Best Practice Life Expectancy: An Extreme Value Approach

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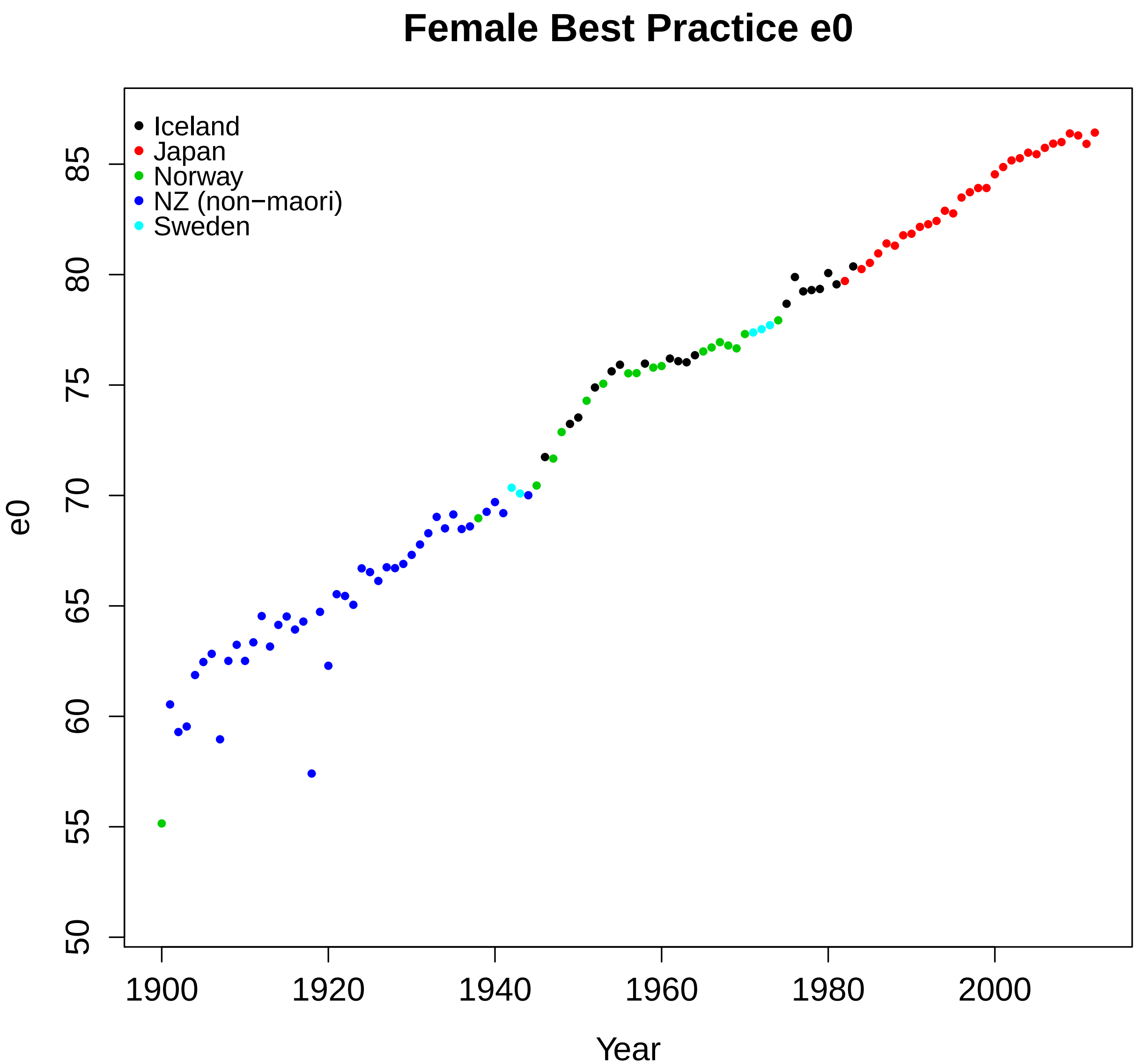
Some Facts

- Best Practice Life Expectancy (BPLE) is the maximum life expectancy observed among nations at a given age.
- At birth, has been increasing almost linearly - beginning in Scandinavia c. 1840 - at about 3 months per year (Oeppen and Vaupel, 2002).
- Life expectancy trends may fit better than individual-country trends in age-standardized (log) death rates (White, 2002).

Some Facts

- Nations experience more rapid life expectancy gains when they are farther below BPLE and tend to converge towards BPLE (Torri and Vaupel, 2012).
- It is sensible to consider national mortality trends in a larger international context rather than individual projections (Lee, 2006; Wilmoth, 1998).

