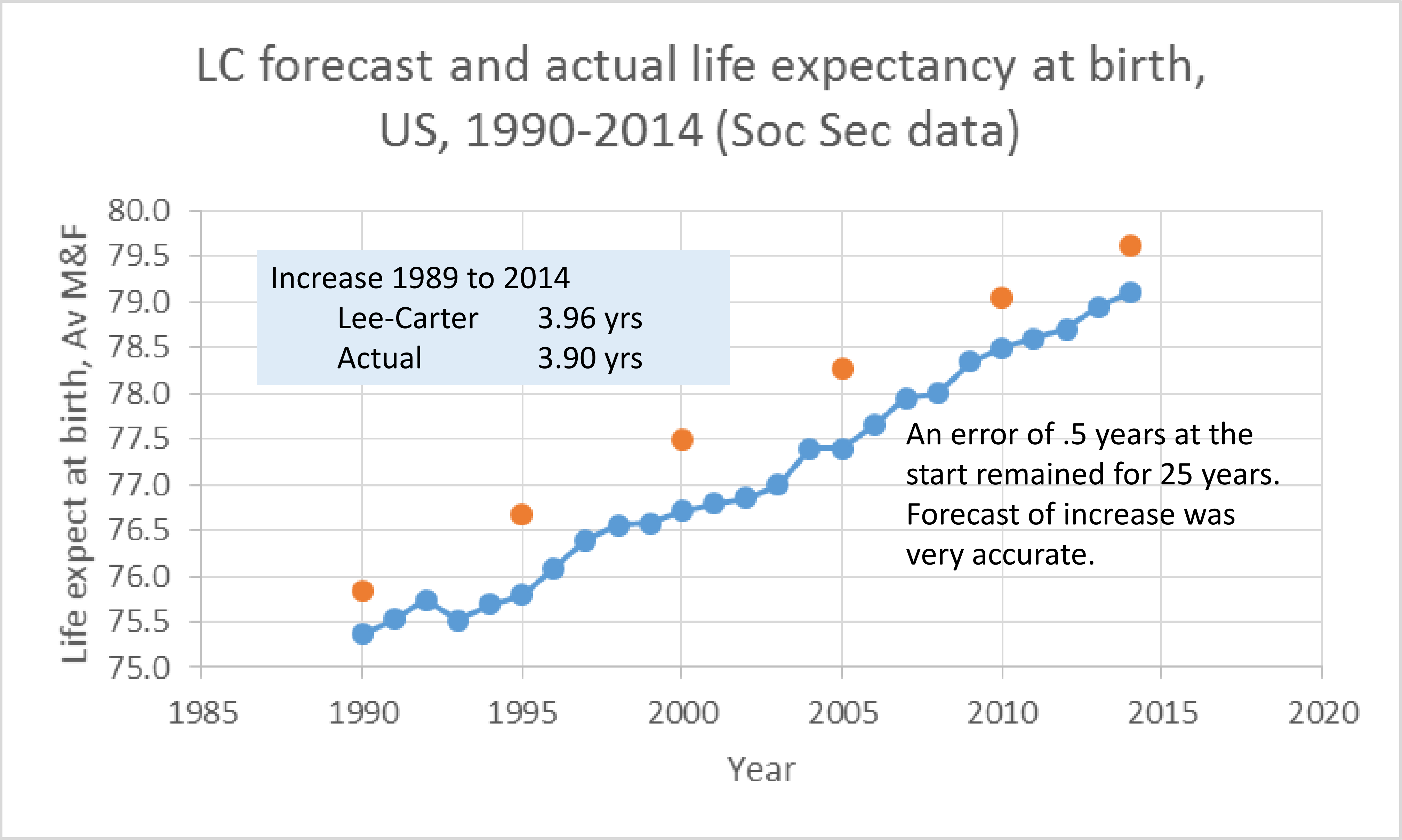


FIGURE 5-7 Mean error in projected life expectancy, across countries and forecasts, by projection length and region.
SOURCE: See Appendix B.

