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Leverage, Value and Firm Scope

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Leverage, Value and Firm Scope*

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Abstract

This paper observes that holding-subsidiary structures (HS) provide a conditional guarantee to their lenders, and compares it to both the unconditional guarantee binding conglomerate divisions and the no-guarantee case of stand alone firms. It then determines the value of their debt and equity claims, when there are bankruptcy costs and taxation in a no-arbitrage setting. We identify conditions ensuring higher value for HS, irrespective of diversification benefits. This stems from their higher debt capacity, in turn due to the conditional guarantee lowering bankruptcy costs, which reduces their tax burden below that of competing organizations. The value of equity in HS is however lower than in stand alone organizations.

Keywords: holding, subsidiary, groups, guarantees, debt, taxes, internal capital market, bankruptcy costs, limited liability

JEL classification numbers: G32, G34, L22

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1 Introduction

Companies are organized as holding-subsidiary structures in several countries (Khanna and Yafeh, 2007; Barca and Becht, 2001). These organizations, consisting of activities which are separately incorporated, rely extensively on debt rather than equity financing (Bae, Kang and Kim, 2002; Chang, 2003; Dewaelheyns et al., 2007). Furthermore, affiliated companies assist each other in distress through informal agreements (Chang and Hong, 2000; Khanna and Palepu, 2000) as well as with contractual guarantees (Deloof and Vershueren, 2006; Samson, 2001). The diffusion of HS (holding-subsidiary) in the world appears at odds with evidence that equity values of their affiliates are often lower than in comparable stand alone firms. This paper proposes a theory of firm scope and leverage explaining why HS emerge despite possibly lower equity values.

For this purpose, we model an entrepreneur who considers organizing his two activities in alternative ways - as stand alone firms, as divisions of a conglomerate or as HS affiliates - taking into account that they can issue debt against their cash flows. Debt allows to deduct interest from taxes, but increases bankruptcy probability. Thus, we depart from the irrelevance theorem (Modigliani and Miller 1958, Stiglitz 1969) and focus on purely financial synergies arising from firm combinations as in Leland (2007).

In this setting, we characterize different organizations depending on guarantees between the two firms. We observe that HS affiliated firms are not responsible for each others' debts, being separately incorporated and therefore enjoying limited liability.[1] However, they share a common management which can transfer cash flows to each other when needed. [2] We therefore model one company - the holding - as supporting its insolvent subsidiary through the internal capital market provided that both can survive, using its limited liability vis-à-vis the subsidiary's debt obligations otherwise. Thus *conditional guarantees* are typical of HS. On the contrary, stand alone firms - being not only separately incorporated but also independently managed - are not liable for the debt of the other units. This is why they are usually modelled as providing *no*

[1] This is a common characteristic across major jurisdictions. See Blumberg (1989) for the US, and Hadden (1996) on Britain, France, Germany and the US.

[2] Khanna and Palepu (2000) observe that Indian group firms assist each other in times of financial distress, while Bertrand et al. (2002) document cash transfers in several forms - from asset sales to internal loans at subsidized rates.

guarantee to each other. At the other extreme there are conglomerate divisions that, being incorporated as one company, are jointly liable vis-à-vis lenders; thus, each offers an *unconditional guarantee* to the other. These alternative guarantee mechanisms determine the overall debt capacity of the organization, its tax burden and expected default costs. These, in turn, affect the value of lenders' and shareholders' claims to cash-flows, which add up to the total value of the organization to the entrepreneur.

We show that the total value of an HS arrangement exceeds that of stand-alone companies when the conditional guarantee is ex post enforceable, because of increased debt capacity. The guarantee - which is an option to save on bankruptcy costs - has non negative value, which is further enhanced by leverage. Indeed, the guarantee reduces the subsidiary insolvency probability, at any given level of debt, relative to the stand-alone arrangement. This implies that the subsidiary can increase debt financing, relative to the stand alone case, so that the higher interest payment reduces its tax burden.

The value of HS also exceeds the conglomerate one in some simple cases. When cash flows in the two activities are equal and perfectly correlated, for instance, the value of the guarantee in a conglomerate is zero: both activities fail in the same contingencies, because they are forced to have the same level of debt. In HS, the level of debt is specific to each activity. As debt gets transferred from the holding (H) onto its subsidiary (S), it becomes possible for H to rescue S: thus default costs fall, the optimal debt increases and the tax burden drops. These results may explain why several companies are separately incorporated under a common financial management. In this way, they are able to increase the tax shield keeping bankruptcy costs under control by optimizing joint capital structure.

Our model thus fits into the literature highlighting the "bright side" of internal capital markets. Our emphasis is however on the trade off between bankruptcy risk and the tax burden, whereas existing research focuses on imperfections arising from asymmetric information (Stein, 1997; Gertner, Scharfstein and Stein, 1994). Previous models also stress the advantages of cross-subsidization (Leland, 2007), arguing that value gains deriving from the internal capital market disappear when cash flow correlation across the two activities is perfect. In our setting holding-subsidiary gains obtain instead irrespective of cash flow correlation, thanks to debt diversity across holdings and subsidiaries.

One type of HS organization is the traditional business group. Group shares often trade at lower values than in stand-alone firms with comparable operating performance (Bennedsen and Nielsen, 2006; Claessens et al., 2002; Masulis et al, 2008). This evidence is puzzling because shareholders seem to give up value that can be simply created by spinning off the subsidiary. In our model equity values, averaged across holding and subsidiaries, are lower than in stand alone arrangements. Yet there is no puzzle as the entrepreneur, who originally owns the activities and issues corporate liabilities, gains with respect to the stand alone case thanks to a higher market value of debt. Two effects generate the lower average value of HS equity. On the one hand, the guarantee implies that the holding shareholders will transfer cash to the subsidiary lenders, should conditions for rescue hold. This reduces the value of equity in the holding below that of a stand alone with the same level of debt, leaving unaffected the value of subsidiary equity. On the other hand, both the holding and the subsidiary modify their leverage in order to optimize the tax-bankruptcy trade-off, leading to higher group debt than in the stand alone case. Since equity value is a call option, increasing debt reduces equity value. Thus, the equity value of two stand alone firms exceeds that of a holding and its subsidiary.

Previous theories of groups focus on fund provision by minority shareholders rather than by lenders. Groups emerge when entrepreneurs prefer to fund activities indirectly, through another company, rather than directly. This is the case when the present value of the activity, net of diversion, is negative. The equity discount thus reflects expropriation of minority shareholders (Almeida and Wolfenzon, 2006). This explanation of low equity values for groups may solve the paradox when affiliated firms are listed on public exchanges with loose enforcement of securities regulation. However, it remains unclear why they thrive in strict-enforcement countries, such as Scandinavian ones, and why unlisted groups are quite common. By shifting the focus from minority shareholders onto lenders our model can account for these situations.

Numerical simulations also indicate that the value differential in favour of holding-subsidiary structures increases when activities differ in risk and bankruptcy costs. Constraining activities to unconditional reciprocal support in a conglomerate is suboptimal, when they bear diverse bankruptcy costs. Furthermore, conglomerates constrain activities to the same level of debt, while we know that shielding income from taxes through debt has higher value in the activity with riskier cash flow, because of the asymmetric nature of taxation. This reasoning also suggests which activity ought to provide - or receive - support in a group, because we know that the supported one has higher leverage. Thus, another contribution of this paper is the endogenous characterization of holding and subsidiaries.

The above results apply to the case of a guarantee which, consistent with our full information setting, is *ex-post* verifiable and enforceable in court. We also numerically analyze a situation when the lender assigns a probability lower than one to the holding honouring the guarantee *ex-post*. This modelling captures the case of informal guarantees, such as comfort letters, which assure subsidiaries' lenders that the holding would assist them in distress but are legally unenforceable. It adds to our understanding of HS structures in three related respects.

First, an informal guarantee lowers optimal subsidiary debt. This, in turn, makes the ownership stake of the holding into the subsidiary relevant for both the leverage choice of the holding and group value: the higher the dividends that the holding receives thanks to its ownership stake, the higher is its optimal debt. Dividends are, in fact, another form of conditional guarantee - which is now provided by the subsidiary on behalf of H lenders - as dividends may support an insolvent holding. The ownership stake of the holding does not matter, instead, when the holding conditional guarantee is *ex-post* enforceable because optimal subsidiary leverage is so large that dividends are always negligible.

Second, our numerical exercise generates a "parent company discount" (Cornell and Liu, 2001), i.e. a situation where the equity value of the holding is lower than the value of its equity share in its subsidiary. The reason has to do with with the higher debt burden onto the shareholders in the holding company. Our model thus hints at an arbitrage-free explanation of this valuation puzzle.

Third, HS may turn out to have lower values than conglomerates. The conditional guarantee becomes less capable than the unconditional one in improving credit conditions, as lenders anticipate an uncertain ex post service of debt even when the holding cash flows are large enough as to make rescue possible. As credit spreads widen, optimal debt falls and group value may fall below conglomerate value. An ex post enforceable conditional guarantee dominates instead the unconditional one - in our numerical simulations - because the latter destroys value when a profitable unit has to support another with negative cash flow.

The paper is organized as follows. Section 2 discusses some closely related literature. Section 3 analyzes the three organizational modes - stand alone, group and conglomerate - for two equal activities, in the case of both ex post enforceability and zero intercorporate ownership. In particular, we indicate how the value of debt and equity (Propositions 2,3,5) and of optimal debt (Corollary 4) change due to the conditional guarantee. Section 4 presents numerical simulations allowing for a comparison of optimal leverage, value of debt and equity across the three organizations, as the correlation between cash flows varies. Section 5 examines activities differing in mean cash flow, volatility and bankruptcy costs. Section 6 examines informal guarantees and positive intercorparate ownership. Section 7 concludes.

2 Related Literature

Our model extends Leland (2007) to the case of holding-subsidiary structures. In so doing, it borrows some key assumptions concerning tax rates and bankruptcy costs with no *ad hoc* modification. In particular, the firm receives no tax refunds when its income is negative. In the real world, companies may carry forward some losses, in order to reduce the asymmetric nature of taxation - which however remains substantial. Bankruptcy costs, which the firm pays only when it does not meet its debt obligations, are proportional to cash-flows. This is a common assumption in structural models of credit risk. We also make our numerical results concerning HS comparable to those of Leland (2007) for stand alone and mergers by using the same calibrations for a BBB stand alone firm.

Our model posits exogenous operating cash flows. On the contrary several papers study how agency problems in internal capital markets affect product market competition and investment choice. Cestone and Fumagalli (2005) analyze the specificities of group internal capital market by both assuming limited liability of the holding and by allowing subsidiaries to raise their own debt, as we do.[3]They study how transfers from the holding impact on the conditions obtained by subsidiaries from its outside financiers, when managerial effort cannot be observed. Higher value is the outcome of increased managerial effort, rather than of a better tax-bankruptcy cost trade-off.

Another related paper, Huizinga et al. (2008), studies tax arbitrage in multinational groups that is engineered by raising more debt in high-tax countries. In our model, groups minimize the tax burden through debt even if there is no tax rate differential between the holding and its subsidiary.

Our model may apply to traditional business groups typical of continental European countries and of emerging markets, as well as to more recent types of HS that are present in innovative industries in the US (Allen, 1998; Sahlman, 1990; Mathews and Robinson, 2006) and in the private equity industry (Jensen, 2007; Kaplan, 1989).

[3]Most other papers on internal capital markets focus on aspects, such as cash-flow pooling, that are typical of both conglomerates and groups. See, among others, Rajan Servaes Zingales (2000), Inderst and Mueller (2003) and Faure Grimaud and Inderst (2005).

3 The common set up

We consider a no arbitrage environment with two dates $t = \{0, T\}$, where T is the length of time spanned by the dates. An entrepreneur owns two production units, and each activity i generates a random future operating (net) cash flow value X_i at time $t = T$. X_i is a continuous random variable that may take both negative values and positive values: having denoted as F_i its distribution function, this implies $F_i(0) < 1$. We assume that $X \in \mathcal{L}^2$, namely that it admits at least the first two moments.The riskfree interest rate over the time period T is $r_T > 0$, and ϕ denotes the corresponding discount factor, $\phi = (1 + r_T)^{-1} < 1$.

No arbitrage implies that the value of the operating cash flow at $t = 0$ is its discounted expected value:

$$X_{0i} = \phi \mathrm{E} X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owner can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = \phi \mathrm{E} X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is

$$L_{0i} = H_{0i} - X_{0i} \geq 0 \tag{3}$$

Now consider a tax rate on future cash flows equal to τ_i. The aftertax value of the unlevered firm, which corresponds to its equity value, is

$$V_{0i} = (1 - \tau_i) H_{0i} \tag{4}$$

The present value of taxes it pays is

$$T_{0i}(0) = \tau_i H_{0i} \tag{5}$$

At time $t = 0$ the entrepreneur can lever the firm by issuing zero-coupon debt so as to maximize the value of his claims to the cash flows. Let its principal value be $P_i \geq 0$, and assume it is due, with absolute priority, at $t = T$. Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of such debt, which is cashed-in by the entrepreneur at issuance. We assume that there is an incentive to issue debt, as interest is a deductible expense. The promised interest payment is equal to:

$$P_i - D_{0i}(P_i) \tag{6}$$

In turn, taxable income is the operating one net of interest payment:

$$X_i - (P_i - D_{0i}(P_i)) \tag{7}$$

and the zero-tax level of cash flow with positive leverage, X_i^Z, is

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{8}$$

Hereafter the argument P_i of D_{0i} and X_i^Z is often suppressed.

We assume that no tax refunds are paid by the tax authority to the owners of the activity if $X_i < X_i^Z$. It follows that operating cash flows, net of tax payments, are[4]

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{9}$$

with $0 < \tau_i < 1$. The present value of future tax payments of the levered firm is equal to:

$$T_{0i}(P_i) = \tau_i \phi \mathrm{E}(X_i - X_i^Z)^+ \tag{10}$$

Clearly, some value gains obtain when (10) is lower than (5). However, issuing debt has costs as well. Similarly to Merton (1974), default occurs when net operating cash flow is smaller than the face value of the debt:

$$X_i^n < P_i \tag{11}$$

Having defined the default threshold X_i^d as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{12}$$

the default triggering condition (11) can be written in terms of the pre tax cash flows as $X_i < X_i^d$. In the event of default, we assume that bondholders will receive a fraction $(1-\alpha_i)$ of operating cash flow, X_i, when this is positive; a fraction $\alpha_i, 0 < \alpha_i < 1$, of cash flows is instead lost upon liquidation. There is then a trade-off between the dissipative default costs, $\alpha_i X_i$, and the tax savings possibly generated by debt.

Let the levered value of equity be denoted as E_{0i}, and D_{0i} be the corresponding value of debt that is cashed in at time-0. The entrepreneur

[4] Having assumed X_i continuous, we omit the boundary values in this and the following inequalities on payoffs.

chooses the face value of debt, P_i, in the two activities, given the tax-bankruptcy cost trade-off, so as to maximize the time-zero combined value of the two units. The value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively, evaluated under the risk neutral measure. Such cash flows vary with the organization- specific guarantees, which we discuss below.