Females e_0


Female Best Practice e0
Iceland
Japan
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NZ (non−maori)
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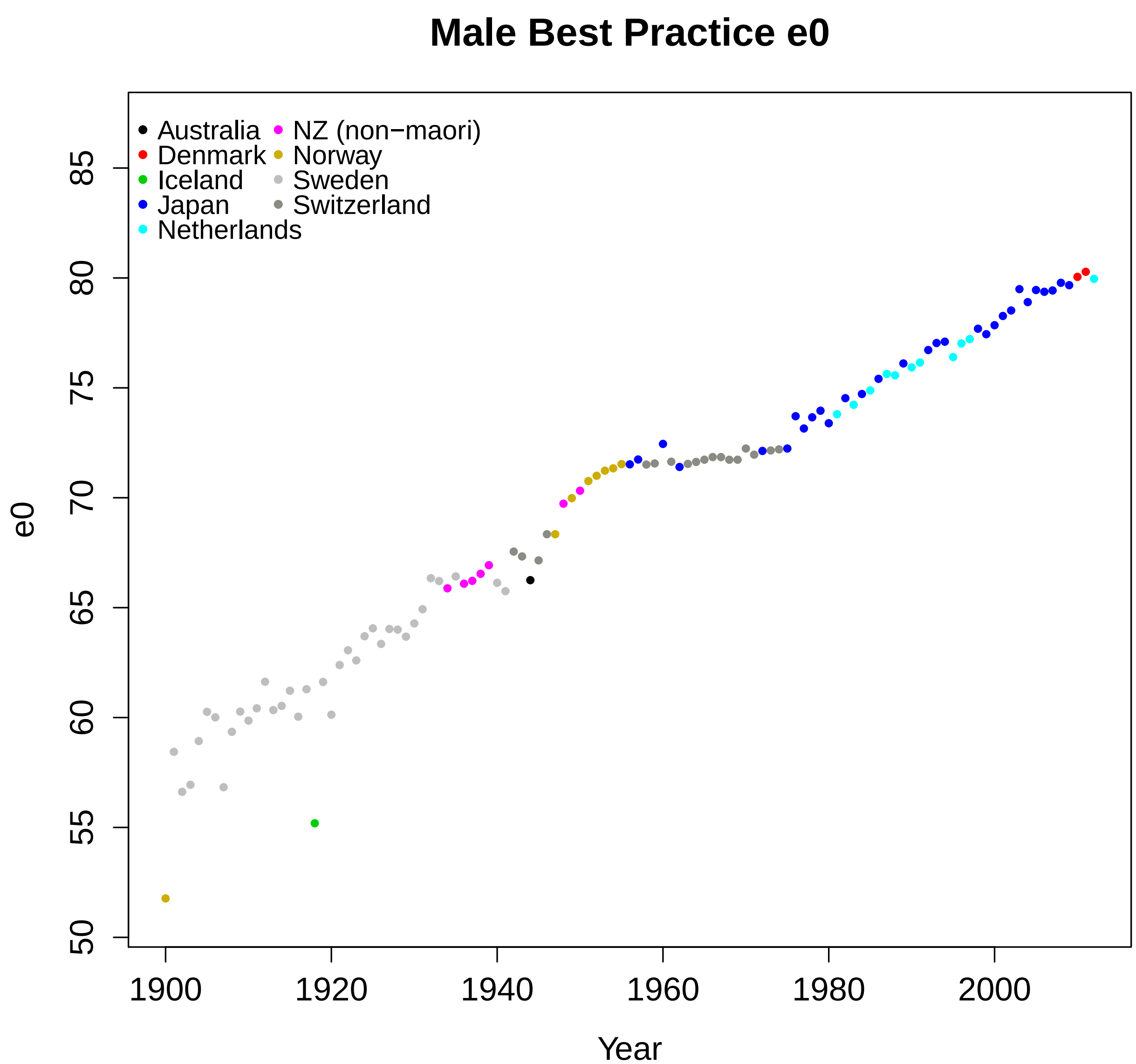
Year

Males e_0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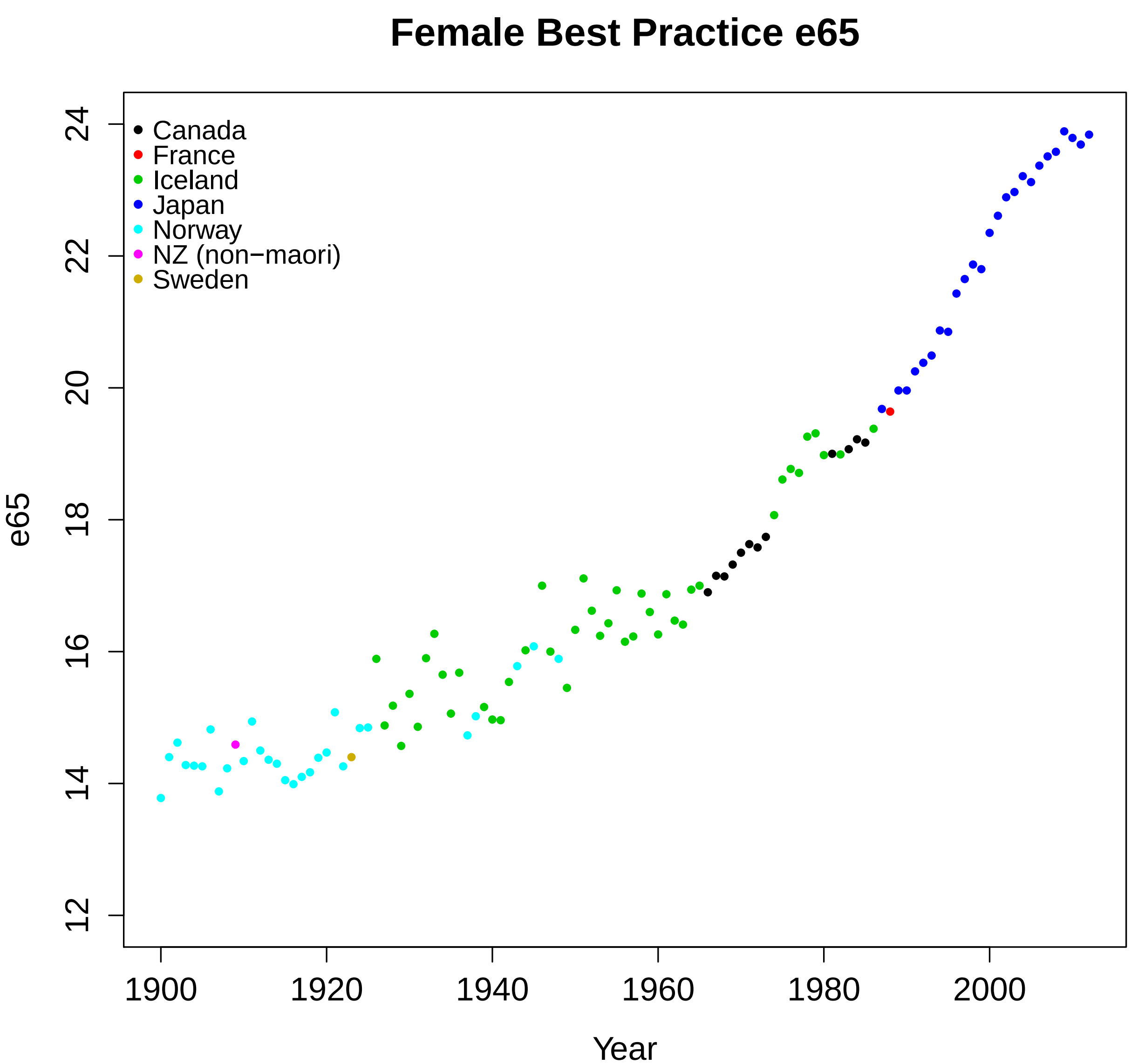
Females e_{65}