Average Bias in Forecasts of Life Expectancy by US Social Security Administration (SSA)




LC forecast and actual life expectancy at birth, US, 1990-2014 (Soc Sec data)
Increase 1989 to 2014
Lee-Carter 3.96 yrs
Actual 3.90 yrs
An error of .5 years at the start remained for 25 years. Forecast of increase was very accurate.
Life expect at birth, Av M&F
80.0
79.5
79.0
78.5
78.0
77.5
77.0
76.5
76.0
75.5
75.0
1985
1990
1995
2000
2005
2010
2015
2020
Year

- No doubt, the success over past 25 years is partly just luck.
- To evaluate performance more systematically, we pretend we had used LC in the past. What errors?
- We used data from 1900 to 1998.
- Our first forecast used data 1900-1920.
- Our next forecast used data 1900-1921.
- For each forecast, we re-estimate a(x), b(x) and k(t).
- To forecast k(t) we always assume the random walk with drift model.

Hypothetical performance of Lee-Carter if had been used in US at all dates since 1913 to all possible horizons. Life Expectancy.

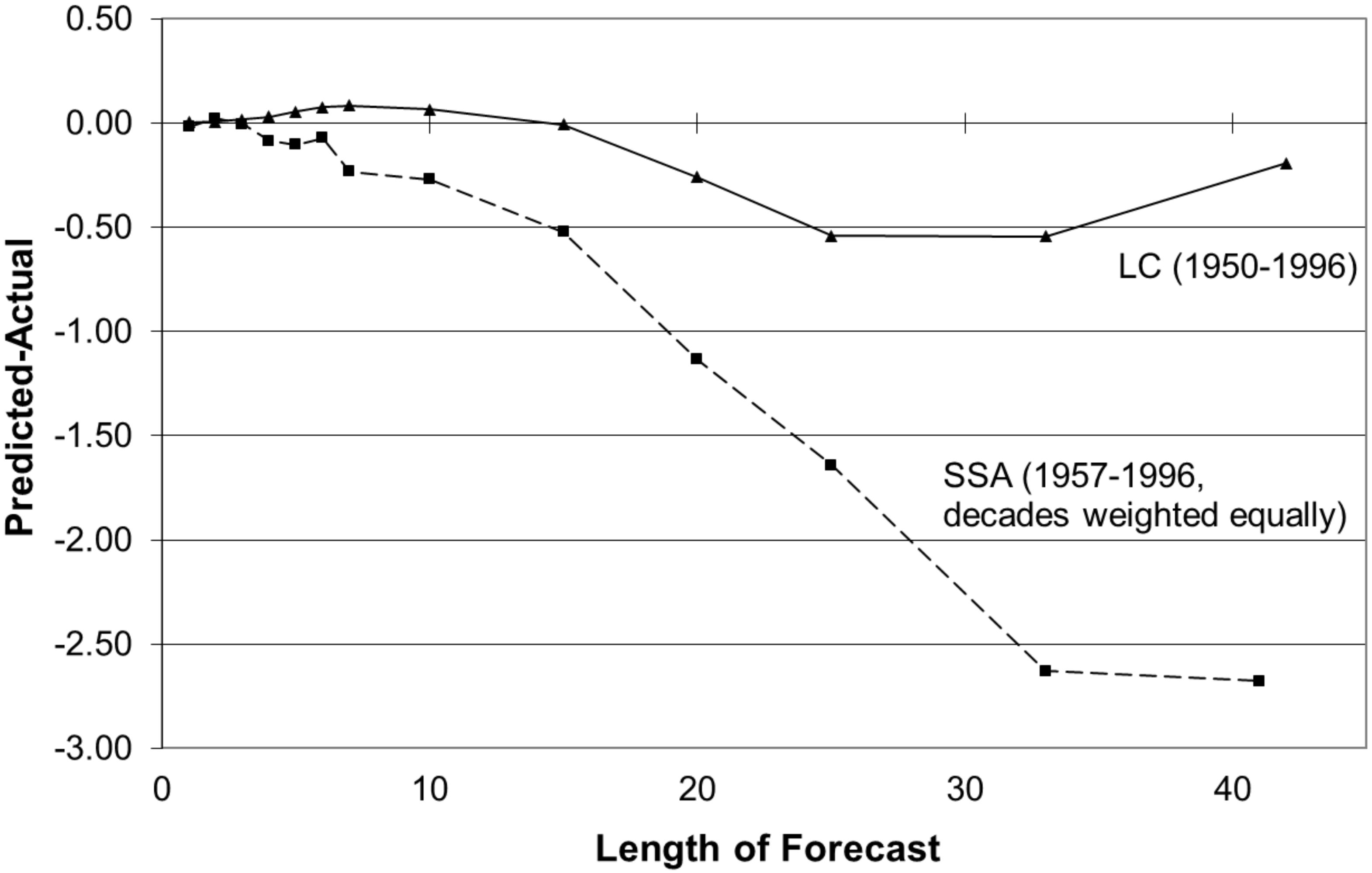
LC projected too low

Forecast Horizon (N)	Average error	MAD	RMSE	MAPE	% under-projected	% within 95% interval
1-5 (380)	-0.11	0.45	0.60	0.16	54	99
6-10 (355)	-0.32	0.82	1.03	0.47	56	100
11-20 (635)	-0.73	1.23	1.60	1.15	67	97
21-30 (535)	-1.37	1.47	1.99	2.03	84	100
31-40 (435)	-1.68	1.73	2.14	2.45	91	100
41-50 (335)	-2.23	2.25	2.75	3.41	96	95
51-60 (235)	-3.54	3.54	3.75	5.07	100	89
61-78 (171)	-4.38	4.38	4.53	5.39	100	80
ALL (3,081)	-1.49	1.76	2.34	2.45	78%	97%

Interval too wide

Interval too narrow

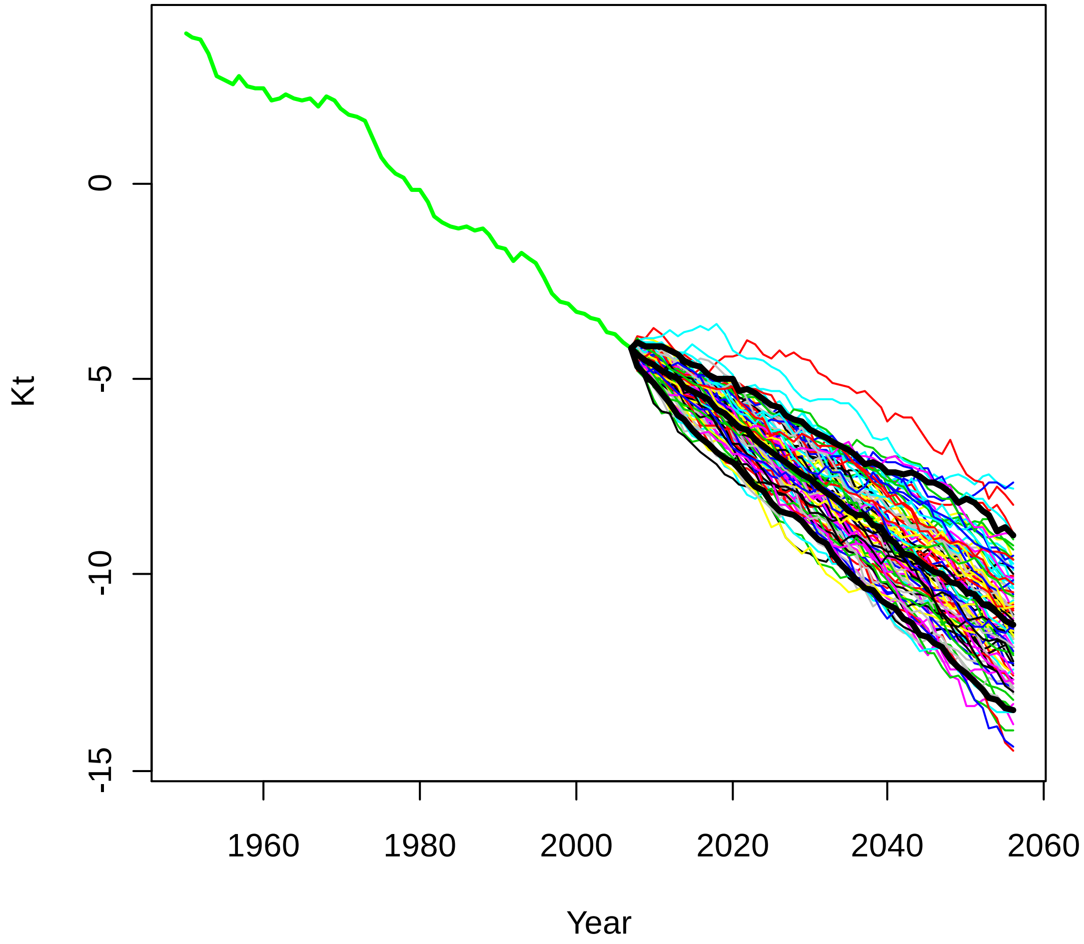
Average Bias in Forecasts of Life Expectancy by SSA compared to Lee-Carter



How can policy makers use these probabilistic mortality forecasts?