3.1 The stand alone case: no guarantee

Stand alone firms - being separately incorporated and independently managed - are typically not liable for each others' debt. We therefore follow Leland (2007) in modelling stand-alone activities as never providing support to each other. Thus, the entrepreneur maximizes the combined levered firm value, $\nu_{0i}(P_i)$, of his two stand alone activities, $(i = 1, 2)$, with respect to the face values of debt:

$$\sum_{i=1}^{2} \nu_{0i}(P_i) = \sum_{i=1}^{2} [E_{0i}(P_i) + D_{0i}(P_i)] \tag{13}$$

We now determine the value of equity and debt, i.e. of two elements on the right-hand side of equation (13), as a function of the payoff to financiers at time T. The cash flow to shareholders at $t = T$, E_i, is operating cash flow less taxes and the repayment of principal, when the difference is positive:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{14}$$

Indeed, limited liability ensures that shareholders bear no responsibility when the difference is negative. By no arbitrage the value of equity is simply[5]

$$E_{0i}(P) = \phi \mathrm{E}(X_i^n - P_i)^+ \tag{15}$$

The cash flows D_i to lenders at time $t = T$ will equal P_i when the firm is solvent, i.e. $X_i > X_i^d$. When the firm is insolvent, debtholders become the residual claimants. They receive cash flows net of bankruptcy costs, $(1 - \alpha_i)X_i$, if cash flows net of interests are lower or equal to zero ($X_i \leq X_i^Z$). Recalling that the government has priority for tax payments before lenders, debtholders will also bear a tax liability $\tau_i(X_i - X_i^Z)$ in default

[5]Notice that E_{0i} is a call option with underlying X_i^n and exercise price P_i. It depends on debt principal both directly and indirectly, through the tax shield X_i^Z that enters the underlying.

when $X_i^Z < X_i < X_i^d$. The payoff to lenders is therefore equal to:

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases} \tag{16}$$

and it can be represented as follows:

Insert here Figure 1

The present value of lenders' payoff (16), $D_{0i}(P_i)$, is the value of zero-coupon debt given the principal P_i:

$$D_{0i}(P_i) = \phi \mathrm{E}\left[\begin{array}{c} (1-\alpha_i)X_i \ \mathbf{1}_{\{0<X_i<X_i^Z\}} + \\ \left[(1-\alpha_i)X_i - \tau_i(X_i - X_i^Z)\right] \mathbf{1}_{\{X_i^Z<X_i<X_i^d\}} + \\ +P_i \ \mathbf{1}_{\{X_i>X_i^d\}} \end{array}\right] \tag{17}$$

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function.[6]

As default costs and taxes approach zero, it is easy to demonstrate that $D_{0i}(P_i)$ increases less than proportionally with respect to its face value P_i, namely

$$0 \leq dD_{0i}(P_i)/dP_i = (1 - F_i(P_i))\phi \leq \phi < 1$$

In particular, we have

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} = \frac{1 - F_i(0)}{1 + r_T} > 0$$

since X_i is not negative for sure ($F_i(0) < 1$). It seems to us that maintaining the same assumption in the costly case - when closed form expressions for $D_{0i}(P_i)$ do not obtain - does not entail loss of generality:

Conjecture 1 *Debt is increasing less than proportionally in the face value of debt:*

$$0 \leq dD_{0i}(P_i)/dP_i < 1$$

with

$$\lim_{P_i \to 0+} \frac{dD_{0i}(P_i)}{dP_i} > 0$$

[6] Due to default costs and tax savings, debt $D_{0i}(P_i)$ is a portfolio of plain vanilla puts and the present value of the principal. Note that (17) is an implicit equation, since X_i^Z and X_i^d are themselves function of D_{0i} through (8) and (12). Numerical methods are necessary for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach in (4) will consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

Intuitively, risky debt should be an increasing function of its face value: differently from riskless debt, it should be increasing less than proportionally to the discount factor, and therefore less than proportionally to one.

It follows that both the tax shield and the default threshold are increasing in the face value of debt:[7]

$$\frac{dX_i^Z}{dP_i} = 1 - \frac{dD_{0i}(P_i)}{dP_i} > 0 \tag{18}$$

$$\frac{dX_i^d}{dP_i} = 1 + \frac{\tau_i}{1-\tau_i}\frac{dD_{0i}(P_i)}{dP_i} > 0 \tag{19}$$

It is possible to show that the gain due to leverage, $\nu_{0i}(P_i) - V_{0i}$, reflects tax savings from interest deduction less default costs. Thus the value of the levered firm is equal to:

$$\nu_{0i}(P_i) = V_{0i} + TS_i(P_i) - DC_i(P_i) \tag{20}$$

where $TS_i(P_i)$ is the present value of tax savings, equal to the differential tax burden of the unlevered and levered firm:

$$TS_i(P_i) = T_i(0) - T_i(P_i) = \tau_i \phi \left[\mathrm{E}X_i^+ - \mathrm{E}(X_i - X_i^Z)^+ \right] \tag{21}$$

and $DC_i(P_i)$ is the present value of the default costs incurred in because of leverage:

$$DC_i(P_i) = \alpha_i \phi \mathrm{E}\left[X_i \ \mathbf{1}_{\{0<X_i<X_i^d\}} \right] \tag{22}$$

Default costs increase in the face value of debt, because the set of default states grows.[8]

It can be shown that under Conjecture 1 - consistently with intuition - it is never optimal to issue zero debt in order to maximize the firm value, unless no taxes exist, i.e. $\tau \to 0$ (see Appendix A). In the sequel, we will assume that $TS_i - DC_i$ is concave in $P_i \geq 0$, and that the the stand alone value is maximized for positive leverage.

[7] In the numerical analysis the properties of both the tax shield and the default threshold are satisfied for all P and all possible parameter values. Plots are availbale from the authors upon request.

[8] Default costs are a barrier call option on X_i with zero strike and barriers equal to zero and X_i^d. Its value is decreasing in the face falue of debt, which determines an increase in the upper barrier.

3.2 Holding-Subsidiary Structures: Conditional Guarantee

The entrepreneur organizes his activities as a HS when one activity - the holding company ($i = h$) - supports its insolvent subsidiary ($i = s$) through a conditional cash transfer. Such a transfer is conditional on the holding company ability to meet both its own and its subsidiary debt obligations. This is the case only if the holding after-tax cash flow, net of debt repayment, exceeds the corresponding difference for the subsidiary, i.e.

$$\begin{cases} X_h > X_h^d \\ X_h^n - P_h > P_s - X_s^n \end{cases} \tag{23}$$

If the second condition does not hold, the holding survives by letting the subsidiary default. In other words, the holding company enjoys limited liability, because - thanks to separate incorporation - is not responsible for its subsidiary's debt obligations.[9]

A necessary condition for the transfer is that the subsidiary is unable to meet its debt obligations. Limited liability also ensures that there is no rescue if the operating cash flows of the subsidiary are negative, as the holding would otherwise bear an operating loss that it could have avoided. Thus, the holding rescues its subsidiary if:

$$0 < X_s < X_s^d \tag{24}$$

Overall, a transfer occurs if and only if both (24) and (23) hold. In what follows we write the support conditions in compact notation as:

$$\begin{cases} 0 < X_s < X_s^d \\ X_h > h(X_s) \end{cases} \tag{25}$$

where we assume for simplicity $\tau_1 = \tau_2 = \tau$ and where $h(X_s)$ denotes the linear function:

$$h(X_s) = \begin{cases} X_h^d + \frac{P_s}{1-\tau} - \frac{X_s}{1-\tau} & X_s < X_s^Z \\ X_h^d + X_s^d - X_s & X_s > X_s^Z \end{cases} \tag{26}$$

The amount of the transfer is accordingly:

$$(P_s - X_s^n)\mathbf{1}_{\{0<X_s<X_s^d, X_h>h(X_s)\}} \tag{27}$$

[9] This assumption is consistent with legal texts (Blumberg, 1989; Hadden, 1996) as well as with some empirical literature. For instance, business groups appear to support struggling subsidiaries, but they tend to terminate such support once group profitability turns negative (Dewaelheyns and Van Hulle (2006)).

The entrepreneur maximizes levered firm value, $\nu_{0i}(P_i)$, of his holding and subsidiary, $(i = h, s)$, with respect to the face values of debt:

$$\nu_{0,HS}(P_h,P_s) = \sum_{i=h}^{s} \nu_{0i}(P_h,P_s) = \sum_{i=h}^{s} \left[E_{0i}(P_h,P_s) + D_{0i}(P_h,P_s)\right] \quad (28)$$

taking into consideration the conditional transfer from the holding to the subsidiary. Observe that, for simplicity, we are assuming zero intercorporate ownership.[10] We will however generalize our results to unrestricted intercorporate ownership.

The choice of debt that maximizes (28) subject to (27) will in general differ from the stand alone one. As a consequence, the default and tax shield thresholds of the holding and subsidiary will be different from their stand alone counterparts.

In the following subsections we compare the total value of HS and stand alone arrangements, when the face value of debt is exogenous. Specifically, we assume that it is equal for the holding and its stand alone counterpart (i.e. $P_h = P_1$) as well as for the subsidiary and its stand alone correspondent (i.e. $P_s = P_2$). For the sake of simplicity, we also assume that tax rates and default costs do not differ between the holding and its subsidiary ($\alpha_s = \alpha_h = \alpha, \tau_s = \tau_h = \tau$). Please notice that, whenever we will assume equality between two cash flows (random variables), it will be equality in distribution.

3.2.1 HS equity

We can now proceed to determine the payoff accruing to shareholders of the holding company at T. This is equal to operating cash flows net of taxes and the service of debt, less the conditional transfer from the holding to its subsidiary:

$$E_h(P_h,P_s) = (X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\left\{0<X_s<X_s^d, X_h>h(X_s)\right\}} \quad (29)$$

Such payoff can be written as:

$$E_h(P_h,P_s) = E_1(P_h) - (P_s - X_s^n)\mathbf{1}_{\left\{0<X_s<X_s^d, X_h>h(X_s)\right\}} \quad (30)$$

where $E_1(P_h)$ is the cash flow accruing to shareholders of a comparable stand alone with the same nominal debt ($P_1 = P_h$). The last term

[10] This situation obtains in HS structures when there is extreme separation between ownership and control, which can be the consequence of multiple voting shares. Alternatively, it can be interpreted as a spin off, when the same entrepreneur/ shareholders control debt choices of both activities.

highlights that the payoff to the holding shareholders never exceeds the payoff of the stand-alone with equal debt, because of the state-contingent support. It follows that also the equity value of a holding company:

$$E_{0h}(P_h,P_s) = \phi \mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\left\{0<X_s<X_s^d, X_h>h(X_s)\right\}}\right] \quad (31)$$

cannot exceed the one of a stand alone with the same capital structure. Equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing debt, and equation (14) still holds for $i = s$. An immediate consequence of the fact that $E_h < E_1, E_s = E_2$, is the following:

Proposition 2 *Consider a holding company, its subsidiary and two stand alone firms with the same face value of debt outstanding ($P_1 = P_h, P_2 = P_s > 0$) and the same operating profits of H and S ($X_1 = X_h, X_2 = X_s$). Then the average equity price of stand-alone firms exceeds that of HS affiliated counterparts.*

This is our first rationale for the observation that equity values are often lower in HS than in stand alone (SA) structures.

3.2.2 HS debt

The value of subsidiary debt, $D_{0s}(P_s, P_h)$, is the present expected value of the following final payoffs:

$$\begin{aligned} & D_s(P_s, P_h) = \quad (32) \\ & = \left[X_s(1-\alpha) + \tau(X_s - X_s^Z)\mathbf{1}_{\{X_s>X_s^Z\}}\right]\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h<h(X_s)\right\}} + \\ & \quad +P_s\left[\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} + \mathbf{1}_{\left\{X_s>X_s^d\right\}}\right] \end{aligned}$$

The first square bracket refers to the case when the subsidiary defaults and the holding does not support its subsidiary because its own cash flow is insufficient ($X_h < h(X_s)$). In this situation, lenders have to pay taxes only if cash flows exceed the tax shield ($X_s > X_s^Z$). The first term in the second square bracket refers to the case when the subsidiary, while defaulting if it were a stand alone firm, is able to reimburse its debt thanks to the holding transfer.

It is easy to show that the payoff to a subsidiary lender, relative to that of its stand alone counterpart with the same nominal debt ($P_2 = P_s$), is equal to:

$$\begin{aligned} D_s(P_s, P_h) = {} & D_i(P_s) + [P_s - X_s(1-\alpha) - \quad (33) \\ & -\tau(X_s - X_s^Z)\mathbf{1}_{\{X_s>X_s^Z\}}\big]\,\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} \end{aligned}$$

Subsidiary lenders obtain, on top of what accrues to stand-alone lenders, the nominal value of debt thanks to the guarantee (first term in square bracket) while losing the cash flow net of bankruptcy costs and taxes (second and third term). In other words, the payoffs to subsidiary lenders is the same as in the stand alone case, outside the states when a transfer takes place. It must instead be augmented by the transfer, as shown in Figure 2, when this occurs.

Insert here Figure 2.

The payoff to lenders of the holding does not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the holding debt. Thus equation (17) holds for $i = h$.

It follows that the average value of debt is higher in a HS arrangement than in stand alone companies, given an exogenous face level of debt.

3.2.3 The value of the conditional guarantee

We can now measure the value of the guarantee as the value of the group less the value of two comparable stand alone units. Dropping the argument of the functions for brevity, this is equal to the discounted expected value of the following payoff:

$$D_s - D_2 + E_h - E_1 = \tag{34}$$
$$= [P_s - X_s(1-\alpha) - \tau(X_s - X_s^Z)\mathbf{1}_{\{X_s > X_s^Z\}} - (P_s - X_s^n))]\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}} =$$
$$= [\alpha X_s]\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}}$$

The value of the guarantee, when the debt burden remains the same in the stand alone and group arrangement, is equal to discounted bankruptcy cost that is avoided:

$$G(P_h, P_s) = \alpha\phi\mathrm{E}\left[X_s\mathbf{1}_{\left\{0<X_s<X_s^d,\ X_h>h(X_s)\right\}}\right] \tag{35}$$

Equation (35) directly implies the following result:

Proposition 3 *Assume that the set of payoffs satisfying (25) has positive probability. Then the value of holding-subsidiary structures exceeds the value of two comparable stand-alone firms with the same distribution of operating profits ($X_1 = X_h; X_2 = X_s$).*

Proof. *When $\alpha > 0$ and the support occurs with positive probability, the subsidiary is levered and the guarantee in (35) is positive. This directly implies that the value of holding-subsidiary structures exceeds the value of two comparable stand-alone firms with the same face value* *of debt outstanding ($P_1 = P_h; P_2 = P_s$) and the same distribution of operating profits ($X_1 = X_h; X_2 = X_s$). This holds for any fixed face values of debt, including the optimal ones for stand alone firms ($P_h = P_1^*, P_s = P_2^*$). A fortiori, the optimized group value, corresponding to the optimal choice of outstanding debt for the holding and subsidiary, will be higher than the sum of two stand alone values.* ■

Thus, subsidizing weaker firms turns out to be value increasing in this case with bankruptcy costs and no endogenous investment choice. On the contrary, such subsidization is value reducing in a setting with agency problems between managers and shareholders which lead to distorted investment choices across activities (Scharfstein and Stein, 2000). Under the maintained assumption of exogenous operating cash-flow distributions, Proposition 3 implies the following:

Corollary 4 *Assume that the set of payoffs satisfying (25) has positive probability. An entrepreneur prefers to incorporate his activities as holding and subsidiary rather than as stand alone companies.*

Observe that HS value exceeds that of comparable stand alone firms even if, from Proposition 2, its equity value is lower. These two propositions thus reconcile the paradoxical findings that groups are common across the globe despite lower equity values associated to the same operating profits.

We now complete our analytical comparison of groups and stand alone firms by endogenously determining the level of debt.