Female Best Practice e65
Canada
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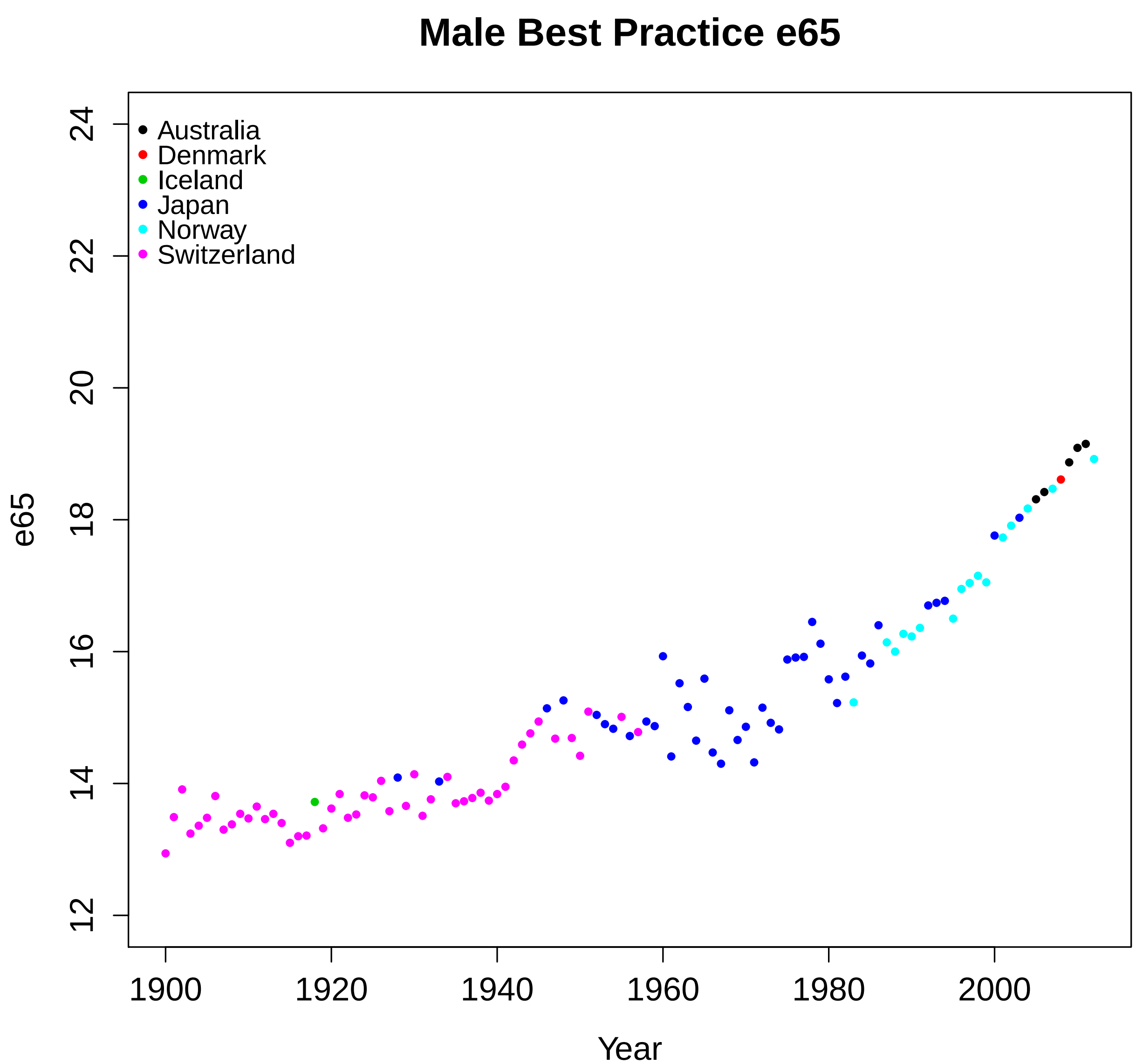
Males e_{65}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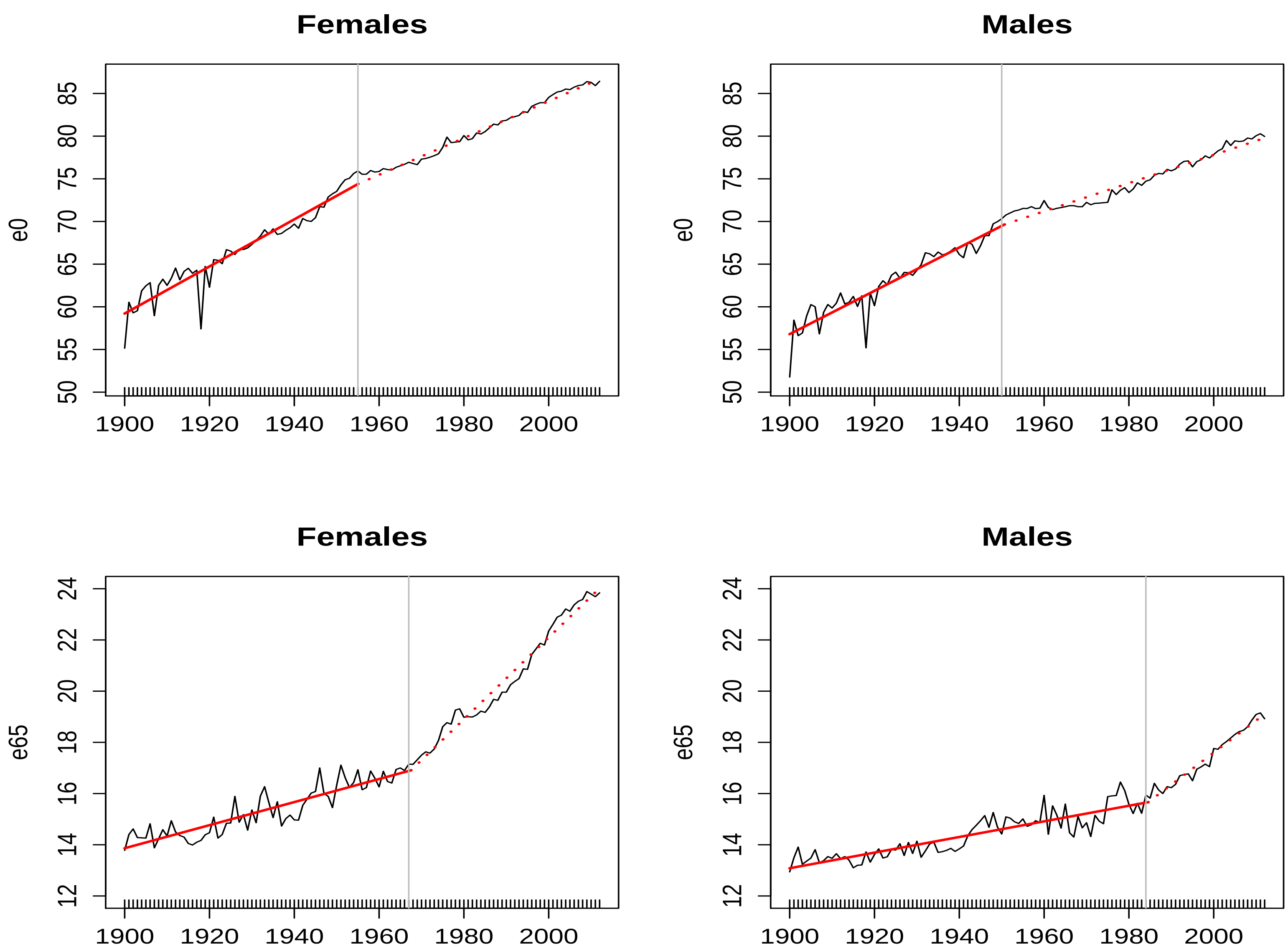
Breakpoints