- Probability distribution of mortality in a given year is mostly not useful by itself.
- Need year to year correlation of forecasting errors.
  - If correlation is high, then for most purposes uncertainty is greater
  - If correlation is low, then errors tend to cancel over time, uncertainty is less.
- Simplest way to use the information in the forecast is to use the fitted model to generate random sample paths, say 1000.
- Suppose you are a pension fund or life insurance company and your fund balance in year t depends on all the mortality outcomes up to year t.
- For each sample path calculate the cumulated outcome of interest for each year along the path.
- Do this 1000 times to get the probability distribution for the fund balance in each year.

k(t) forecasted trajectories for US – random sample paths with 95% bounds



Often (e.g. public pensions) it is population age distributions that matter

- Mortality is one source of uncertainty
- Fertility matters too
- So does immigration
- Need a stochastic population forecast for which mortality is only one ingredient.
- There is a large literature in this area.

5. The future of mortality – deeper issues

Oeppen-Vaupel (2002), Record female life expectancy since 1840

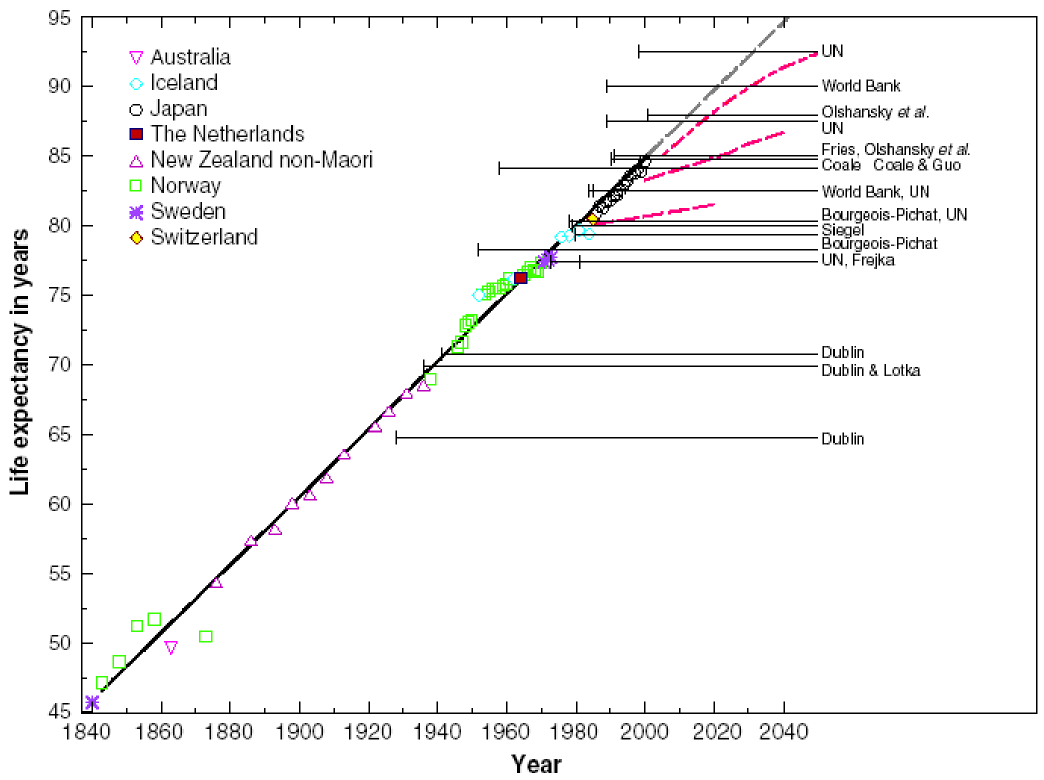
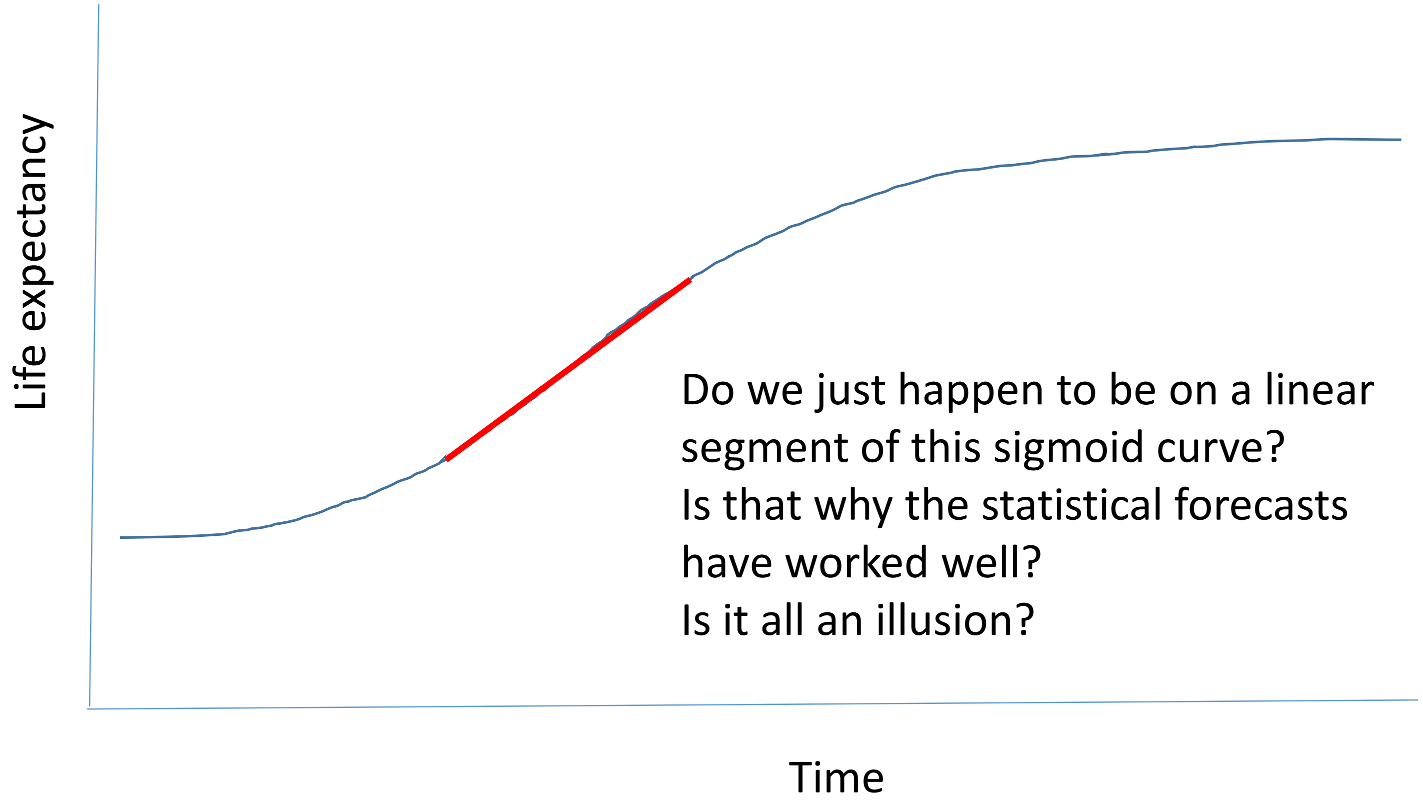


Fig. 1. Record female life expectancy from 1840 to the present [suppl. table 2 (*1*)]. The linear-regression trend is depicted by a bold black line (slope = 0.243) and the extrapolated trend by a dashed gray line. The horizontal black lines show asserted ceilings on life expectancy, with a short vertical line indicating the year of publication (suppl. table 1). The dashed red lines denote projections of female life expectancy in Japan published by the United Nations in 1986, 1999, and 2001 (*1*): It is encouraging that the U.N. altered its projection so radically between 1999 and 2001.

Oeppen and Vaupel, “Broken Limits to Life Expectancy”
http://user.demogr.mpg.de/jwv/pdf/scienceMay2002.pdf

- Highest female life expectancy in world from 1840 to 2000.
- Rises on a straight line.
  - 25 years per century
  - 2.5 years per decade
  - 15 minutes every hour.
- Pattern has continued up to present.
- Will it slow down in 21st century?