3.3 Leverage and holding-subsidiary value

Observe that the tax burden on group affiliated companies coincides with the tax burden of their stand-alone counterparts with equal debt outstanding ($P_1 = P_h, P_2 = P_s$):

$$\begin{aligned} T_{HS}(P_h, P_s) &= \tau\phi \left[\mathrm{E}(X_s - X_s^Z)^+ + \mathrm{E}(X_h - X_h^Z)^+\right] = \\ &= T_1(P_h) + T_2(P_s) \end{aligned} \tag{36}$$

On the contrary, default costs can be lower because of the conditional guarantee:

$$DC_{HS}(P_h, P_s) = \alpha\phi\mathrm{E}\left[X_s \mathbf{1}_{\{0<X_s<X_s^d,\ X_h<h(X_s)\}} + X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right] = \tag{37}$$

$$= DC_1(P_h) + DC_2(P_s) - G(P_h, P_s)$$

When the two firms have the same capital structure as two stand alone counterparts, subsidiary default costs are indeed lower than the sum of the stand alone costs, the difference arising from the guarantee. Thus the choice of debt, P_h, P_s, that maximizes group value solves:

$$\min_{P_h, P_s} [T_{HS}(P_h, P_s) + DC_{HS}(P_h, P_s)] = \tag{38}$$

$$= \min_{P_h, P_s} [T_1(P_h) + T_2(P_s) + DC_1(P_h) + DC_2(P_s) - G(P_h, P_s)]$$

The proposition below establishes that an optimum in which the subsidiary is unlevered does not exist. This optimum would entail no guarantee and an unlevered holding too: therefore the HS structure would reduce to two unlevered stand alone firms. Such a situation is dominated by the levered stand alone arrangement, which can be replicated by the HS structure. The proposition establishes also that - if a minimum exists - the minimum is such that the holding is unlevered and total debt in the group is larger than total debt in two stand alone firms.

Proposition 5 *Assume Conjecture 1 holds. Then a) there cannot be a local minimum in which the subsidiary is unlevered; b) if the value of default costs plus tax burden, net of the guarantee, is convex in debts and a technical convergence condition holds for $xf(x, y)$, there exists a level of the tax rate above which the holding is optimally unlevered ($P_h^* = 0$) and total debt in the holding-subsidiary organization - which coincides with the subsidiary one - is higher than in two stand alone companies ($P_s^* > P_1^* + P_2^*$).*

Proof. See Appendix B ■

Please notice that part a) can be equivalently stated as: minima, if they exist, entail a levered subsidiary ($P_s^* > 0$). Forming a group in which the subsidiary is unlevered is not value-enhancing, since the guarantee is never used. A corollary in Appendix B shows that the subsidiary's optimal debt is positive also if Conjecture 1 does not hold.

The key result is that book debt is larger in HS than in two stand alone firms. The intuition is as follows. At P_1^*, P_2^* the marginal default cost in the subsidiary is lower[11] than the marginal default cost in a

[11] It is equal only for $\rho = 1$.

stand alone activity thanks to the guarantee, while the tax savings are the same. The marginal default cost and tax savings of the holding, when evaluated at P_1^*, P_2^*, are equal for the holding and the stand alone. If subsidiary default costs are convex in the face value of debt, debt in the subsidiary must be larger in order to reestablish equality with tax savings. It follows that total default costs in groups exceed total default costs in stand alone units. At the same time, tax savings are also higher.

This Proposition relies on the assumption of convexity of the sum of tax burden and default costs, which ensures the existence of a solution to value maximization. The assumption cannot be guaranteed in general, but do hold in the numerical cases which follow the presentation of the conglomerate case. [12]

3.4 The Conglomerate Merger Case: Unconditional Guarantee

In the conglomerate-merger case, the two activities are incorporated as one firm - with cash flow $X_m = X_1 + X_2$ - and are jointly liable vis-à-vis lenders. Thus, we propose to represent the conglomerate merger as equivalent to two stand alone activities, each enjoying an unconditional guarantee issued by the other one; that is, each having a cash flows equal to one half of the poold cash-flow X_m.

We can therefore write the entrepreneur objective as maximizing

$$\sum_{i=1}^{2} \nu_{0m}(P_i) = \sum_{i=1}^{2} [E_{0i}(P_i) + D_{0i}(P_i)] \tag{39}$$

with respect to the face value of debt in each the two activities, P_i, when each activity lender has a claim against one half the sum of the cash flows:

$$X_i = 0.5(X_1 + X_2) \tag{40}$$

This problem corresponds to the one in Leland (2007)[13] which has a unique choice variable, the face value of merger debt, P_m, maximizing the entrepreneurs' wealth, i.e. the merger value:

$$\nu_{0m} = \nu_0(P_m) = E_0(P_m) + D_0(P_m) \tag{41}$$

[12] A detailed, numerical characterization of optimal capital structure is necessary as the value of debt depends on debt face values of both subsidiary and holding companies, since the transfers, tax shields and default thresholds do as well. Thus both E_{0h} and D_{0s} depend on both principals P_h and P_s, which are simultaneously determined.

[13] When cash flows are Normally distributed, firm value is homogeneous of degree 1 in P (Leland, 2007).

where $E_0(P_m)$ and $D_0(P_m)$ are computed according to (14) and (16) with $i = m$.

As in the case of other organizations, the problem of value maximization can be equivalently stated as the minimization of tax burden:

$$T_m = \tau\phi \left[\mathrm{E}(X_m - X_m^Z)^+\right] \tag{42}$$

plus default costs:

$$DC_m = \alpha\phi\mathrm{E}\left[X_m \mathbf{1}_{\{0<X_m<X_m^d\}}\right] \tag{43}$$

where X_m^Z and X_m^d are defined as in (8) and (12).

3.4.1 Comparison with HS

The value differential between HS and a conglomerate merger is given by:

$$\Delta\nu_{0HS} = \nu_{0HS}(P_h, P_s) - \nu_{0m}(P_m) = \underbrace{-\Delta V_{0m}}_{\text{unlevered, pretax difference}} \underbrace{-\Delta T_{HS} - \Delta DC_{HS}}_{\text{leverage effect}} \tag{44}$$

where

$$\Delta V_{0m} = \phi\left[\mathrm{E}(X_m)^+ - \mathrm{E}X_h^+ - \mathrm{E}X_s^+\right] \tag{45}$$

is the loss to the unlevered conglomerate due to the pooling of cash flows, which reduces value when one activity is unprofitable (Leland, 2007). Pooling of cash flows is avoided both in HS and stand alone structures, because activities are separately incorporated. The second is the differential tax burden, i.e. equation (36) minus equation (42), while the third term is the differential default costs, i.e. equation (37) minus equation (43).

We cannot obtain general results concerning $\Delta\nu_{0HS}$. However, we can establish the following:

Proposition 6 *(a) Let $X_1 = X_2$, implying $\rho = 1$; or (b) let $X_1 - EX_1 = -(X_2 - EX_2)$, implying $\rho = -1$, together with $EX_1 = EX_2 > 0$. Then $\Delta\nu_{0HS}^* > 0$, provided that we ignore second order effects on the value of debt, i.e. that we assume*[14] $D_{0s}(P) = D_{0i}(P)$.

[14] Please notice that $D_{0h}(P) = D_{0i}(P)$, since the expression for debt does not differ between an holding and a stand alone; in principle, instead, $D_{0s}(P) \neq D_{0h}(P)$. Both have first derivative positive and smaller than one, by conjecture one. Here we ignore their difference in concavity.

Proof. We will examine each term in $\Delta\nu_{0HS}$ and consider the case $X_1 = X_h, X_2 = X_s$

(a) Let us start with $\Delta V_{0m} = \phi\left[\mathrm{E}(2X)^+ - 2\mathrm{E}X^+\right] = 0$.This result obtains because X_m is equal in distribution to $X_h + X_s = 2X$ when $X_h = X_s = X$. In order to examine the last two terms, let $P_h = P_s = \frac{P_m^*}{2}$. The differential tax burden becomes:

$$\begin{aligned}\Delta T_{HS} &= \tau\phi[\mathrm{E}(X_h - X_h^Z)^+ + \mathrm{E}(X_s - X_s^Z)^+ - \mathrm{E}(X_m - X_m^Z)^+] = \\ &= \tau\phi[2\mathrm{E}(X - (\frac{P_m}{2} - \frac{D_m}{2}))^+ - \mathrm{E}(2X - (P_m - D_m))^+] = 0\end{aligned} \tag{46}$$

where $X_h^Z = \frac{P_m}{2} - \frac{D_m}{2}$ under homogeneity of degree one of debt with respect to its face value, while $X_s^Z = \frac{P_m}{2} - \frac{D_m}{2}$ under the assumption that the holding and subsidiary debt function do not differ when they correspond to the same face value of debt. Since both functions have been assumed increasing in debt, what we are indeed ignoring when setting $X_s^Z = X_h^Z$ is mainly their difference in concavity, as stated in the proposition assumptions[15].

As for differential default costs, these are equal to:

$$\Delta DC_{HS} = DC_h + DC_s - DC_m =$$

$$\begin{aligned}&= \alpha\phi[E\left(X_h\mathbf{1}_{\{0<X_h<X_h^d\}}\right) + E\left(X_s\mathbf{1}_{\{0<X_s<X_s^d, X_h<h(X_s)\}}\right) - E(X_m\mathbf{1}_{\{0<X_m<X_m^d\}})] = \\ &= \alpha\phi[2EX\mathbf{1}_{\{0<X<\frac{X_m^d}{2}\}} - E(2X\mathbf{1}_{\{0<2X<X_m^d\}})] = 0\end{aligned} \tag{47}$$

since the set $\{X_h < h(X_s)\}$ has probability 1 when $\rho = 1$ and $P_h = P_s = \frac{P_m^*}{2}$. It follows that the value of HS and merger coincide when evaluated at $P_h = P_s = \frac{P_m^*}{2}$. Now we use the fact that a merger is equal to two stand alone firms when $\rho = 1$,as there are no diversification opportunities (Leland, 2007). Thus $P_m^* = P_1^* + P_2^*$. Recall that $P_s^* > P_1^* + P_2^*, P_h^* = 0$ by Proposition (5). Thus $\nu_{0HS}(P_h^*, P_s^*) \geq \nu_{0HS}(\frac{P_m^*}{2}, \frac{P_m^*}{2}) = \nu_{0m}(P_m^*)$.

(b) Under the assumptions of part b, since

$$X_m = X_1 + X_2 = E(X_1) + E(X_2) = 2E(X) > 0$$

and X_1 is equal to $2E(X)$–X_2 in distribution, the first term becomes

$$\Delta V_{0m} = \phi\left[2E(X) - 2\mathrm{E}\left(X^+\right)\right].$$

[15] In the numerical cases which follow this assumption has been checked to have no impact on the results.

Let $P_h = 0, P_s = P_m^*$. Observe that the conglomerate defaults with probability 1 if debt is higher than $2E(X)$, it never defaults for smaller debt. Correspondingly, it has $P_m^* = 2E(X)$

$$X_m^Z(P_m^*) = 2E(X)\,(1-\phi)$$
$$X_m^d(P_m^*) = 2E(X)\left(1+\frac{\tau\phi}{1+\tau}\right)$$

As a consequence, it never defaults if $P_m < P_m^*$, but its tax savings could be increased. Considering again that, under the assumptions of the theorem, $X_s^Z = X_m^Z$, the differential tax burden becomes:

$$\begin{aligned}\Delta T_{HS} &= \tau\phi[\mathrm{E}(X_h)^+ + \mathrm{E}(X_s - X_m^Z)^+ - \mathrm{E}(X_m - X_m^Z)^+] = \\ &= \tau\phi[\mathrm{E}(X_h)^+ + \mathrm{E}(X_s - 2E(X)\,(1-\phi))^+ - \\ &\quad -\mathrm{E}(2E(X) - 2E(X)\,(1-\phi))^+] = \qquad (48)\\ &= \tau\phi[\mathrm{E}(X_h)^+ + \mathrm{E}(2E(X) - X_h - 2E(X)\,(1-\phi))^+ - 2E(X)\phi] \leq \qquad (49)\\ &\leq \tau\phi[\mathrm{E}(X_h)^+ - \mathrm{E}(X_h) + 2E(X)\phi - 2E(X)\phi] \leq 0 \qquad (50)\end{aligned}$$

Finally, differential default costs become:

$$\Delta DC_{HS} = \alpha\phi[X_s\mathbf{1}_{\{0<X_s<X_s^d, X_h<h(X_s)\}} - E(X_m\mathbf{1}_{\{0<X_m<X_m^d\}}] =$$
$$= \alpha\phi[X_s\mathbf{1}_{\{0<X_s<X_m^d, X_h<h(X_s)\}} - E(X_m\mathbf{1}_{\{0<X_m<X_m^d\}}] =$$
$$= -\alpha\phi[X\mathbf{1}_{\{0<X_m<X_m^d\}}] \leq 0$$

since the set $X_h < h(X_s)$ has zero probability and

$$X_m^d = 2E(X)\left(1+\frac{\tau\phi}{1+\tau}\right)$$

It follows that

$$\begin{aligned}\Delta\nu_{0HS} &= -\Delta V_{0m} - \Delta T_{HS} - \Delta DC_{HS} = \\ &= \phi\left[2\mathrm{E}\left(X^+\right) - 2E(X)\right] - \Delta T_{HS} - \Delta DC_{HS} \geq \\ &\geq \phi\left[2\mathrm{E}\left(X^+\right) - 2E(X)\right] > 0\end{aligned}$$

Recall this holds in $P_s = P_m^*$. A fortiori, it holds in $P_{s.}^*$ ■

The case $\rho = 1$ highlights why debt diversity enhances the value of the conditional guarantee. If cash flows in the two activities are the same and debt is also the same - as it must be when the two activities are merged - the holding cannot rescue its subsidiary ever. As one unit of debt gets transferred from the holding onto its subsidiary, it is possible for H to rescue S: thus default costs fall, the optimal debt increases and the tax burden drops. At the other end of the spectrum, when $\rho \to -1$, ΔV_{0m} captures the costs of cash flow pooling in conglomerates: an unprofitable activity forces a profitable one out of business. While cash flow pooling reduces the tax burden on the conglomerate relative to HS, there is still a value differential in favour of the latter.

We now turn to a numerical study of optimal firm scope with endogenous leverage.

4 Numerical analysis

This section analyzes the properties of different organizational modes through numerical methods, assuming that the annual cash flow distribution is Normal. The parameters are equal to the base case in Leland (2007), which is consistent with a typical firm that issues BBB-rated unsecured debt. Table 1 reports parameter values. Expected operating cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is $X_0 = 100$. Operating cash flow at the end of 5 years has standard deviation (Std) of 49.2, consistent with an annual standard deviation of cash flows equal to 22.0 ($= 49.2/\sqrt{5}$) if annual cash flows are independently distributed in time. Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice.

Insert here Table 1

Table 2 shows the optimal capital structure and value for a firm with base-case parameters. The first column reports results for a stand alone. The second, third and fourth columns refer to holding, subsidiary and average affiliated company respectively, while the last column to half of a conglomerate. The correlation coefficient between the units cash flows is set equal to 0.2, as in Leland, so as to allow for comparison.

Insert here Table 2

4.1 HS versus stand alone

The optimal face value of debt, which is equal to 57.2 for a stand alone company, reaches 109.5 in the "average" HS affiliate, consistent with our analytical results. Accordingly, both expected tax savings and default costs are smaller in the former (2.33 and 0.90) than in the latter case

(7.27 and 3.99). The value increase due to leveraging jumps from 1.18 for the stand alone firm to 3.15 for the representative HS affiliate. This jump is the effect of differential tax savings and differential default costs, and captures the value of the internal capital market in groups. As a result, HS affiliate value (82.95) exceeds that of a SA (81.23). The beneficiary of the internal capital market is the initial owner - or the initial shareholders - of the two activities, who can sell them for more.