Empirical motivation

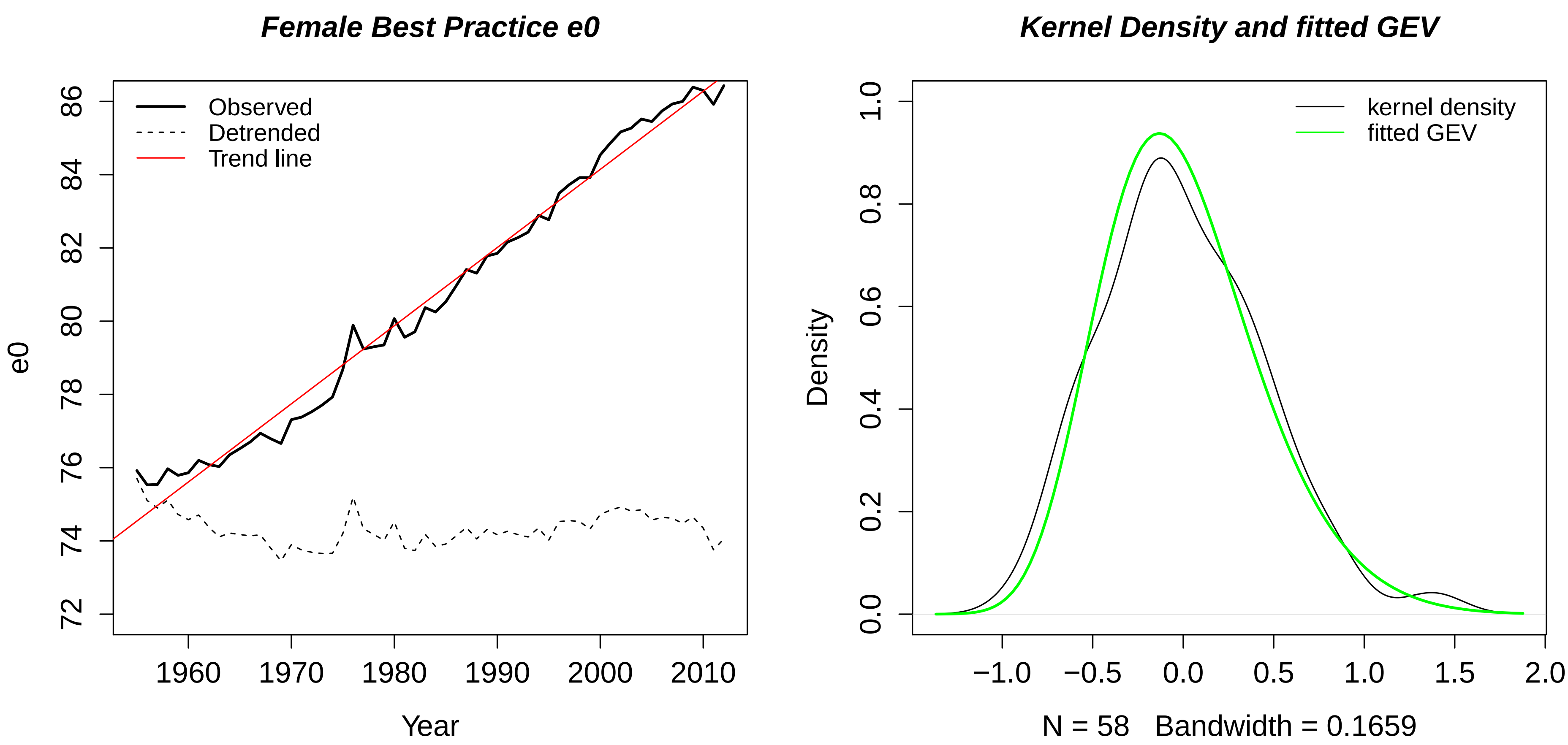


Figure: Left panel: raw and detrended data. Right panel: kernel density and fitted GEV distribution.

Theoretical motivation

Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independent, identically distributed random variates all having a common distribution function $F(x)$.

Let $M_n = \max\{X_1, X_2, \ldots, X_n\}$.

The distribution of the maxima, M_n, converges (for large n) to the Generalized Extreme Value (GEV) Distribution.

The Generalized Extreme Value Distribution

$$G(z) = \exp\left\{ -\left[1 + \xi\left(\frac{z-u}{\sigma}\right)\right]^{\frac{-1}{\xi}} \right\}$$

