Cohort and Value-Based Multi-Country Longevity Risk Management

Michael Sherris[†], Yajing Xu[⋆] and Jonathan Ziveyi[†]

[†] School of Risk & Actuarial Studies
Centre of Excellence in Population Ageing Research
UNSW Business School, UNSW Sydney
[⋆]Southwestern University of Finance and Economics, Chengdu, China

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Outline of the Presentation

- Motivation
- Aims of the Paper
- Model Construction
- Value-based Longevity Indexes
- Numerical Illustrations
- Conclusion

Motivation

- Longevity risk is an important risk factor for life insurance companies and defined benefit (DB) pension plans that provide lifetime annuity-type payouts.
- This risk cannot be averaged out by applying the law of large numbers.
- Liquidity constraints due to absence of a market to offload the risk.
- A need for a dedicated market for facilitating longevity risk transactions.
- There has been a gradual increase in the number of transactions involving longevity risk transfer occurring among pension funds as presented in the next slides.
- Most of these transactions have been customized indemnity-based hedges.
- However, index-based hedges are more desirable due to greater liquidity potential and lower transaction costs (Lin and Cox 2005; Coughlan et al. 2011; Cairns and El Boukfaoui 2017).

Recent Transactions involving longevity-linked instruments

Date	Hedger	Provider	Type	Size
March-2018	Unknown scheme	Aon	Longevity Swap	£2bn
Feb-2018	Scottish Widows	Prudential	Longevity Reinsurance	$1.8bn
Jan-2018	Pension Insurance Corp	PartnerRe	Longevity Reinsurance	$750m
Dec-2017	Liverpool Victoria	RGA	Asset & Longevity Reinsurance	£900m
Dec-2017	Legal & General	Prudential	Index-Based Longevity Hedge	£800m
Nov-2017	NN Group	HannoverRe	Longevity Reinsurance	€3bn
Nov-2017	Pension Insurance Corp.	Prudential	Longevity Reinsurance	$1.2bn
Nov-2017	MMC UK	PICA & Canada Life Re	Longevity Swap & Reinsurance	£3.4bn

Table: Source:
`http://www.artemis.bm/library/longevity_swaps_risk_transfers.html`

- These transactions suggest a huge potential market in trading longevity risk.

Earlier Transactions involving longevity-linked instruments

Date	Hedger	Provider	Type	Description	Size (£m)
Jan-08	Lucida	J.P. Morgan	Value hedge	10-year q-Forward (LifeMetrics Index)	N/A
Jul-08	Canada Life	J.P. Morgan	Cash flow hedge	40-year survivor swap	500
Feb-09	Aviva	Royal Bank of Scotland	Cash flow hedge + value hedge	10-year collared survivor swap + final commutation payment	475
Jan-11	Pall UK Pension Fund	J.P. Morgan	Value hedge	10-year q-Forward (LifeMetrics Index)	70
Jul-14	British Telecom Pension Scheme	Prudential Insurance Co of America	Cash flow hedge	Longevity swap	16,000
Jan-15	Merchant Navy Officers Pension Fund	Towers Watson	Cash flow hedge	Longevity swap	1,500

Table: Earlier transactions involving longevity linked securities. Source: `http://www.longevity-risk.org/Pres_Coughlan.pdf`

Motivation cont...

Prudential in $1.8 billion longevity reinsurance deal for Scottish Widows

by ARTEMIS *on* FEBRUARY 5, 2018

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Prudential Insurance Company of America (PICA), part of the Prudential Financial, Inc. group, has completed a $1.8 billion longevity reinsurance arrangement for Scottish Widows Limited, a subsidiary of Lloyds Banking Group plc.

The direct longevity risk transfer and reinsurance arrangement saw PICA assume the longevity risk on approximately $1.8 billion (£1.3 billion) of annuity liabilities held by Scottish Widows within its life and annuities book.

"Prudential is privileged to be chosen by Scottish Widows as it seeks to efficiently manage its longevity risk over the coming decades," commented David Lang, Prudential's lead negotiator for this transaction. "With our new partnership, Scottish Widows attains more flexibility for managing longevity risk, trusting that, with PICA, it has chosen a strong counterparty with shared values and a long-term commitment to the longevity risk transfer business."

Michael Downie, finance director, Annuities and Investment Strategy at Scottish Widows, added, "I am delighted to have completed our first longevity reinsurance transaction with Prudential Financial. Their financial strength and long-term commitment to the market was a key consideration for Scottish Widows when selecting a counterparty. Throughout the negotiations, PICA took the time to understand our needs and actively tailored their offering to meet our requirements."

Motivation cont: Existing Indexes

- An initial effort to construct a longevity index was undertaken by Credit Suisse in 2005 which launched the first market-wide longevity index based on historical and projected life expectancy for the US population.
- This index is no longer being actively marketed by Credit Suisse.
- Two influential longevity indexes are still operational; the LLMA index by Life & Longevity Markets Association and the Xpect index by Deutsche Börse.
- LLMA publishes indexes for US, England & Wales, Netherlands and Germany.
- J.P. Morgan developed an internal toolkit called LifeMetrics now intellectual property of LLMA designed for pension plans, their sponsors, insurers, reinsurers and investors.
- Xpect provides mortality data and indexes for Germany, Netherlands and England & Wales on a monthly basis and uses the Lee-Carter model as the underlying mortality model.