They benefit despite the fact that intercorporate guarantees reduce the market value of equity - namely, $E_h^* + E_s^* < E_1^* + E_2^*$. Proposition 2 emphasizes the transfer from H to S, while in this exercise we appreciate the interplay betwen the transfer and the level of debt. Specifically, in Table 2 we see that the value of equity in the stand alone is larger than in the subsidiary (39.01 instead of 0.037), because of its much lower level of debt (57.1 versus 219). On the contrary, the value of equity in the holding is larger than the stand alone one (49.2 versus 39.01), even if part of its cash flow is being transferred to the subsidiary lenders, because its debt is lower (0 versus 57.1). We will see in later sections that the opposite may happen, i.e. $E_h^* < E_1^* < E_s^*$. However, it will still be the case that the average value of SA equity exceeds that of HS. This fact could mistakenly be interpreted, by an outside observer such an econometrician, as the consequence of group inefficiency.

Figure 2 helps understanding the optimal leverage strategy of the HS in this case. By setting debt to zero in the holding, its default threshold coincides with the horizontal axis. This maximizes the transfer area A, given P_s and X_s^d. Raising more debt in the subsidiary ensures that its tax threshold moves to the right - making it less likely that taxes will be paid both when the firm is bankrupt and when it is not. Clearly also the subsidiary default threshold, X_s^d, moves to the right, but the transfers will often make sure that the subsidiary does not default - while enjoying tax privileges.

It is worth noting the similarity between HS and leveraged buy-outs. In HS, the tax burden of debt drops from 17.62% to 5.42% of operating cash flow in Table 2. In firms taken private through MBOs, the tax burden dropped from 20% to 1% in the first two years and to 4.8% in the third year (Kaplan, 1989). In Table 2, the default threshold for a subsidiary is 248, way above the mean operating income. In a sample of distressed highly leverage transactions, all sample firms had operating margins in excess of the industry median (Andrade and Kaplan, 1998). Last but not least, the model implies leverage in excess of 95% in subsidiaries. This is also observed in the private equity industry, where our assumption of no agency costs applies well (Jensen, 2007).[16]

[16] Private equity partners often need to raise new funds in the market because of

4.2 HS versus conglomerate mergers

Merging two symmetric activities allows the resulting conglomerate to increase debt capacity above the one of the two stand alone firms, because cash flow pooling between the two activities reduces the probability of default. This brings enhanced tax advantages because of interest deductions, as predicted by Lewellen (1971). It also allows to use the losses from one unit to offset taxable income from the other unit, thus reducing the negative impact of tax asymmetries (Majd and Myers, 1987). However, one unprofitable conglomerate division may absorb the cash flows of a profitable one, reducing the value of limited liability for the unlevered conglomerate - the "Sarig effect". Despite this, the value of conglomerates turns out to exceed that of two stand alone activities for a correlation coefficient between cash flows equal to 0.2. These results, due to Leland (2007), can be visualized by comparing the first with the last column of Table 2.

We now turn to the numerical comparison between half of a conglomerate and a representative affiliate of a holding-subsidiary structure. The optimal debt in HS is far greater than in conglomerates (109.5 versus 58.7). This sugges that, at 58.7, the marginal costs of default is lower in groups than in conglomerates, implying that the effect of support conditionality exceeds that of missing support from S to H. As a consequence of higher debt, the tax burden in HS falls to 12.69 as opposed to 17.81 in the conglomerate, while expected default costs respectively rise to 3.99 as opposed to 0.62. Furthermore, the value increase thanks to leveraging is twice as large in groups (3.15) than in conglomerates (1.53). As a consequence, the value of a HS affiliate, 82.95, exceeds that of half a conglomerate, 81.57.

The possibility of raising different debt levels in the two affiliates (0 and 219) versus an equal debt level of 58.7 in divisions enhances the value of HS, by allowing it to better exploit both conditional support and the asymmetry of taxation. Indeed, raising more debt from the subsidiary increases its no tax threshold, which reaches 102.32 , against a mean cash flow of 127.63. Its tax savings are as large as 14.53, which compare to 2.19 in the conglomerate division. However, the default threshold of the subsidiary is pushed up to 248.169. Such a burdensome debt service can be sustained because of conditional transfers from the holding company.

The extant literature argues that the allocation of capital among divisions improves in conglomerates, relative to the case of stand-alone companies, because monitoring incentives are stronger (Gertner, Scharf-

the limited temporal commitments of financiers, and this is possible only if their reputation is good. Moreover, subsidiary managers receive bonuses only when they repay their debt obligations.

stein and Stein, 1994) and the well informed headquarter is able to allocate scarce financial resources to the best projects (Stein, 1997). These arguments may also apply to HS since they also have an internal capital market. However, they cannot explain why HS – as opposed to conglomerates – seem to be a very common organizational form. Our results suggest that the value increase is due to the specificities of HS - namely conditionality of the guarantee and the possibility of choosing diverse debt levels in the two activities. Both are allowed for by the limited liability linked to separate incorporation.

In the following section we assess whether these patterns hold when diversification opportunities change.

4.3 Capital structure and value with changing correlation

Leland (2007) shows that gains from a merger (M) disappear together with diversification: as the correlation coefficient between activities' cash flows tends to 1, the default costs of the merger converge to those of stand alone firms, so does its debt level and overall value. The HS structure also exploits diversification. One may thus expect that, as correlation among cash flows increases, the transfers from the H to S will become less likely and the optimal face value of debt will converge to the stand alone level.

The first part of this reasoning is correct: as the correlation coefficient ρ increases from -0.8 to 0.8, the (unreported) probability of a transfer from H to S halves. The second part of the argument is however incorrect: debt in HS continues to be much larger than in SA, and very diverse between H and S. Figure 3 - which still refers to an ex post enforceable guarantee - displays such result in the bottom right panel.[17]

The optimal face value of debt in HS actually increases in ρ. In conglomerates the opposite holds: debt falls as the probability of paying twice bankruptcy costs increases. In other words, the unconditional guarantee has no value in the conglomerate at $\rho = 1$, while the conditional guarantee still works in HS thanks to debt diversity which ensures that H and S have different levels of earnings after interest. Thus, lower debt in H ensures that H can still rescue S even if they have the same operating cash flows - the reason being that H has larger earnings after interests. Furthermore, H never incurs into bankruptcy costs having zero optimal leverage. Thus, debt diversity enhances the value of the conditional guarantee, the more so the larger is ρ. Consider in fact that

[17] Contrary to the face value, the market value of S debt falls as correlation increases (see upper right panel of Figure 3): lenders required spread grows in ρ in anticipation of reduced support by H.

expected default costs $E\left[\alpha X_s \mathbf{1}_{\ 0<X_s<X_s^d}\right]$ are increasing in X_s^d, not only because the probability of default increases but also because conditional default costs, αX_s, are larger. The larger is ρ, the likelier it is that H cash flow suffices to rescue S, i.e. $X_h > h(X_s)$, when conditional default costs are also large.

Consistent with implications of our Proposition 2, HS equity value $(E_1^* + E_2^*)$ is lower than in the case of both stand alone and conglomerates for all correlation coefficients (upper right panel of Figure 3). This is due to both higher debt levels and the cash flow transfer from H shareholders to the benefit of S debt-holders. The wedge between conglomerate and HS equity value increases in ρ, as transfers to debtholders in the conglomerate fall and reach zero at $\rho = 1$, while they are still positive in HS thanks to debt diversity across affiliates.

Finally, and most importantly, the value differential between HS and M - and stand alone firms - is always positive (top left panel of Figure 3), and achieves a maximum for $\rho = 1$. Cash flows after interest are reduced in the subsidiary thanks to its high debt level. On the contrary, tax asymmetries hit the conglomerate the most because there is no profit smoothing between the two activities, which obtain the same operating cash flows and the same after tax cash flows.

5 HS, M and SA when activities differ

So far results refer to two symmetric activities, that differ only - in the HS structure - because one is assumed to support the other. The analysis below refers to cases when activities differ in either cash flow volatility (σ), or in size (Mu) or in proportional bankruptcy costs (α).

This investigation deserves attention for at least two reasons. First, Rajan et al. (2000) point to potential inefficiencies stemming from diversity - in size and investment opportunities - across conglomerate activities. While they focus on inefficiencies arising from capital budgeting, Leland (2007) highlights that cash flow diversity has a cost in conglomerates because of the larger foregone value of limited liability: conglomerate may turn out to have lower value than stand alone organizations only when activities are asymmetric. We now assess whether HS are more or less valuable when activities are diverse, in the BBB case of Leland.

Second, we observe that strategic alliances (Robinson, 2008), venture capital funds (Sahlman, 1990) and innovative firms (Allen, 1998) often adopt a HS structure, with riskier ventures incorporated as subsidiaries. The same is true in traditional business groups (Bianco and Nicodano, 2006; Masulis et al., 2008). This suggests that HS value is not invariant to the relative features of H and S. Below, we endogenously derive the characteristics of holding and subsidiaries that maximize HS value, which is a further contribution of this paper relative to prior literature.

The following proposition summarizes our main numerical findings, under our parametric assumptions:

Proposition 7 *Assume asymmetric cash-flow distributions for BBB calibrated companies. Then (i) the optimal HS structure has higher value than competing organizations, and value gains increase with risk and bankruptcy costs asymmetries between activities; (ii) in the optimal HS structure, the default costs and the size of the holding are at least as large as those of its subsidiary. The subsidiary, in turn, is at least as risky as the holding.*

These results hold for the case, displayed in Tables 3 to 5, when correlation between cash-flows is equal to 0.2, as well as for the unreported range {-0.8, +0.8}.

Table 3 displays numerical results when the two activities have proportional bankruptcy costs respectively equal to 23%, as in the base case, and 75%. With larger bankruptcy costs, the optimal value of a stand alone firm drops from 81.23 (see the second column) to 80.83 (first column) as its face value of debt reduces from 57.2 to 33. In the case of HS, by contrast, it turns out that the activity with larger bankruptcy costs should be the holding company, because - under the optimal capital structure - H never pays them. Both the optimal capital structure and group value do not change as default costs in the subsidiary increase from 23% to 75%. It follows that value gains from group structure increase (from 3.44 in Table 2 to 3.85 in Table 3) with asymmetries in default costs across activities.

The value of a conglomerate merger case also falls from 163.14 to 162.47 with asymmetric default costs, as joint incorporation of activities constrains divisions with diverse bankruptcy costs to the same face value of debt. The value gains from a HS, relative to the M, structure grow from 2.76 in the symmetric case to 3.45 in the current, asymmetric one.

Unreported optimizations assess the cost of a suboptimal HS structure, associated with the subsidiary bearing higher proportional default costs than its holding. Its face value of debt falls to 107 (as opposed to 219). Due to a reduced tax shield, HS value is now lower (162.37) than the one of the merger (162.47), and this holds for all correlations between -0.8 and +0.8. However, even a suboptimal HS dominates the stand alone organization.

Table 4 concerns the case of different risk, with one unit having an annualized cash flow volatility of 44% as opposed to 22% of the other. We know that the tax shield has higher value with a riskier cash flow, because the firm pays taxes when earnings after interests are positive, but does not get a comparable tax refunds in the opposite situation. It is therefore unsurprising to find higher optimal debt, and associated tax shield, in all organizations. A comparison between the first and the second column reveals that the higher volatility unit, when incorporated as a stand alone, has higher face value of debt (83 instead of 57), and is accordingly charged a much higher spread (6.2% as opposed to 1.2%) by lenders. This implies increased tax savings (4.66 versus 2.31), and a higher value of the riskier stand alone (84.84 versus 81.23) - even though its equity value drops from 39.32 to 36.1. The total value of these two stand alone is equal to 166.07.

The conglomerate value is 163.24, only marginally higher than in the base case of equal risk across divisions, and lower than the stand alone value. This result echoes Leland (2007) observation that merging two diverse activities may reduce firm value, despite diversification gains. On the one hand the riskier division is more likely to drag the safer one in default, while on the other the tax shield is constrained to be equal across the two diverse activities.

This is not a problem for the HS structure, as the holding can use its limited liability to avoid joint default and diverse debt levels to tailor the tax shields to each activity needs. Consistent with empirical evidence, we find that subsidiaries are riskier than their holding companies in the optimal group structure. A riskier holding incurs into larger losses more often than a safer one. Hence it would not be able to rescue its subsidiary as often as a safer one. Moreover, a riskier holding would suffer more than a riskier subsidiary from the asymmetric nature of taxation, as it uses less the tax shield of debt. Going back to Table 4, subsidiary debt reaches 223 (up from 219 in the base volatility case), ensuring that interests shield the larger profits - which are now more likely - from taxes. Total group value is now equal to 170.13, exceeding the value of the group in the base case (163.14), when the subsidiary is less risky. Importantly, value gains relative to SA (M) increase from 3.44 (2.76) to 4.06 (6.89). [18]

The last case we examine is the one of differing size. Simulation results in Table 5 refer to a situation where the expected cash flow of one activity is five times the other's. In all the organizations, value falls relative to the symmetric case. Fora SA and HS structures, the reduction is small (SA from 162.46 to 162.44; HS from 165.9 to 165.65) provided

[18] The size of these gains makes it more likely that they will outweigh costs associated with HS structures, which are not included in the present analysis. This may explain why HS structures are widespread in innovative industries, where the risk of subsidiary activities is large relative to that of holding companies.

that the smaller activity is the subsidiary. On the contrary, M value drops from 163.14 to 162.98. Once again, the larger company is more likely to drag the smaller, healthy one into insolvency; moreover both are constrained to the same tax shield.

In the HS case, smaller activities should be the ones that receive conditional support. The smaller subsidiary - see the fourth column - raises less debt than the one of the base case (121.33 as opposed to 219), with two consequences. On the one hand the holding company wants to raise debt as well so as to reduce taxation, on the other it can do so without compromising the provision of support to its subsidiary- thanks to its size. Debt appears in the holding company (63.33). A HS with a large subsidiary would be suboptimal, as the large subsidiary could hardly be rescued by a small holding. Consequently, it would raise less debt with a corresponding reduction in both the tax shield and value (163.25). This is however still higher than in the competing organizations.

All the previous qualitative results under asymmetry hold true when correlation varies: we conclude that - at least in Leland's set up - the ability of groups to create value by optimally trading off taxes and bankruptcy costs is a strikingly robust result - absent incentive problems.

6 Relaxing some simplifying assumptions

Subsidiary lenders may expect a holding company to support its subsidiary with some probability even when there is no legally binding guarantee or assumption of debt by Boot et al. (1993) argue that the holding trades-off the benefits from improved future credit conditions with the costs of reduced financial integrity in deciding whether to honor comfort letters, that are legally unenforceable promises of rescue sent by the parent to its subsidiary's lenders. It is also the case that rating agencies consider the strategic importance of the subsidiary, the percentage ownership of the holding, shared names, common sources of capital, financial capacity of providing support as relevant factors is assessing the probability of *ex post* rescue (Samson, 2001). In this section we will assume that lenders attribute an exogenous probability $\pi < 1$ to ex post rescue, whereas we had implicitly set such probability to 1 in the previous ones. In other words, we will allow for an informal guarantee whereas the previous analysis deals with the legally binding case.