- u is the location parameter
- σ is the scale parameter
- ξ is the shape parameter, which determines the tail behaviour

  $\xi > 0$: polynomial tail decay and the Fréchet Distribution
  $\xi = 0$: exponential tail decay and the Gumbel Distribution
  $\xi < 0$: bounded upper finite end point and the Weibull Distribution

Inference

Quantiles

Inverting the GEV distribution function:

$$z_p = \mu - \frac{\sigma}{\xi}\left[1 - \{-log(1-p)\}^{-\xi}\right],$$

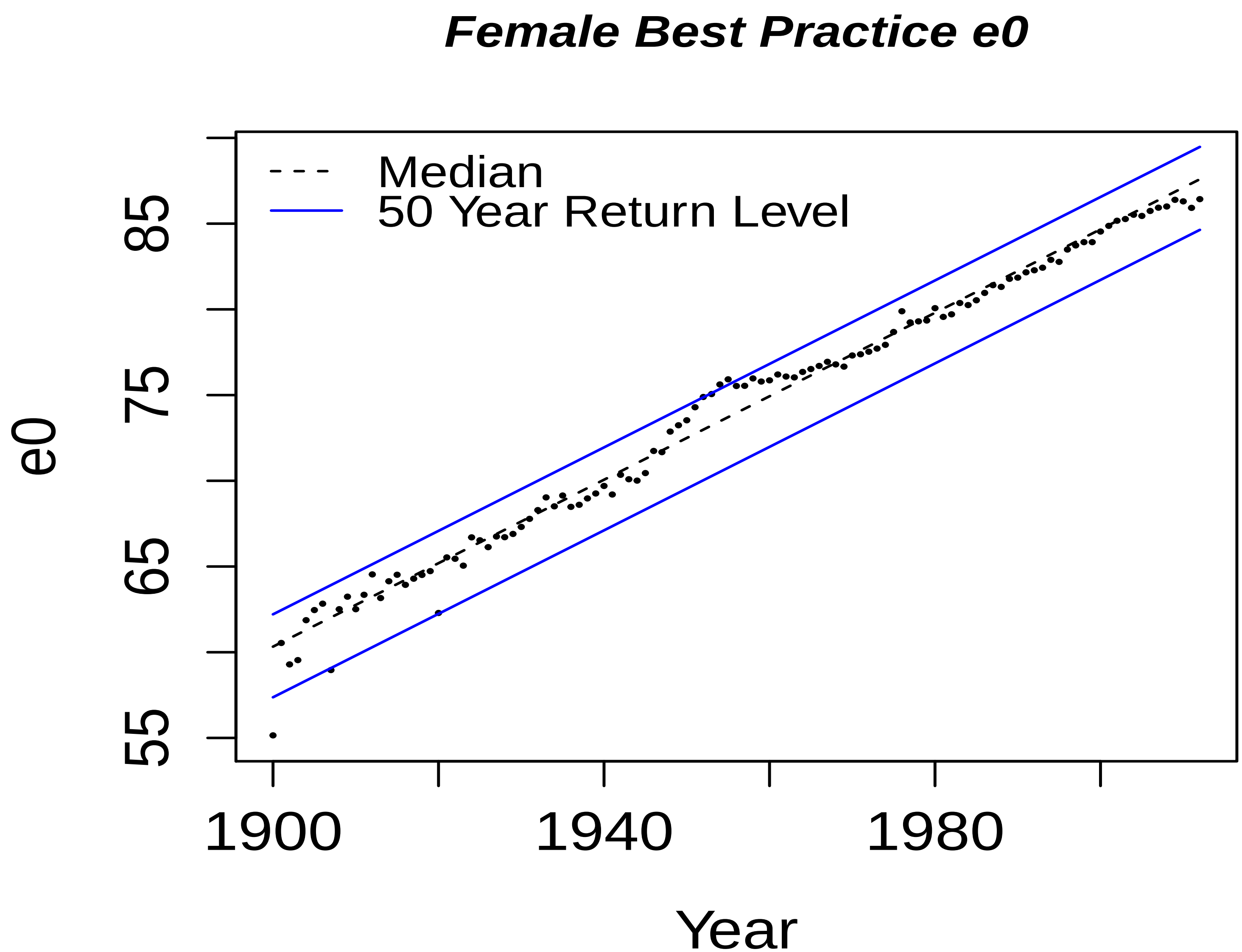
where p is the tail probability and $G(z_p) = 1 - p$