Motivation cont: Existing Indexes cont...

- Developing a robust value-based longevity index, which can be used as a reference by market participants in designing more effective financial instruments for managing longevity risk remains an important impediment.
- Non of the existing indexes has been universally effective as basis for financial market transactions as they are all based on bespoke groups of lives rather than population data.
- There is need for indexes capable of mitigating both mortality and interest rate risk inherent in liabilities of insurance companies.
- Such indexes would provide benefits for both providers and pension schemes alike in terms of improving the liquidity of longevity risk transfer solutions

Aims

- Filling the gap by developing value-based longevity indexes for cohorts in different countries which can be used as basis for pricing and hedging longevity-linked instruments.
- Such indexes will be handy for quantifying changes in the costs and risks of longevity at the same time.
- An index is defined as the discounted value of lifetime income of a unit of currency per annum for a specific cohort in a domestic country and a foreign country.
- We use a multi-country model for the mortality dynamics and an arbitrage-free Nelson-Siegel (AFNS) model to describe interest rate evolutions for both the domestic and foreign countries.
- We present a framework for assessing basis risk in index-based longevity hedges using a graphical risk metric which provides visual interpretations on the interplay between the portfolio to be hedged and the hedging instruments.

Mortality Model

- We propose a three-factor affine term structure model with one common factor and two local country-specific factors.
- Assume that the domestic and foreign cohort instantaneous mortality intensities, $\mu^d(x,t)$ and $\mu^f(x,t)$ respectively of an initial age x at time zero are affine functions of latent state variables

$$\begin{aligned} \mu^d(x,t) &= \delta_0 + \delta_1' \mathbf{Y}_x^d(t), \\ \mu^f(x,t) &= \delta_0 + \delta_1' \mathbf{Y}_x^f(t), \end{aligned} \tag{1}$$

where $\delta_0 \in \mathbb{R}$ and $\delta_1 \in \mathbb{R}^2$.

- $\mathbf{Y}_x^d(t)$ and $\mathbf{Y}_x^f(t)$ are vectors of two factors each and can be divided into two parts as

$$\mathbf{Y}_x^d(t) = \begin{pmatrix} X(t) \\ Z_x^d(t) \end{pmatrix} \text{ and } \mathbf{Y}_x^f(t) = \begin{pmatrix} X(t) \\ Z_x^f(t) \end{pmatrix}, \tag{2}$$

Mortality Model cont...

- Here $X(t)$ is the common factor that affects both countries while $Z_x^d(t)$ and $Z_x^f(t)$ are country specific factors evolving according to

$$\begin{pmatrix} dX(t) \\ dZ^d(t) \\ dZ^f(t) \end{pmatrix} = -\begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix} \begin{pmatrix} X(t) \\ Z^d(t) \\ Z^f(t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} dW_\mu^{\bar{Q}}(t) \\ dW_\mu^{d,\bar{Q}}(t) \\ dW_\mu^{f,\bar{Q}}(t) \end{pmatrix}.$$

- where ϕ_1, ϕ_2, ϕ_3, σ_1, σ_2 and σ_3 represent the best-estimate parameters, $W_\mu^{\bar{Q}}(t)$, $W_\mu^{d,\bar{Q}}(t)$ and $W_\mu^{f,\bar{Q}}(t)$ are independent standard Wiener processes.
- This model is said to be decomposable as it can be decomposed into two single-country mortality models as the common factors do not depend on local factors (Egorov et al. 2011).

Mortality Model cont...

- Under the best estimate probability measure, $\bar{Q}$, the domestic cohort survival probabilities at initial age x and time t can be represented as

$$\begin{aligned} S^d(x,t,T) &= E^{\bar{Q}}[e^{-\int_t^T \mu^d(x,s)ds}|\mathcal{F}_t] \\ &= e^{B_1^d(t,T)X(t)+B_2^d(t,T)Z^d(t)+A^d(t,T)}, \end{aligned} \tag{3}$$

where $B_1^d(t,T)$, $B_2^d(t,T)$ and $A^d(t,T)$ are Riccati ODEs

$$\begin{pmatrix} \frac{dB_1^d(t,T)}{dt} \\ \frac{dB_2^d(t,T)}{dt} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \begin{pmatrix} B_1^d(t,T) \\ B_2^d(t,T) \end{pmatrix}, \tag{4}$$

$$\frac{dA^d(t,T)}{dt} = -\frac{1}{2}\sum_{j=1}^{2}((\Sigma^d)'B^d(t,T)B^d(t,T)'(\Sigma^d))_{j,j}, \tag{5}$$

- The ODEs are solved subject to boundary conditions $B_1^d(T,T) = B_2^d(T,T) = A^d(T,T) = 0$ and Σ^d being a diagonal matrix with elements, σ_j for $j = 1, 2$.

Mortality Model cont...