Next, we will investigate the case when the holding keeps a fraction $0 < \omega < 1$ of the subsidiary equity capital. When the HS structure is unlisted, this fraction is usually close to 1. It often exceeds 0.5 even when the subsidiary is listed on a public exchange if the entrepreneur wishes to keep majority control. So far we maintained that control is kept with a zero ownership share ($\omega = 0$) for tractability. Interestingly, our results are unaffected when we allow for positive ω as long as $\pi = 1$: dividends paid out to H by S are anyway negligible under the optimal capital structure. We will highlight the interaction between informal guarantees and positive ownership share in what follows, assuming again that cash flow distributions are equal across the two activities.

6.1 Unenforceable conditional guarantees

Table 6 reports numerical results when the probability, π, that the guarantee is honoured ex post is 0.5 instead of 1, which is the value maintained so far in our analysis and implicit in Tables 2 to 5.[19]

Broadly speaking, the observations concerning the comparison between stand alone units and HS hold true (and carry over to other positive values of the probability π). The subsidiary is still able to raise a larger amount of debt (69) relative to its stand alone counterpart (57.2). This is because, at 57.2, the marginal tax benefit of debt is equal across the two organizations while the expected marginal bankruptcy cost is still lower in the S due to the guarantee issued by the parent. Total debt capacity also increases in a HS (123) relative to the case of two stand alone firms (114.4) thanks to its lower default probability at a level of debt equal to 57.2. This still allows to obtain proceeds from a group sale (163.56) higher than those from the sale of two stand alone firms (162.46).

However, total debt capacity is drastically smaller than in the contractual case (162.56 instead of 219) and its distribution in the holding and in the subsidiary is more balanced (55 as opposed to zero; 69 as opposed to 219), due to the uncertainty regarding the actual rescue. Correspondingly, total value falls from 165.9 to 163.23 as the guarantee becomes informal. The holding now keeps more debt than in the contractual case because lenders would charge too high a spread if the entrepreneur shifted all HS debt in the subsidiary. This may explain why both holding and subsidiaries raise debt in Belgian and Italian groups (Dewaelheyns and Van Hulle, 2007; Bianco and Nicodano, 2006).

Thanks to its lower debt and the lower spreads associated with the guarantee, now the subsidiary equity prices are higher (41.01) than in the stand alone company. It is still the case, however, that average equity prices in group affliated firms (35.98) fall short of stand-alone equity valuation (39.1), because share prices in the holding (30.95) are now *lower* than in the stand alone firms (39.1). This is due to the combined effect of the guarantee, that implies a transfer from holding sharehold-

[19] We do not allow π to change with corporate organization, as modelled by Inderst and Mueller (2003).

ers to subsidiary lenders, and of its relatively high level of debt (53). Such equity discounts in the holding company are often documented in the literature (Khanna and Yafeh, 2007), and they are interpreted as the outcome of inefficiencies or moral hazard in complex organizations. Here, lower equity valuations in the holding are the counterpart of higher obligations vis-à-vis both its own and its subsidiary lenders, absent any inefficiency.

6.2 Capital structure and intercorporate dividends

When the holding keeps a fraction $0 < \omega < 1$ of the subsidiary equity, the face values of debt in the two companies maximize the proceeds from the sale of the HS structure, less the sum kept by the holding, ωE_{0s}:

$$\nu_{0g} = \nu_0(P_h, P_s) = E_{0h} + D_{0h} + (1-\omega)E_{0s} + D_{0s} \tag{51}$$

Thus $\omega = 1$ corresponds to a wholly owned subsidiary, while $\omega = 0$ is the case we analyzed above, namely extreme separation between ownership and control.

The holding is entitled to a share ω of the earnings after interest and taxes, which may allow H to avoid default in some states. Thus dividends function as another type of conditional guarantee, in favour of the holding. Appendix C characterizes the state contingent flow of dividends to the holding, as well as its new default thresholds.

Unreported results for $\omega = 0.5$ and $\omega = 1$ show that - when the guarantee is contractual ($\pi = 1$)- ownership and the associated dividends do not affect at all optimal group capital structure, as the highly levered subsidiary pays negligible dividends. Thus, state contingent transfers like dividends - that may support a defaulting company but are not targeted to that purpose - do not alter equilibrium tax savings and default costs. Results on comparative value of groups, which are summarized in Proposition 2, carry over to this generalized case. Ownership levels matter for firm value when incentive problems plague relationships between majority and minority shareholders (see Almeida and Wolfenzon, 2006, among others). In our set up without agency problems, ownership levels are irrelevant for group leverage and value unless the guarantee is informal.

When the guarantee is informal and $\pi = 0.5$, Table 7 reveals that the larger is ω, the larger is total group value. For $\rho = 0.2$, total value grows from 162.56 (for $\omega = 0$) to 162.93 (for $\omega = 1$). This value gain stems from a reduction in the probability that the holding defaults, at any given level of its debt, thanks to the transfer from its subsidiary. Consistent with this conjecture, we observe an increase in the optimal face value of debt in the holding - which restores equality between the marginal tax benefit and the marginal bankruptcy cost. P_h now ranges from 53 at $\omega = 0$ to 81 at $\omega = 1$.

Total group debt increases at a slower pace (from 123 to 133), as the level of debt in the subsidiary falls (from 70 to 52). This reduction, in turn, stems from the higher holding leverage which reduces both its net cash flow after interest and its ability to rescue its subsidiary.

The behavior of tax savings, default costs and equity prices for the subsidiary are non-linear. However, tax savings in S are lower at $\omega = 1$ than at $\omega = 0$ because of decreasing debt which translates in decreasing deductible interest expenses. This holds true for all values of ρ. The opposite occurs for tax savings in the holding.

Perhaps counterintuively, default costs in the holding are higher at $\omega = 1$ than at $\omega = 0$ for $\rho = 0.8$. This is because the holding has higher debt and, at the same time, dividends are almost useless for avoiding insolvency - both firms being profitable in similar contingencies. On the contrary, default costs in the holding are lower at $\omega = 1$ than at $\omega = 0$ for $\rho = -0.8$. In other words, the HS structure becomes more similar - in terms of capital structure - to a conglomerate when $\omega > 0$ and $\pi < 1$. This implies that the gains from debt diversity, that show up in Tables 2 through 5, are reduced and the typical reasoning relating to diversification dominates again.

Unreported results show that conglomerate value (163.15) exceeds HS value in the base case, for $\pi = 0.1$. The optimal HS debt (116) is now lower than the merger debt (117.4), as lenders - anticipating reduced - charge higher spreads both to the holding (1.22%) and to the subsidiary (1.24%) relative to the merger (0.6%). The tax burden is almost equal (35.31 versus 35.61) but the expected default cost are higher in HS (1.81 versus 1.24) because the unconditional guarantee always works while the conditional one is less reliable.

We can summarize these numerical findings as follows.

Proposition 8 *Assume a BBB calibrated company. (a)Let $\pi = 1$. Then the entrepreneur always prefers HS over SA and M. Moreover, HS value is independent of ω and achieves its maximum value for $\rho = 1$.*

(b)Now let $0 < \pi < 1$. Then HS achieves its maximum value, which is decreasing in ρ, for $\omega = 1$. The entrepreneur always prefers HS over SA. There exists a $\pi'(\omega = 0, \rho)$ such that the entrepreneur prefers M to HS for all $\pi < \pi'$.

One last remark concerns the value of the holding company. Higher dividend transfers, which range from 0 to 38.97 as ω varies between 0 and 1, keeping $\rho = 0.2$, translate into higher equity value for the holding, which varies from 30.952 to 63.223. Observe however that the equity appreciation is less than one for one, because of the higher debt burden onto the holding shareholders. This opens up the possibility to explain the holding company discount (Cornell and Liu, 2001). Observe in fact that the value of equity in a stand alone is 39.1. One may expect the equity value of a firm that owns 100% of a clone to be twice as much, i.e. 78.2. We see instead that, in the base case $\rho = 0.2$, the value of the holding company with $\omega = 1$ is only 63.22, only 1.61 higher than the capitalization of one stand-alone. The comparison fares even worse when the benchmark is the value of the subsidiary equity, which is 42.39, leading to a ratio of 1.49 instead of 2. Such "holding company discount" gets worse for more extreme cash flow correlations. The relative values of equity in the holding and in the subsidiary, given 100% ownership, is 1.3 for $\rho = 0.8$ but falls to 0.88 when $\rho = -0.8$.

7 Concluding comments

This paper contributes to our understanding of firm scope, by clarifying the role of guarantees in determining debt and value creation. It shows that holding- subsidiary structures, which allow for the provision of a conditional guarantee, are value maximizing arrangements in environments where bankruptcy costs and taxation make firm debt policy relevant. The value of conditionality, which is lost in conglomerates, proves especially valuable in contexts with asymmetric cash flows deriving from the two activities. This sheds some light on the reason why similar structures are common in innovative industries, where the risk of subsidiary activities is large relative to that of holding companies.

The paper also shows that the value of a conditional guarantee is enhanced by the possibility of choosing diverse levels of debt in the affiliated activities, especially when diversification opportunities vanish. However, a crucial determinant of capital structure and associated value gains turns out to be the probability that the lenders attribute to ex post rescue. When this is low, an unconditional guarantee provided by a merger may prove superior to a conditional one provided by a HS structure.

Our pricing model clarifies the effects of dividends and guarantees on the value of debt and equity of affiliated companies- albeit in a simplified setting without agency problems. This allows to shed some light on some pricing puzzles, such as the holding company discount. When lenders do not fully trust the guarantee pledged by the holding, larger dividends from the subsidiary to the holding are associated with lower equity value - because its optimal debt increases with dividend transfers. Similarly, our pricing model explains the lower equity value of HS structure relative to SA, when the firms have equal cash flows, in a no-arbitrage setting. Finally, it paves the way for predicting the value effects of corporate restructurings that generate HS, such as carve-outs.

We leave several other developments for further work. A relevant one deals with the welfare properties of different organizations: the HS structure, while being value maximizing, appears to incur into too large bankruptcy costs, beside deriving value gains from tax avoidance. A second extension may assess whether accounting for HS structures may contribute to the solution of capital structure puzzles. Last but not least, this model may turn out to be useful to the understanding of other institutions which are characterized by separate incorporation and yet linked by an internal capital market, such as financial conglomerates and project financing.

Appendix A

This appendix studies the stand alone firm optimization problem, namely

$$\max_{P_i} \nu_{0i} = \max_{P_i} \left[V_{0i} + T_i(0) - T_i(P_i) - DC_i(P_i)\right]$$

- when $i = 1, 2$ - through its equivalent problem, namely

$$\min_{P_i} \left[T_i(P_i) + DC_i(P_i)\right]$$

The Kuhn-Tucker (KT) necessary conditions for such a problem are

$$\begin{cases} \frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i} \geq 0 \\ P_i^* \geq 0 \\ \left[\frac{dT_i(P_i^*)}{dP_i} + \frac{dDC_i(P_i^*)}{dP_i}\right] P_i^* = 0 \end{cases} \tag{52}$$

The derivative of tax burdens and default costs is

$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} =$$

$$= -\tau_i(1 - F_i(X_i^d)) \left[1 - \frac{dD_{0i}(P_i)}{dP_i}\right] \phi +$$
$$+ \alpha X_i^d f_i(X_i^d) \left[1 + \frac{\tau_i}{1 - \tau_i} \frac{dD_{0i}(P_i)}{dP_i}\right] \phi$$

where f_i is the density of X_i. If no taxes exist ($\tau_i = 0$), then, considering that $X_i^d = 0$ if and only if $P_i = 0$,

$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} > 0 \qquad \text{when } P_i > 0$$
$$\frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} = 0 \qquad \text{when } P_i = 0$$

so that at the origin the KT conditions are all satisfied as equalities and a minimum can exist. If taxes exist ($\tau_i > 0$), then a minimum at the origin cannot exist, since

$$\frac{dT_i(0)}{dP_i} + \frac{dDC_i(0)}{dP_i} = \tag{53}$$

$$= -\tau_i(1 - F_i(0)) \left[1 - \frac{dD_{0i}(0)}{dP_i}\right] \phi < 0$$

If the tax rate tends to 100% ($\tau_i \to 1$), the necessary conditions are satisfied by finite P_i and D_i. Indeed, even for finite debt X_i^d diverges (with order of infinity 1) when τ_i goes to one and

- the existence of the first moment for X_i entails that, even if X_i^d diverges, $X_i^d f_i(X_i^d)$ converges to zero with order of infinitesimal greater than one with respect to X_i^d, and therefore with respect to $1 - \tau_i$;

- at the same time, the ratio $\frac{\tau_i}{1-\tau_i}$- and consequently $\frac{\tau_i}{1-\tau_i}\frac{dD_{0i}(P_i)}{dP_i}$- diverges with order of infinity one.

It follows that

$$\lim_{\tau_i \to 1} X_i^d f_i(X_i^d) \left[1 + \frac{\tau_i}{1-\tau_i}\frac{dD_{0i}(P_i)}{dP_i}\right] = 0 \tag{54}$$

holds. As a consequence

$$\lim_{\tau_i \to 1} \frac{dT_i(P_i)}{dP_i} + \frac{dDC_i(P_i)}{dP_i} = \lim_{\tau_i \to 1} -\tau_i(1 - F_i(X_i^d)) \left[1 - \frac{dD_{0i}(P_i)}{dP_i}\right] \phi$$

and the latter limit is zero, since

$$\lim_{\tau_i \to 1} F_i(X_i^d) = \lim_{X_i^d \to +\infty} F_i(X_i^d) = 1$$

As soon as $TS_i - DC_i$ is concave in $P_i \geq 0$, $T_i + DC_i$ is convex in $P_i \geq 0$, and the above necessary conditions are also sufficient.