Return Levels

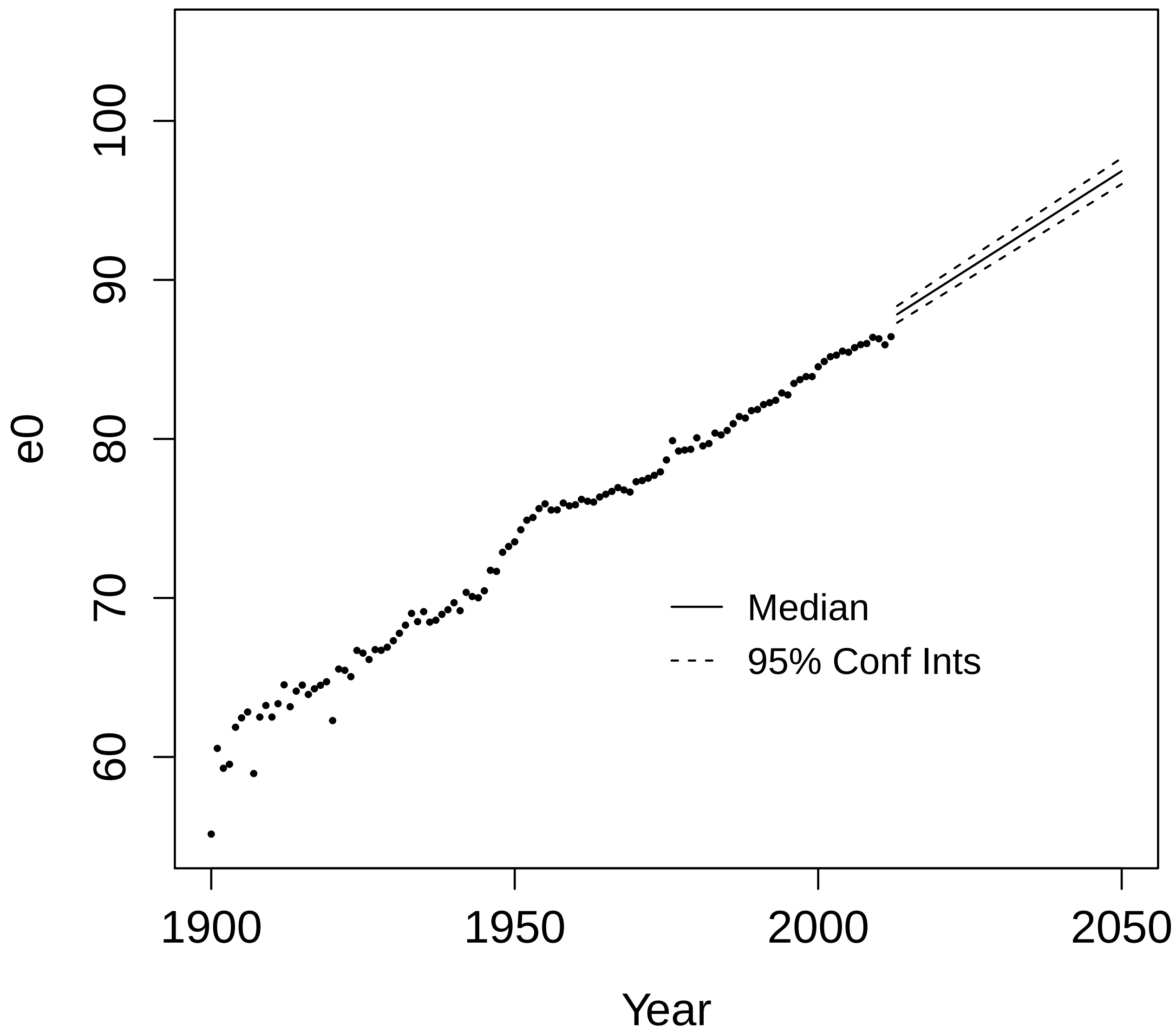
- Simply a different way of thinking about the quantiles.
- If data are annual the $(1-p)$th quantile would be exceeded on average once every $1/p$ years.