My prior expectation for the long term trajectory for life expectancy would be a sigmoid curve, not a straight ever-rising line



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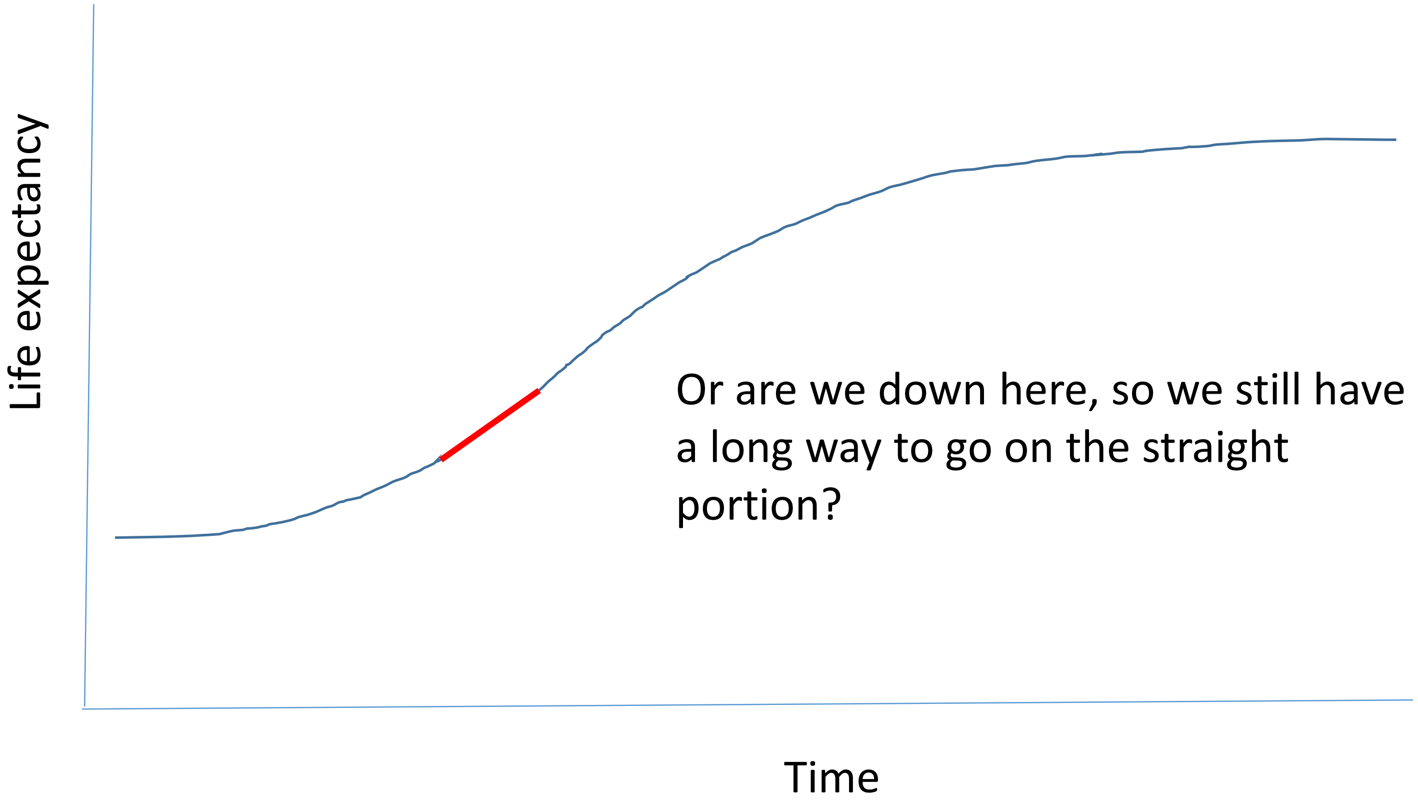