8 Appendix B

In order to demonstrate the main theorem, we first introduce a lemma, which characterizes the guarantee G. Notice that, letting $f(x,y)$ be the joint density of the cash flows (X_s, X_h), we can write the guarantee G as

$$G(P_s, P_h) = \alpha\phi \int_0^{X_s^d} \int_{h(x)}^{+\infty} xf(x,y)dxdy =$$

$$= \alpha\phi \left[\int_0^{X_s^Z} x \int_{X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}}^{+\infty} f(x,y)dydx + \int_{X_s^Z}^{X_s^d} x \int_{X_h^d + X_s^d - x}^{+\infty} f(x,y)dydx \right]$$

Lemma 9 *If proportional bankrupcty costs are positive ($\alpha > 0$), then a) the guarantee is non increasing in P_h :*

$$\frac{\partial G(P_h, P_s)}{\partial P_h} \leq 0$$

and has a null derivative if and only if $P_s = 0$:

$$\frac{\partial G(P_h, 0)}{\partial P_h} = 0$$

b) the guarantee has a null derivative wrt P_s at $P_s = 0$; c) Under a technical convergence condition for $f(x,y)$, the guarantee is decreasing in P_s when the latter diverges:

$$\lim_{P_s \to +\infty} \frac{\partial G(P_h, P_s)}{\partial P_s} < 0$$

Proof. Part (a) follows from the fact that

$$\frac{\partial G}{\partial P_h} = -\alpha\phi \times$$

$$\times \left[\int_0^{X_s^Z} xf\left(x, X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}\right) dx + \int_{X_s^Z}^{X_s^d} xf(x, X_h^d + X_s^d - x)dx \right] \times$$

$$\times \left[1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} \right] \leq 0 \tag{55}$$

since - according to (19)- we have

$$1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} > 0$$

Equality in (55) holds if and only if

$$X_s^d = X_s^Z = 0$$

which in turn happens if and only if $P_s = 0$. As concerns part (b), we compute:

$$\frac{\partial G}{\partial P_s} = \alpha\phi\times \tag{56}$$

$$\times\left\{\left[-\frac{1}{1-\tau}\int_0^{X_s^Z} xf\left(x, X_h^d+\frac{P_s}{1-\tau}-\frac{x}{1-\tau}\right)dx - \frac{dX_2^d}{dP_2}\int_{X_s^Z}^{X_s^d} xf(x, X_h^d+X_s^d-x)dx\right]+\right.$$

$$+\frac{dX_2^d}{dP_2}\int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y)dy =$$

$$= \frac{\alpha}{(1-\tau)(1+r_T)}\times\left\{-\int_0^{X_s^Z} xf\left(x, X_h^d+\frac{P_s}{1-\tau}-\frac{x}{1-\tau}\right)dx+\right.$$

$$\left.+\left(1-\tau+\tau\frac{dD_{02}(P_2)}{dP_2}\right)\left[-\int_{X_s^Z}^{X_s^d} xf(x, X_h^d+X_s^d-x)dx+\int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y)dy\right]\right\}$$

When $P_s = 0$, then $X_s^d = X_s^Z = 0$, all the integrals vanish and the previous derivative is null.
As concerns part (c), when $P_s \to +\infty$, definition (12) implies that

$$\lim_{P_s\to+\infty} X_s^d = +\infty$$

For fixed y, the convergence condition

$$\lim_{x\to+\infty} xf(x,y) = 0 \tag{57}$$

- which follows from the fact that f is a density - implies that, for any sequence x_n which goes to $+\infty$, then

$$f_n(y) := x_n f(x_n, y)$$

converges to zero. Let us suppose that the function $f_n(y)$ satisfies the dominated convergence property (this is the technical condition in the statement of the lemma). This allows us to exchange integration and limit:

$$\lim_{n\to+\infty}\int_{X_h^d}^{+\infty} x_n f(x_n, y)dy =$$

$$= \lim_{n\to+\infty}\int_{X_h^d}^{+\infty} f_n(y)dy =$$

$$= \int_{X_h^d}^{+\infty}\lim_{n\to+\infty} f_n(y)dy = 0$$

and, as a consequence,

$$\lim_{X_s^d \to +\infty} \int_{X_h^d}^{+\infty} X_s^d f(X_s^d, y) dy = 0$$

Together with (19) this entails

$$\lim_{P_s \to +\infty} \frac{\partial G}{\partial P_s} =$$

$$= \lim_{P_s \to +\infty} \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ -\int_0^{X_s^Z} xf\left(x, X_h^d + \frac{P_s}{1-\tau} - \frac{x}{1-\tau}\right) dx - \right.$$

$$\left. - \left(1 - \tau + \tau \frac{dD_{02}(P_2)}{dP_2}\right) \int_{X_s^Z}^{X_s^d} xf(x, X_h^d + X_s^d - x) dx \right\} < 0$$

and proves part (c). ■

We are now ready for the proof of the main theorem.

Proof. Part a): let us examine the Kuhn Tucker necessary conditions

for a minimum of $T_{HS} + DC_{HS}$ with respect to the subsidiary debt, considering that the latter cannot be negative.

$$\left\{ \begin{array}{c} \frac{\partial T_{HS}(P_h^*, P_s^*)}{\partial P_s} + \frac{\partial DC_{HS}(P_h^*, P_s^*)}{\partial P_s} = \frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_s} \geq 0 \\ P_s^* \geq 0 \\ \left[\frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*, P_h^*)}{\partial P_s} \right] P_s^* = 0 \end{array} \right. \tag{58}$$

As a consequence of part (b) of the previous lemma, when $P_s^* = 0$ we have

$$\frac{dT_2(0)}{dP_s} + \frac{dDC_2(0)}{dP_s} - \frac{\partial G(P_h^*, 0)}{\partial P_s} = \tag{59}$$

$$= \frac{dT_2(0)}{dP_s} + \frac{dDC_2(0)}{dP_s}$$

The derivative of tax burdens and default costs wrt the subsidiary's debt is

$$\frac{dT_2(P_s)}{dP_s} + \frac{dDC_2(P_s)}{dP_s} =$$

$$= -\tau(1 - F_2(X_s^d)) \left[1 - \frac{dD_{02}(P_s)}{dP_2}\right] \phi +$$

$$+ \alpha X_s^d f_2(X_s^d) \left[1 + \frac{\tau}{1-\tau} \frac{dD_{02}(P_s)}{dP_2}\right] \phi$$

where F_2 and f_2 are respectively the distribution and density functions of $X_2 = X_s$.Since $X_s^d = 0$ when $P_2 = 0$, its value with zero debt is

$$\frac{dT_2(0)}{dP_s} + \frac{dDC_2(0)}{dP_s} =$$

$$= -\tau(1 - F_2(0)) \left[1 - \frac{dD_{02}}{dP_2} |_{P_2=0} \right] \phi$$

which is negative, since $F_2(0) < 1$ and

$$\lim_{P_2 \to 0+} \frac{dD_{02}}{dP_2} < 1 \tag{60}$$

by Conjecture 1.
This concludes the proof of part a) of the theorem.
Let us consider now part (b) of the theorem. In order to demonstrate it, we can write down the KT conditions for a minimum of $T_{HS} + DC_{HS}$ under non negativity of debt for both the holding and the subsidiary, as well as under the constraint

$$P_h^* + P_s^* \geq P_1^* + P_2^* := K \tag{61}$$

They are

$$\begin{cases} \frac{\partial T_{HS}(P_h^*, P_s^*)}{\partial P_h} + \frac{\partial DC_{HS}(P_h^*, P_s^*)}{\partial P_h} = \frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_h} = \mu_1 + \mu_3 (i) \\ P_h^* \geq 0 \qquad (ii) \\ \mu_1 P_h^* = 0 \qquad (iii) \\ \frac{\partial T_{HS}(P_h^*, P_s^*)}{\partial P_s} + \frac{\partial DC_{HS}(P_h^*, P_s^*)}{\partial P_s} = \frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_s} = \mu_2 + \mu_3 (iv) \\ P_s^* \geq 0 \qquad (v) \\ \mu_2 P_s^* = 0 \qquad (vi) \\ P_h^* + P_s^* \geq K \qquad (vii) \\ \mu_3 (P_h^* + P_s^* - K) = 0 \qquad (viii) \\ \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0 \qquad (ix) \end{cases} \tag{62}$$

We want to demonstrate that - under the stated conditions - there exists a point $(0, P_s)$, with $P_s > K$, which solves them. We are then interested in a solution for which the constraint (61) is not binding: $\mu_3 = 0$. Consider first that, if $P_h = 0$ and $\mu_3 = 0$, condition (i) is satisfied, provided that we choose $\mu_1 > 0$ such that

$$\frac{dT_1(0)}{dP_h} + \frac{dDC_1(0)}{dP_h} - \frac{\partial G(0, P_s^*)}{\partial P_h} = \mu_1$$

We know that the first two addenda in the left hand side sum to zero, while the third is negative by part a) of the lemma. Thus $\mu_1 > 0$ and conditions (i, ii, iii) - including the complementary slackness for P_h and its multiplier- are satisfied. If later we choose $P_s > K$, also conditions (v, vi, vii) are satisfied, provided that we select $\mu_2 = 0$. Then condition (iv) becomes

$$\frac{dT_2(P_s^*)}{dP_s} + \frac{dDC_2(P_s^*)}{dP_s} - \frac{\partial G(0, P_s^*)}{\partial P_s} = 0 \tag{63}$$

The left hand side of such equality represents the partial derivative of the objective function wrt P_s, evaluated at $(0, P_s^*)$, and its first two addenda are the derivative of the stand-alone tax burdens and default costs, $T_2 + DC_2$, at P_s^*. Considered as a function of the subsidiary's debt, they vanish at $P_s = P_2^*$ and their sum is positive when $P_s > P_2^*$. Evaluate the left hand side at $P_s = P_1^* + P_2^*$ and denote it as $h(\tau)$

$$\begin{aligned}
h(\tau) &:= \frac{dT_2(P_1^* + P_2^*)}{dP_s} + \frac{dDC_2(P_1^* + P_2^*)}{dP_s} - \frac{\partial G(0, P_1^* + P_2^*)}{\partial P_s} = \\
&= \frac{1}{(1 + r_T)} \left\{ -\tau(1 - F_2(X_s^{d**})) \left[1 - \frac{dD_{02}(P_1^* + P_2^*)}{dP_2} \right] + \right. \\
&\quad + \alpha X_s^{d**} f_2(X_s^{d**}) \left[1 + \frac{\tau}{1-\tau} \frac{dD_{02}(P_1^* + P_2^*)}{dP_2} \right] + \\
&\quad + \frac{\alpha}{(1-\tau)} \times \left[- \int_0^{X_s^{Z**}} x f\left(x, \frac{P_1^* + P_2^*}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right. \\
&\quad + \left(1 - \tau + \tau \frac{dD_{02}(P_1^* + P_2^*)}{dP_2}\right) \Bigg] \times \\
&\quad \times \left. \left[- \int_{X_s^{Z**}}^{X_s^{d**}} x f(x, X_s^{d**} - x) dx + \int_0^{+\infty} X_s^{d**} f(X_s^{d**}, y) dy \right] \Bigg] \right\}
\end{aligned}$$

where X_s^{d**} and X_s^{Z**} are the default and tax shield thresholds corresponding to $P_s = P_1^* + P_2^*$. The function h vanishes with taxes, since, as demonstrated in Appendix A,

$$\begin{aligned}
\lim_{\tau \to 0+} P_i^* &= 0+ \qquad\qquad i = 1, 2 \\
+ \lim_{\tau \to 0+} X_s^{d**} &= \lim_{\tau \to 0+} X_s^{Z**} = 0 \\
\lim_{\tau \to 0+} F_2(X_s^{d**}) &= \lim_{X_s^{d**} \to 0} F_2(X_s^{d**}) < 1
\end{aligned}$$

$$\lim_{\tau \to 0+} h(\tau) =$$

$$= \lim_{X_s^{d**}\to 0} \alpha \left\{ -\int_0^{X_s^{Z**}} xf\left(x, \frac{P_1^*+P_2^*}{1-\tau} - \frac{x}{1-\tau}\right)dx + \left(1-\tau+\tau\frac{dD_{02}(P_1^*+P_2^*)}{dP_2}\right) - \left[-\int_{X_s^{Z**}}^{X_s^{d**}} xf(x, X_s^{d**}-x)dx + \int_0^{+\infty} X_s^{d**} f(X_s^{d**}, y)dy \right] \right\} = 0$$

while, when the tax rate increases ($\tau \to 1-$), then the limit (54), together with dominated convergence, guarantees that

$$\lim_{\tau\to 1-} h(\tau) = \lim_{\tau\to 1-} \frac{\alpha}{(1-\tau)(1+r_T)} \left\{ -\int_0^{X_s^{Z**}} xf\left(x, \frac{P_1^*+P_2^*}{1-\tau} - \frac{x}{1-\tau}\right)dx + \right.$$

$$\left. + \left(1-\tau+\tau\frac{dD_{02}(P_1^*+P_2^*)}{dP_2}\right) \left[-\int_{X_s^{Z**}}^{X_s^{d**}} xf(x, X_s^{d**}-x)dx + \int_0^{+\infty} X_s^{d**} f(X_s^{d**}, y)dy \right] \right\} =$$

$$= \lim_{\tau\to 1-} \frac{\alpha}{(1-\tau)(1+r_T)} \left\{ -\int_0^{X_s^{Z**}} xf\left(x, \frac{P_1^*+P_2^*}{1-\tau} - \frac{x}{1-\tau}\right)dx + \right.$$

$$\left. + \frac{dD_{02}(P_1^*+P_2^*)}{dP_2} \left[-\int_{X_s^{Z**}}^{X_s^{d**}} xf(x, X_s^{d**}-x)dx \right] \right\} = -\infty$$

Denote as τ_0 the smallest tax rate level such that $h(\tau) < 0$. We know from the above that $\tau_0 \geq 0$. Notice also that the whole left hand side of (63) is positive, for no matter which tax rate, when the subsidiary debt diverges, namely

$$\lim_{P_s\to +\infty} \left(\frac{dT_2(P_s)}{dP_s} + \frac{dDC_2(P_s)}{dP_s} - \frac{\partial G(0, P_s)}{\partial P_s} \right) > 0$$

Such limit behavior follows from the previous lemma, together with ii) above.

Since such left hand side can be easily shown to be continous, it follows that for any $\tau \geq \tau_0$ there exists a point $P_2^*(\tau_0) > P_1^* + P_2^*$ where it vanishes, as needed by (iv). All the KT conditions are then satisfied at $P_2^*(\tau_0)$. Under the assumed convexity of the objective function, the program is concave and such conditions are also sufficient for a minimum. This proves part (b). ■

One can wonder whether the results in the previous theorem depend on Conjecture 1. The following corollary asserts that absence of leverage for the subsidiary holds even without Conjecture 1.

Corollary 10 *Assume proportional bankruptcy costs are positive ($\alpha > 0$). Then, even if Conjecture 1 does not hold and*

$$\lim_{P_i\to 0+} \frac{dD_{0i}}{dP_i} = 1 \qquad\qquad i = 1, 2$$

there cannot be a local minimum in which the subsidiary is unlevered and the group optimal value is greater than the one of two stand alone companies.