Fitted Model

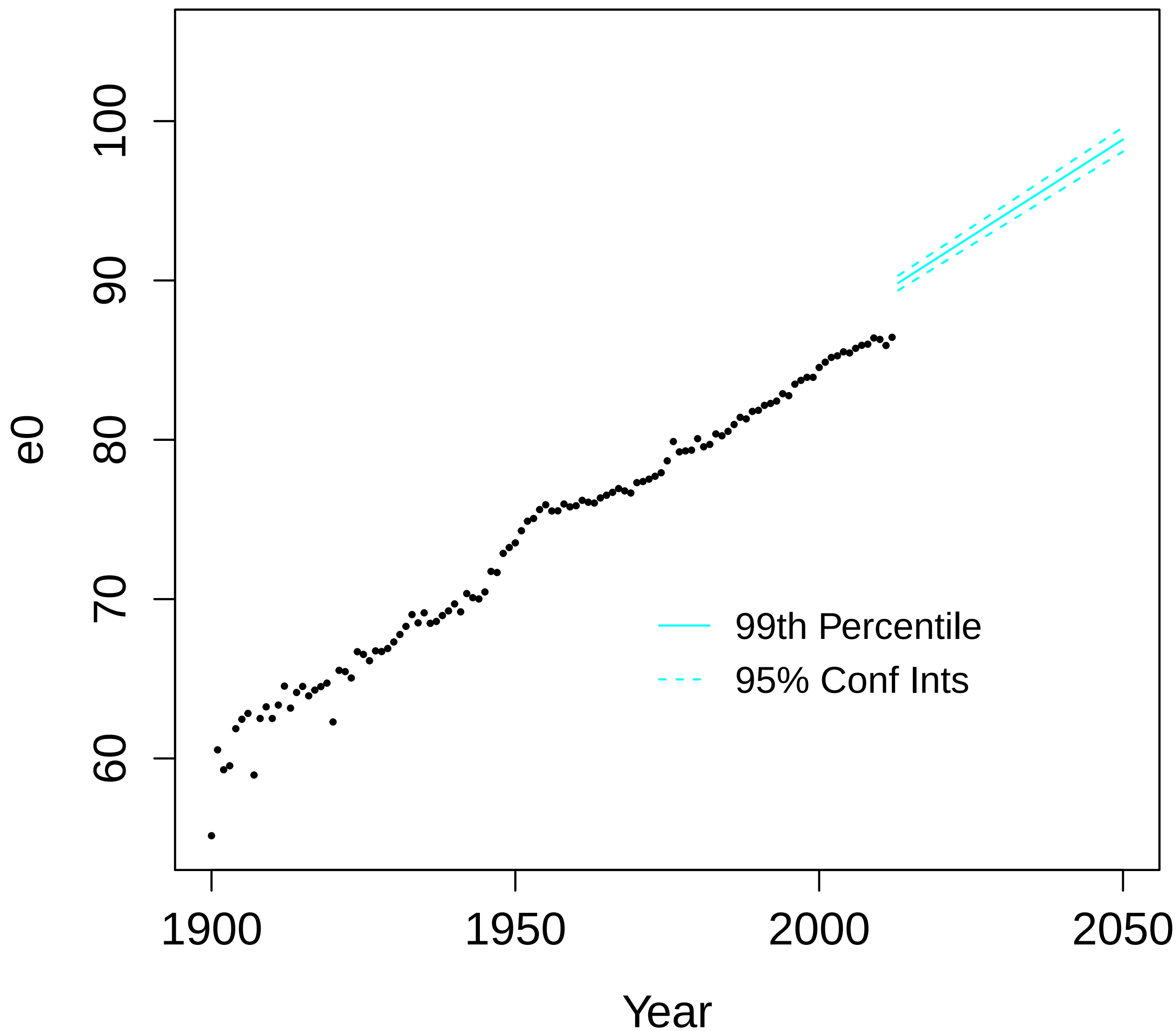
$$GEV(u_t = 59.6 + 0.24t, \quad \sigma = 1.31, \quad \xi = -0.48)$$



Projections, Females e_0



Projections, Females e_0



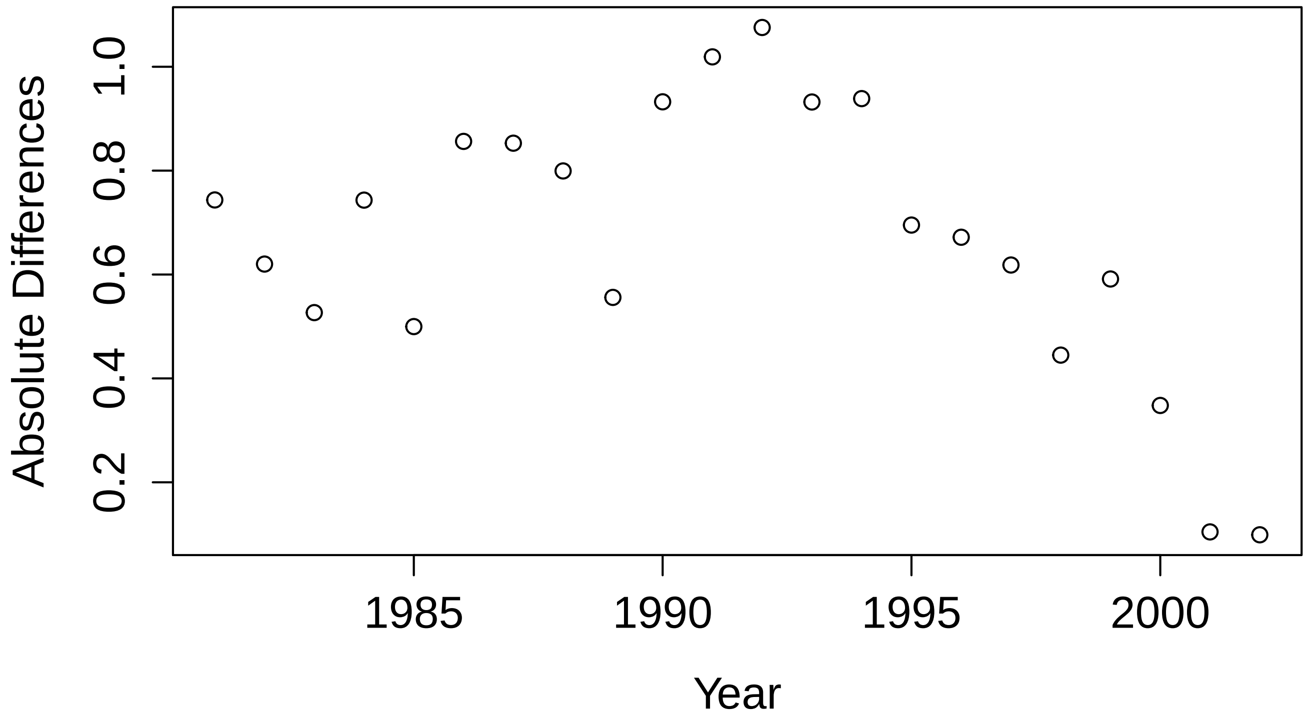
Other Inference

A probability distribution has been fit so the usual tools are available.