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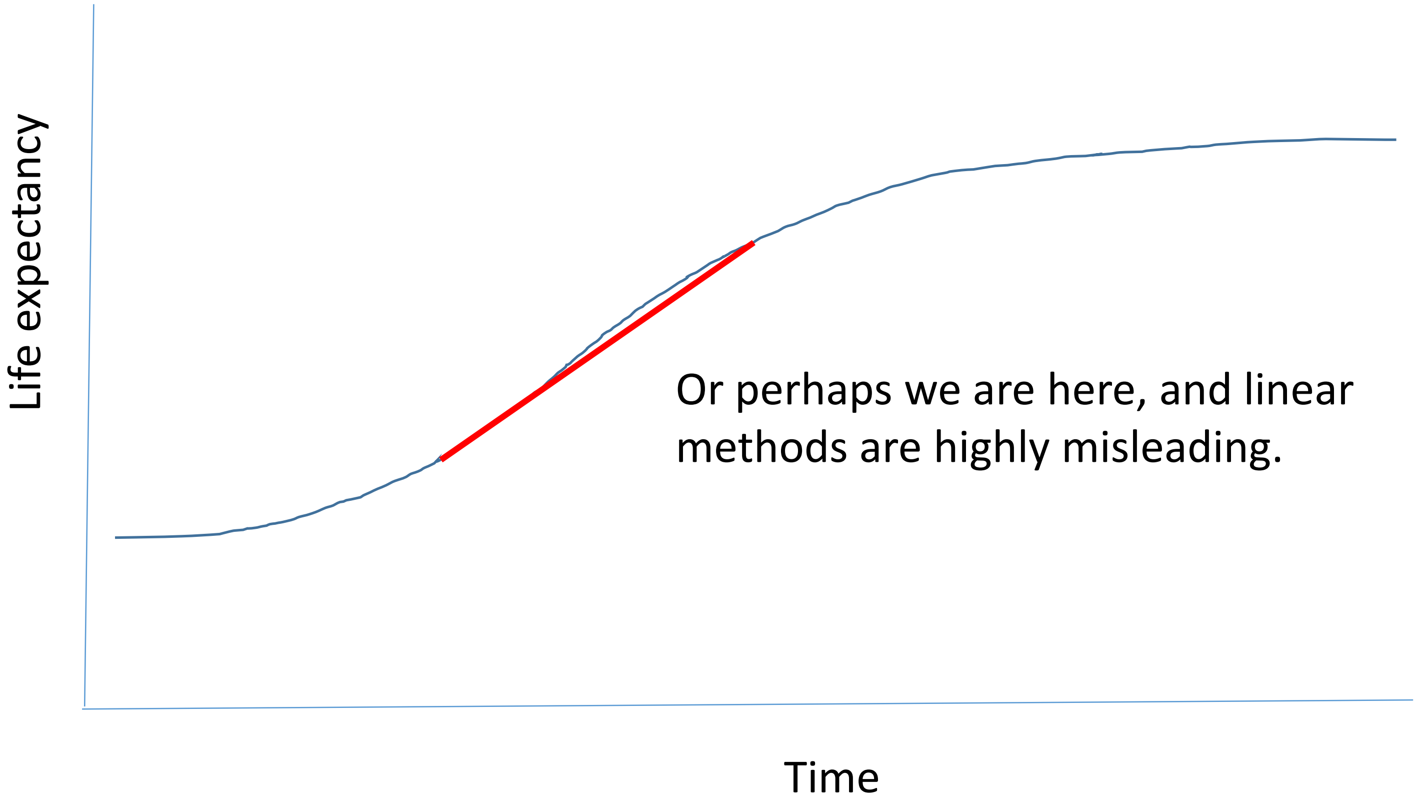

- There is no indication yet that the trend has begun to slow, and the line to bend.

Fundamentally, mortality is biological

- Fortunately, Tom Kirkwood will be speaking to us.
- There is a great deal of research on mortality in other organisms such as mice, medflies, nematodes, and yeast
- Turns out it is possible to modify life span quite substantially through varying the environment (including diet) or by genetic interventions.
- We have already greatly doubled or tripled our life expectancy through environmental change.

Still lots of room for further break-throughs

- Gene therapy, stem cell therapies, Genetic engineering, SENS (Strategies for Engineered Negligible Senescence)
- All raise major uncertainties about future mortality trends
- All get attention from at least some serious researchers
- Their importance for mortality trends remains unproven.

- Lee-Carter and other statistical methods are based on “business as usual” extrapolation of historical trends, both for central projections and for constructing probability intervals.
- The probability is unknown for a major trend break due to emerging technologies.
- Personally, though, I expect past trends to continue for a number of decades to come.

Some references

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