Proof. If Conjecture 1 does not hold and

$$\lim_{P_2 \to 0+} \frac{dD_{02}}{dP_2} = 1, \tag{64}$$

then the first KT necessary condition (58) is satisfied for a null subsidiary debt, $P_s = 0$, as an equality. The other KT conditions for P_s are satisfied too. In this case we should consider also the necessary conditions for the holding debt, namely

$$\begin{cases} \frac{\partial T_{HS}(P_h^*,P_s^*)}{\partial P_h} + \frac{\partial DC_{HS}(P_h^*,P_s^*)}{\partial P_h} = \frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*,P_s^*)}{\partial P_h} \geq 0 \\ P_h^* \geq 0 \\ \left[\frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*,P_h^*)}{\partial P_h}\right] P_h^* = 0 \end{cases} \tag{65}$$

The first of these conditions can be written as

$$\begin{gathered} \frac{dT_1(P_h^*)}{dP_h} + \frac{dDC_1(P_h^*)}{dP_h} - \frac{\partial G(P_h^*, P_s^*)}{\partial P_h} = \\ = -\tau(1 - F_1(X_h^{d*})) \left[1 - \frac{dD_{01}(P_1)}{dP_1} |_{P_1 = P_h^*}\right] \phi + \\ + \alpha X_h^{d*} f_1(X_h^{d*}) \left[1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} |_{P_1 = P_h^*}\right] \phi \times \\ + \alpha\phi \times \\ \times \left[\int_0^{X_s^{Z*}} xf\left(x, h(x)\right) dx + \int_{X_s^{Z*}}^{X_s^{d*}} xf(x, h(x)) dx\right] \times \\ \times \left[1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} |_{P_1 = P_h^*}\right] \geq 0 \end{gathered} \tag{66}$$

where F_1 and f_1 are respectively the distribution and density functions of X_h, and X_i^{d*} is the default threshold corresponding to $P_i^{*\cdot}, i = h, s$. When it is evaluated at $P_s^* = 0$, it reduces to

$$\begin{gathered} -\tau(1 - F_1(X_h^{d*})) \left[1 - \frac{dD_{01}(P_1)}{dP_1} |_{P_1 = P_h^*}\right] \phi + \\ + \alpha X_h^{d*} f_1(X_h^{d*}) \left[1 + \frac{\tau}{1-\tau} \frac{dD_{01}(P_1)}{dP_1} |_{P_1 = P_h^*}\right] \phi \geq 0 \end{gathered} \tag{67}$$

since the integrals in the last addendum vanish. Such condition can be satisfied by $P_h^{*\cdot} = 0$ and is sastisfied by $P_h^{*\cdot} = P_1^{*\cdot}$. Consider $P_h^{*\cdot} = 0$: the left hand side of (67) is null when $P_h^* = 0$, if, by similarity to the subsidiary case, Conjecture 1 is violated and

$$\lim_{P_1 \to 0+} \frac{dD_{01}}{dP_1} = 1 \tag{68}$$

In such case (67) is sastified as an equality. It is satisfied as an equality also by $P_h^* = P_1^*$, since it is the necessary condition for an interior minimum of tax burdens and default costs in firm 1. It follows that all the KT conditions (for the subsidiary and the holding) are satisfied at $(0, P_1^*)$ if and only if (64) holds, and at the origin $(0, 0)$ if and only if (64) and (68) hold. However, even if the points $(0, 0)$ and $(0, P_1^*)$ satisfy the Kuhn-Tucker conditions, and even if they are minima, they are dominated ones. Indeed, since no guarantee can be offered if the subsidiary is unlevered, they both correspond to couples of stand alone companies: the first point corresponds to two unlevered stand alone firms, the second to an unlevered firm with the subsidiary's cash flows and to an optimally levered stand alone with the holding's cash flows. For given cash flows, both would be dominated, in the sense of producing a smaller overall value, by a couple of optimally levered stand alone firms. This completes the proof of the corollary. ■

Appendix C - Groups with finite ownership

Let ω be the ownership share of the holding in the subsidiary ($\omega \in (0, 1]$). The cash flows of the holding must be augmented for dividends, net of intercorporate taxation. If there is no such taxation, these cash flows are

$$X_h^n + \omega(X_s^n - P_s)^+ \tag{69}$$

Observe that dividends are zero when the subsidiary defaults, i.e. when $X_s < X_s^d$, or equivalently $X_s^n < P_s$. If taxation exists, we distinguish between four contingencies, depending on whether dividends are smaller or greater than the tax shield and on whether they are being paid out or not:

$$\begin{cases} X_h & X_s^n < P_s, X_h < X_h^Z \\ X_h - \tau(X_h - X_h^Z) & X_s^n < P_s, X_h > X_h^Z \\ X_h + \omega\left[X_s - \tau(X_s - X_s^Z) - P_s\right] & X_s^n > P_s, X_h < X_h^Z \\ X_h - \tau(X_h - X_h^Z) + \omega\left[X_s - \tau(X_s - X_s^Z) - P_s\right] & X_s^n > P_s, \\ & X_h > X_h^Z \end{cases} \tag{70}$$

The payoff to H lenders is accordingly equal to:

$$\begin{cases} 0 & X_h^n + \omega(X_s^n - P_s)^+ < 0 \\ (1-\alpha)\left[X_h^n + w(X_s^n - P_s)^+\right] & 0 < X_h^n + \omega(X_s^n - P_s)^+ < P_h \\ P_h & X_h^n + \omega(X_s^n - P_s)^+ > P_h \end{cases} \quad (71)$$

In the first case cash flows, gross of any dividends, are negative. Therefore lenders lose all their capital. In the second case, the holding defaults and lenders receive all cash flows by absolute priority. In the last case, the holding is solvent.

When dividends are paid out, the holding default threshold, $\bar{X}_h^d$, depends on the subsidiary cash flow X_s. It is the level of operating cash flows, net of taxes but gross of dividends, that equals P_h:

$$\bar{X}_h^d - \tau(\bar{X}_h^d - X_h^Z)^+ + \omega(X_s - \tau(X_s - X_s^Z) - P_s) = P_h \quad (72)$$

As a result, the holding defaults with the following combinations of $\{X_s, X_h\}$, given $\{P_s, P_h\}$:

$$\begin{cases} X_h < X_h^d,\ X_s < X_s^d \\ X_h^Z < X_h < X_h^d,\ X_s^d < X_s < X_s^d - (X_h - X_h^d)/\omega \\ 0 < X_h < X_h^Z,\ X_s^d < X_s < X_s^d - \frac{X_h - P_h}{\omega(1-\tau)} \end{cases} \quad (73)$$

We visualize the default threshold and the lenders' payoff in Figure 4.

Insert here Figure 4

The holding shareholders receive no dividend below the default threshold. Above it, they receive cash flows gross of any dividends.

The new value of the holding equity and debt value obtain by discounting the expectation of these new cash flows to shareholders and lenders, respectively. The problem is complicated by the fact that they now depend on the face value of the subsidiary debt. The values of subsidiary debt and equity are instead unaffected by the payment of dividends to the holding: therefore, they can be represented as in (16) and (14).

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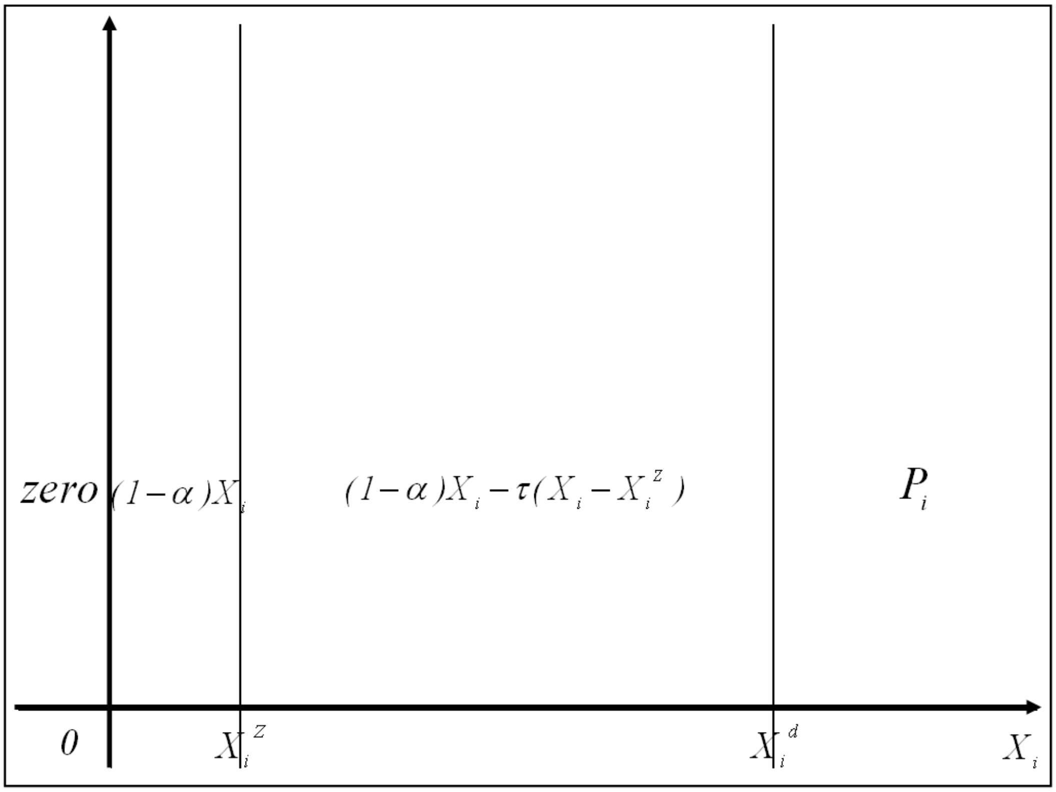

Figure 1: This figure represents the payoff to the lenders of a stand alone company. Lenders are fully reimbursed (P_i) when cash flow X_i exceeds the default threshold X_i^d. Otherwise, they receive a fraction $(1-\alpha)$ of positive cash flows. However, when the stand alone defaults and its cash flow exceeds the no tax level X_i^z, debtholders pay taxes on top of bankruptcy costs.

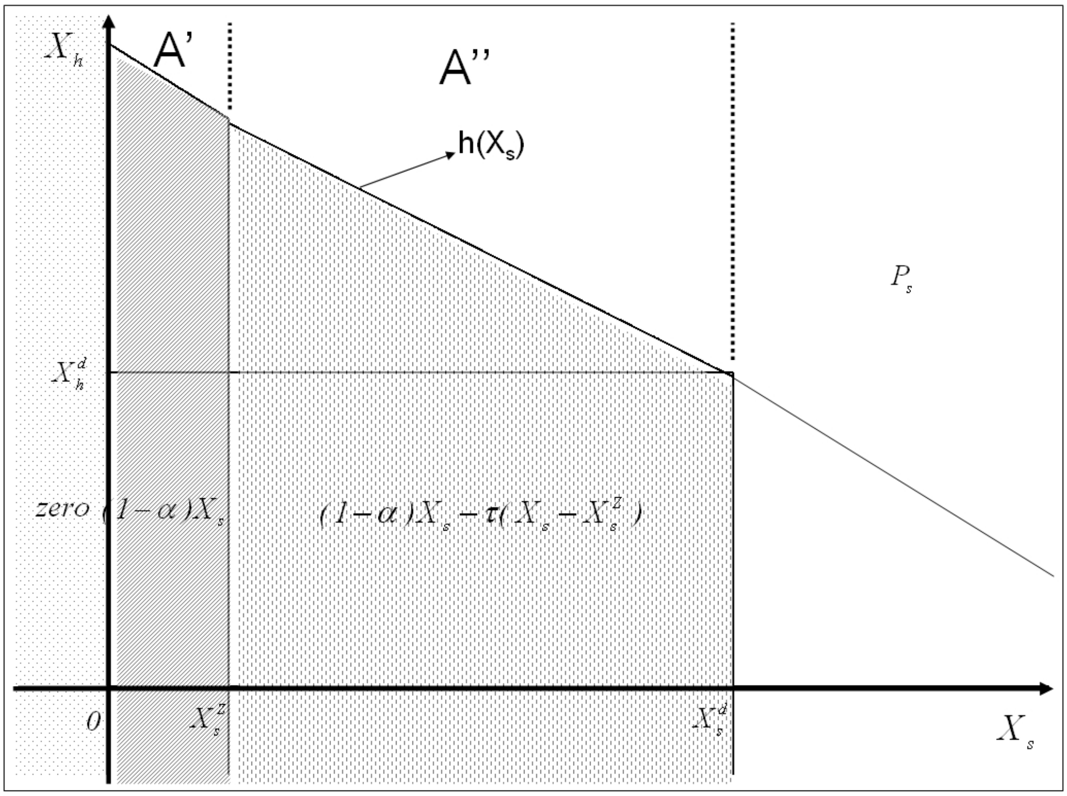

Figure 2: This figure represents the payoffs to subsidiary lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of infinitesimal ownership share. The figure reproduces the one for stand alone firms when X_h is lower than the holding default threshold X_h^d, as in this case the holding is unable to help its subisdiary. The area of the transfer is $A = A' \cup A''$ and is bounded by the linear function $h(X_s)$. In A'', the subsidiary does not default thanks to the transfer, but it pays taxes. In A', the subsidiary saves on both default costs and taxes thanks to the transfer. The payoff $(1-\alpha)X_s$ applies to the whole dark grey zone, while the payoff $(1-\alpha)X_s - \tau(X_s - X_s^Z)$ applies to the whole pale grey zone.

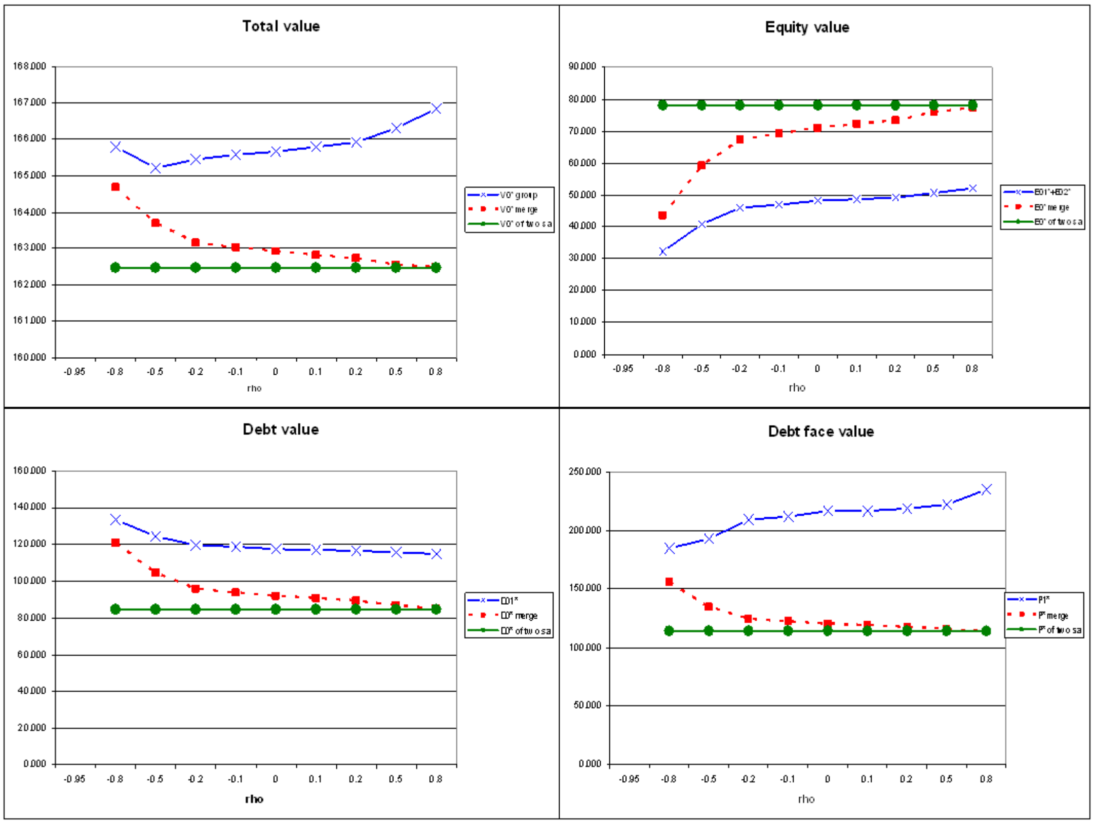

Figure 3: The upper left panel displays the value of an HS (stars), a conglomerate (big and small dots) and two stand alone firms (dotted) as the correlation coefficient between the activities cash glows varies between -0.8 and +0.8. Similarly, the upper right panel displays the value of equity, the lower left panel the market value of debt and the last one the face value of debt.