Year	$P(e_0^{max} > 90)$	$P(e_0^{max} > 95)$
2020	35%	$< 0.001\%$
2050	$> 99.99\%$	91%

In Sample Comparison

Fit model using data up to 1980.
Compare Observed 10 Year Maxima vs 10 Year return Levels .



Mean Absolute Difference(MAD)= 0.67 years
Mean Absolute Percentage Error = 0.8%

ARIMA model residuals

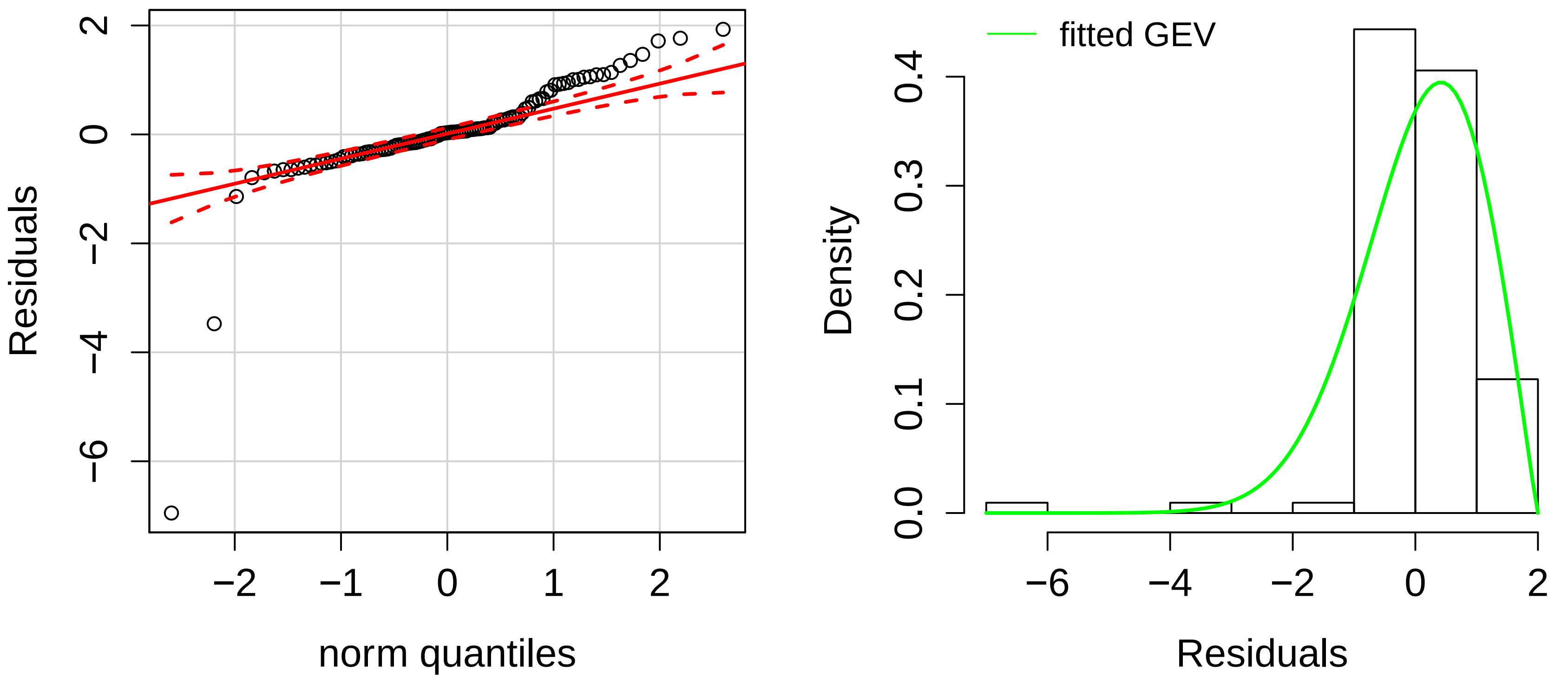


Figure: Normality tests for residuals of ARIMA(2,1,1) fitted to female e_0 BPLE. Left panel: QQ Plot; Right panel: histogram.

Innovations Process

- Assumption of Gaussian errors is often arbitrary and can be poorly fitting.
- GEV is more flexible and is able to capture the shape of different error distributions - not just symmetric.
- In practice Gaussian often provides a reasonable fit but GEV should be considered as an alternative for the innovations process.

Conclusion

- Method can be used similarly to the Torri and Vaupel (2012) approach to forecasting life expectancy:
  - Either through projecting BPLE directly, which is preferable
  - Or using the GEV as the innovations process in an ARIMA model
- EVT can identify in an objective way whether life expectancy is actually at an extreme level rather than just "high"
- EVT can be used to obtain probabilities and/ or levels of extreme longevity

References

Lee, R. (2006). *Perspectives on Mortality Forecasting. III. The Linear Rise in Life Expectancy: History and Prospects*, Volume III of *Social Insurance Studies*. Swedish Social Insurance Agency, Stockholm.

Oeppen, J. and J. W. Vaupel (2002). Broken limits to life expectancy. *Science 296*(5570), 1029–1031.

Torri, T. and J. W. Vaupel (2012). Forecasting life expectancy in an international context. *International Journal of Forecasting 28*(2), 519–531.

White, K. M. (2002). Longevity advances in high-income countries, 1955–96. *Population and Development Review 28*(1), 59–76.

Wilmoth, J. R. (1998). Is the pace of Japanese mortality decline converging toward international trends? *Population and Development Review*, 593–600.