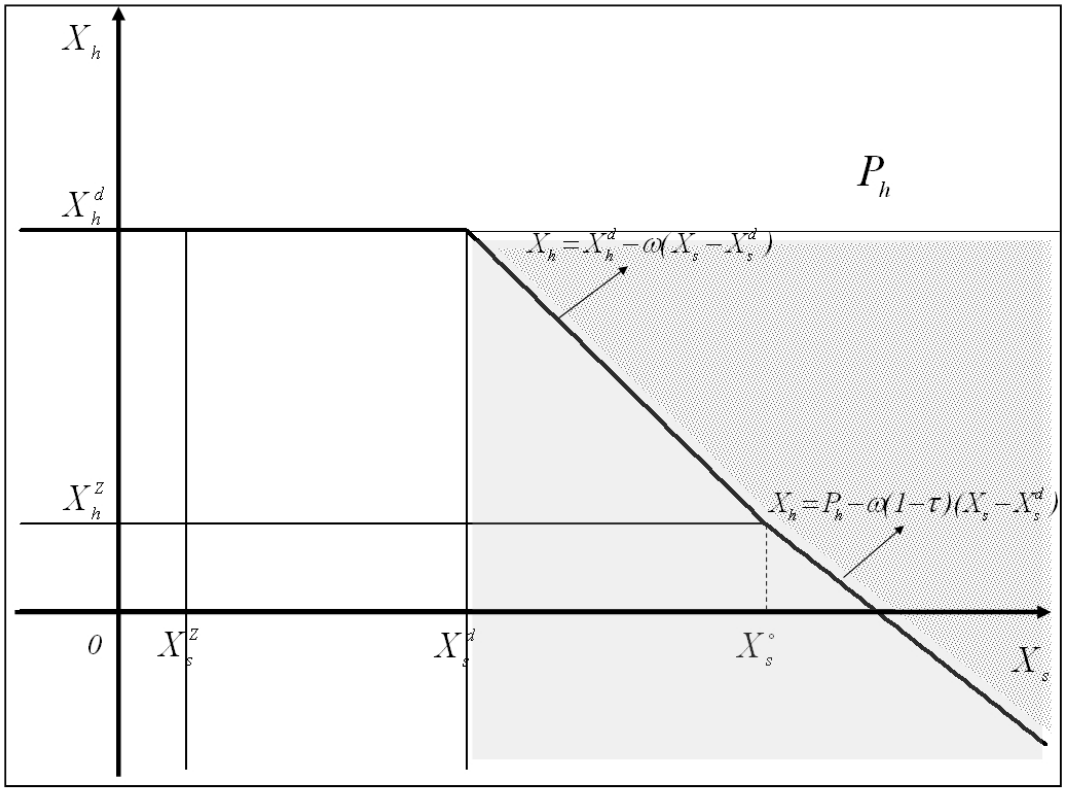

Figure 4: This figure represents the payoffs to holding lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of positive ownership share. Consider the shaded area where the holding would default ($X_h < X_h^d$) and the subsidiary does not default ($X_s < X_s^d$). The holding now receives dividends proportional to its ownership share, ω, that allow its survival in the shaded area with dots.

Table 1: Base Case Parameters

Variables	Symbols	Values
Annual Riskfree Rate	r	5.00%
Time Period/Debt Maturity (yrs)	T	5.00
T-period Riskfree Rate	$r_T = (1+r)^T - 1$	27.63%
Capitalization Factor	$Z = (1+r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected Future Operational Cash Flow at T	Mu	127.63
Expected Operational Cash Flow Value (PV)	$X_0 = Mu/(1+r)^T$	100.00
Cash Flow Volatility at T	Std	49.19
Annualized operating Cash Flow Volatility	$\sigma = Std/T^{0.5}$	22.00
Tax Rate	τ	20%
Value of Unlevered Firm w/Limited Liability	V_0	80.05
Value of Limited Liability	L_0	0.057

Table 2: Optimal Capital Structure and Value

	Symbols	Values				
		Stand Alone	Holding	Subsidiary	1/2 HS	1/2 Conglom
Default Costs	α	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	57.20	0	220	110	58.5
Default Threshold	X^{d*}	67.75	0	249.2663	-	69.64
No Tax Profit Level	X^{Z*}	14.98	0	102.93	-	13.94
Value of Optimal Debt	D_0^*	42.22	0	117.06	58.53	44.56
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99.9%	70.26%	54.62%
Annual Yield Spread of Debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.26%	//	8.45%	-	0.6%
Value of Optimal Equity	E_0^*	39.01	49.46	0.07	24.76	37.01
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.62	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.33	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.90	0	8.13	4.07	0.61
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	-30.59	37.08	3.24	1.57
Capitalized Value of Optimal Leverage	$Z(\nu_0^* - V_0)/V_0$	6. 81%	-1.77	2.14	20.23%	9.06%

Table 3: Asymmetric alfas: capital structure and value across organizational forms, $\rho = 0.2$

Variables	Symbols	Values					
		S. Alone	S. Alone	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	75%	23%	75%	23%	23%	75%
Optimal Face Value of Debt	P^*	33	57.20	0	219	109.5	46.5
Default Threshold	X^{d*}	39.247	67.75	0	248.17	-	55.43
No Tax Profit Level	X^{Z*}	8.01	14.98	0	102.32	-	10.79
Value of Optimal Debt	D_0^*	24.99	42.22	0	116.68	58.34	35.71
Value of Optimal Equity	E_0^*	55.84	39.01	49.2	0.037	24.62	45.52
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80.83	81.23	49.2	116.71	82.95	81.23
Optimal Leverage Ratio	D_0^*/ν_0^*	30.92%	52%	0	99.9%	70.3%	44%
Annual Yield Spread of Debt (%)	y	0.7%	1.26%	//	8.4%	-	0.4%
Tax Burden	T_0^*	18.76	17.62	19.95	5.42	12.69	18.31
Tax Savings of Leverage	TS_0^*	1.25	2.33	0	14.53	7.27	1.69
Expected Default Costs	DC_0^*	0.46	0.90	0	7.98	3.99	0.455
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.78	1.18	-	-	3.15	1.18
Capitalized Optimal Value of Leveraging	$Z(\nu_0^* - V_0)/V_0$	4.50%	6.81%			18.24%	6.81%

Table 4: Asymmetric volatilities: capital structure and value across organizational forms, $\rho = 0.2, \sigma_s = 44\%, \sigma_h = 22\%$

Variables	Symbols	Values					
		S.A.($\sigma = 44\%$)	S. A.($\sigma = 22\%$)	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	83	57.20	0	223	111.5	59
Default Threshold	X^{d*}	95.19	67.75	0	248.169	-	34.75
No Tax Profit Level	X^{Z*}	34.25	14.98	0	102.32	-	8.50
Value of Optimal Debt	D_0^*	48.75	42.22	0	106.83	53.41	41. 98
Value of Optimal Equity	E_0^*	36.10	39.01	60.29	3.01	31.65	39. 64
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	84.84	81.23	60.29	109.84	85.06	81. 62
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	52%	0	97.3%	62.8%	51.4%
Tax Burden	T_0^*	16.05	1.26%	19.95	7.01	13.48	17.45
Tax Savings of Leverage	TS_0^*	4.66	17.62	0	13.59	6.80	2.60
Expected Default Costs	DC_0^*	2.64	2.33	0	5.53	2.765	1.18
Annual Yield Spread of Debt (%)	y	6.2%	0.90	//	10.9%	-	2%
Value of Optimal Leveraging	$\nu_0^* - V_0$	4.79	1.18	-	-	5.26	1.58
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	27.6 4%	6. 81%			30.45%	9. 12%

Table 5: Asymmetric size: capital structure and value across organizational forms, $\rho = 0.2,\ V_{h0} = 167, V_{s0} = 33.$

Variables	Symbols	Values					
		S. Alone(1/3)	S. Alone(5/3)	Holding	Subsidiary	1/2 HS	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	19	95	63.33	121.33	92.33	57
Default Threshold	X^{d*}	22.50	112.54			-	67.81
No Tax Profit Level	X^{Z*}	4.98	24.85			-	13.765
Value of Optimal Debt	D_0^*	14.02	70.15	48.25	70.26	59.25	43.24
Value of Optimal Equity	E_0^*	13.04	65.24	47.15	0	23.58	38.255
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	27.06	135.38	95.39	70.26	82.83	81.49
Optimal Leverage Ratio	D_0^*/ν_0^*	51.81%	51.81%	50.57%	100%	71.54%	53.06%
Annual Yield Spread of Debt (%)	y	1.2%	1.2%	0.6%	6.5%	-	1.1%
Tax Burden	T_0^*	5.90	29.49	30.90	0.48	15.69	17.78
Tax Savings of Leverage	TS_0^*	0.77	3.87	2.35	6.17	8.52	2.23
Expected Default Costs	DC_0^*	0.30	1.48	0.37	2.10	1.23	0.75
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.38	1.96	-	-	3.03	1.44
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	6.58%	6.79%			17.54%	8.31%

Note: HS principal is calculated as the sum of holding and subsidiary principals.

Table 6: Optimal Capital Structure and Value, rho 0.2 different levels of π, $\omega = 0$

	Symbols	Values							
	π			0.5			1		
		Stand Alone	Holding	Subsid	1/2 HS	Holding	Subsid	1/2 HS	1/2 M
Default Costs	α	23%	23%	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	57.20	55	69	62	0	220	110	58.5
Default Threshold	X^{d*}	67.75	65.21	81.69	-	0	249.2663	-	69.64
No Tax Profit Level	X^{Z*}	14.98	14.16	18.23	-	0	102.93	-	13.94
Value of Optimal Debt	D_0^*	42.22	40.84	50.85	45.84	0	117.06	58.53	44.56
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	50.58%	61.56%	56.13%	0	99.9%	70.26%	54.62%
Annual Yield Spread of Debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.26%	1.13%	1.33%	-	//	8.45%	-	0.6%
Value of Optimal Equity	E_0^*	39.01	31.70	39.91	35.80	49.46	0.07	24.76	37.01
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	80.75	82.48	81.62	49.46	117.13	83. 29	81.57
Tax Burden	T_0^*	17.62	17.81	17.18	17.49	20.01	5.39	12. 70	17.77
Tax Savings of Leverage	TS_0^*	2.33	2.20	2.83	2.51	0	14.62	7.31	2.18
Expected Default Costs	DC_0^*	0.90	0.77	1.11	0.94	0	8.13	4.07	0.61
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.43	0.70	2.33	1.57	-30.59	37.08	3.24	1.57
Cap. Value of Optimal Leverage	$Z(\nu_0^* - V_0)/V_0$	6. 81%	4.06%	14.01%	9.07%	-1.77	2.14	20.23%	9.06%

Note: both the stand alone and the conglomerate cases are invariant to π. Holding and subsidiary figures coincide with stand alone figures for $\pi = 0$.HS figures obtain by summing up the holding and the subsidiary figures.

Table 7: Capital structure and value with intercorporate dividends, different values of rho, $\pi = 0.5$

Variables	Symbols	ρ								
		-0.8			-0.2			0		
Omega	ω	0	0.5	1	0	0.5	1	0	0.5	1
Face Value of Subs. Debt	P^*_{0s}	74	59	48	72	62	52	71	63	53
Face Value of Parent Debt	P^*_{0h}	56	82	119	55	71	88	58	70	86
Value of Subs. Debt	D^*_{0s}	54.90	44.69	36.69	53.12	46.40	39.37	52.23	46.89	39.96
Value of Parent Debt	D^*_{0h}	41.49	63.39	92.37	40.84	53.81	67.13	42.79	52.62	64.93
Levered Group Value	ν^*_{0g}	163.34	164.13	165.03	163.29	163.64	163.97	163.27	163.55	163.82
HS Lever. Ratio	D^*_{0g}/ν^*_{0g}	59.00%	65.85%	78.21%	57.54%	61.23%	64.95%	58.20%	60.84%	64.02%
Subsid. Optimal Lever. Ratio	D^*_{0s}/ν^*_{0s}	65.71%	54.14%	44.80%	64.00%	56.31%	48.06%	63.14%	56.99%	48.83%
Equity Value of Parent	E^*_h	38.31	37.86	35.96	39.45	45.44	57.47	37.76	46.35	58.92
Equity Value of Subsid.	E^*_s	28.65	37.12	45.22	29.87	35.99	42.53	30.49	35.85	41.88
Equity Value of Group	E^*_g	66.96	74.98	81.18	68.32	81.43	100.00	68.25	82.20	90.80
Default Threshold	X^d_g	154.10	168.02	199.26	150.49	158.05	166.62	152.75	157.88	165.22
No Tax Profit Level	X^Z_g	33.62	32.92	37.94	33.04	33.04	33.50	33.98	34.82	34.11
Tax Burden	T_0	34.80	34.91	34.14	34.89	34.92	34.82	34.85	34.82	34.72
Parent Tax Savings of Lever	TS^*_{0h}	2.26	2.89	4.13	2.20	2.67	3.24	2.36	2.70	3.27
Subsid. Tax Savings of Lever	TS^*_{0s}	2.97	2.23	1.76	2.93	2.43	1.97	2.92	2.50	2.03
Parent Exp. Default Costs	DC^*_{0h}	0.82	0.51	0.68	0.77	0.78	0.86	0.93	0.88	1.04
Subsid. Exp. Default Costs	DC^*_{0s}	1.13	0.51	0.27	1.16	0.73	0.42	1.18	0.82	0.49
Parent Yield Spread	y_h	1.18%	0.28%	0.20%	1.13%	0.70%	0.56%	1.27%	0.87%	0.78%
Subsidiary Yield Spread	y_s	1.15%	0.71%	0.52%	1.27%	1.97%	0.72%	1.33%	1.08%	0.81%
Value of Optimal Leverage	$\upsilon^*_0 - V_0$	3.25	4.04	4.94	3.20	3.55	3.88	3.18	3.46	3.72
Capital. Value of Opt. Lever	$Z(\upsilon^*_0 - V_0)/V_0$	9.37%	11.66%	14.24%	9.23%	10.25%	11.20%	9.18%	9.99%	10.75%

Table 7: Capital structure and value with intercorporate dividends, different values of rho, $\pi = 0.5$

Variables	Symbols	ρ					
		0.2			0.8		
Omega	ω	0	0.5	1	0	0.5	1
Face Value of Subs. Debt	P^*_{0s}	69	63	52	68	58	49
Face Value of Parent Debt	P^*_{0h}	55	68	83	51	66	89
Value of Subs. Debt	D^*_{0s}	50.77	46.73	39.17	38.15	42.98	36.81
Value of Parent Debt	D^*_{0h}	40.84	50.85	62.26	49.53	48.45	64.02
Levered Group Value	ν^*_{0g}	163.23	163.48	163.71	163.10	163.22	163.55
HS Lever. Ratio	D^*_{0g}/ν^*_{0g}	56.13%	59.69%	61.96%	53.75%	56.01%	61.65%
Subsid. Optimal Lever. Ratio	D^*_{0s}/ν^*_{0s}	61.56%	56.89%	47.93%	46.58%	52.65%	45.20%
Equity Value of Parent	E^*_h	39.91	48.19	62.30	43.04	52.46	62.72
Equity Value of Subsid.	E^*_s	31.70	35.41	42.56	32.38	38.65	44.62
Equity Value of Group	E^*_g	71.61	83.60	104.86	75.42	91.11	107.34
Default Threshold	X^d_g	146.90	155.40	160.36	140.92	146.86	163.21
No Tax Profit Level	X^Z_g	32.38	33.42	33.57	31.32	32.57	37.17
Tax Burden	T_0	34.99	34.83	34.81	35.15	34.96	34.25
Parent Tax Savings of Lever	TS^*_{0h}	2.20	2.66	3.22	1.99	2.73	3.87
Subsid. Tax Savings of Lever	TS^*_{0s}	2.83	2.53	2.00	2.86	2.34	1.90
Parent Exp. Default Costs	DC^*_{0h}	0.77	0.91	1.09	0.60	1.10	1.86
Subsid. Exp. Default Costs	DC^*_{0s}	1.11	0.87	0.49	1.25	0.83	0.51
Parent Yield Spread	y_h	1.13%	0.98%	0.92%	negative	1.38%	1.81%
Subsidiary Yield Spread	y_s	1.33%	1.16%	0.83%	7.25%	1.18%	0.89%
Value of Optimal Leverage	$\upsilon^*_0 - V_0$	3.14	3.39	3.62	3.01	3.13	3.45
Capital. Value of Opt. Lever	$Z(\upsilon^*_0 - V_0)/V_0$	9.07%	9.79%	10.46%	8.70%	9.03%	9.98%