- The solution to the ODE system can be represented as

$$B_1^d(t,T) = -\frac{1-e^{-\phi_1(T-t)}}{\phi_1},$$
$$B_2^d(t,T) = -\frac{1-e^{-\phi_2(T-t)}}{\phi_2},$$
$$A^d(t,T) = \frac{1}{2}\sum_{i=1,2}\frac{\sigma_i^2}{\phi_i^3}[\frac{1}{2}(1-e^{-2\phi_i(T-t)}) - 2(1-e^{-\phi_i(T-t)}) + \phi_i(T-t)].$$

- The corresponding domestic average force of mortality curve is an affine function of the state variables $X(t)$ and $Z^d(t)$ which can be represented as

$$\begin{aligned}\bar{\mu}^d(x,t,T) &= -\frac{1}{T-t}\log[S^d(x,t,T)] \qquad (6)\\ &= \frac{1-e^{-\phi_1(T-t)}}{\phi_1(T-t)}X(t) + \frac{1-e^{-\phi_2(T-t)}}{\phi_2(T-t)}Z^d(t) - \frac{A^d(t,T)}{T-t}.\end{aligned}$$

Mortality Model cont...

- Using similar manipulations, the foreign average force of mortality curve can be represented as

$$\begin{aligned}\bar{\mu}^f(x,t,T) &= -\frac{1}{T-t}\log[S^f(x,t,T)] \\ &= \frac{1-e^{-\phi_1(T-t)}}{\phi_1(T-t)}X(t) + \frac{1-e^{-\phi_3(T-t)}}{\phi_3(T-t)}Z^f(t) - \frac{A^f(t,T)}{T-t},\end{aligned} \tag{7}$$

  where

$$A^f(t,T) = \frac{1}{2}\sum_{i=1,3}\frac{\sigma_i^2}{\phi_i^3}[\frac{1}{2}(1-e^{-2\phi_i(T-t)})-2(1-e^{-\phi_i(T-t)})+\phi_i(T-t)].$$

- The corresponding foreign cohort survival function can as well be represented as

$$S^f(x,t,T) = E^{\bar{Q}}[e^{-\int_t^T \mu^f(x,s)ds}|\mathcal{F}_t].$$

Interest Rate Model

- It is important to factor in uncertainty associated with financial risk which can be explain by interest rate movements.
- We adopt the arbitrage-free Nelson-Siegel (AFNS) model developed in Christensen et al. (2011) which combines the affine features and good empirical fit to the term structure of interest rates.
- AFNS models have proved to better capture full effects of shifts in the term structure as they allow yields of all maturities to be stochastic.
- We also assume that the risk factors in the interest rate model are independent from those of the mortality model (Biffis 2005).
- AFNS model belongs to the affine term structure models while maintaining the yield curve representation introduced by Nelson and Siegel (1987).
- The AFNS model combines the no-arbitrage property of ATSMs and good empirical fit of Nelson-Siegel models.

- Let $P^d(t, T)$ and $P^f(t, T)$ denote the domestic and foreign zero-coupon bond prices contracted at time t with $T - t$ to maturity, then by definition we have

$$P^d(t, T) = e^{-(T-t)y^d(t,T)}, \tag{8}$$
$$P^f(t, T) = e^{-(T-t)y^f(t,T)}, \tag{9}$$

where $y^d(t, T)$ and $y^f(t, T)$ are corresponding domestic and foreign zero-coupon yield rates

Interest Rate Model cont...

- Christensen et al. [2011] show that the domestic yield function of AFNS model is given by

$$y^d(t,T) = L^d(t) + \frac{1 - e^{-\lambda^d (T-t)}}{\lambda^d (T-t)} S^d(t) + \left[\frac{1 - e^{-\lambda^d (T-t)}}{\lambda^d (T-t)} - e^{-\lambda^d (T-t)} \right] C^d(t) - \frac{V^d(t,T)}{T-t},$$

  where λ^d is the Nelson-Siegel parameter, $L^d(t)$, $S^d(t)$ and $C^d(t)$ are the level, slope and curvature which have been shown to be sufficient to account for the time variation in the cross section of nominal Treasury yields.

- Under the risk-neutral Q-measure, $L^d(t)$, $S^d(t)$ and $C^d(t)$ have the following dynamics

$$\begin{pmatrix} dL^d(t) \\ dS^d(t) \\ dC^d(t) \end{pmatrix} = - \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^d & -\lambda^d \\ 0 & 0 & \lambda^d \end{pmatrix} \begin{pmatrix} L^d(t) \\ S^d(t) \\ C^d(t) \end{pmatrix} dt + \begin{pmatrix} s_1^d & 0 & 0 \\ 0 & s_2^d & 0 \\ 0 & 0 & s_3^d \end{pmatrix} \begin{pmatrix} dW_1^{d,Q}(t) \\ dW_2^{d,Q}(t) \\ dW_3^{d,Q}(t) \end{pmatrix}, \quad (10)$$

The Value-Based Longevity Indexes

- We develop value-based indexes for both domestic and foreign countries.
- We utilise a graphical risk metric to assess the basis risk arising from hedging both longevity risk and interest rate risk in domestic country with liquid foreign indexes.
- The indexes for the domestic and foreign countries are denoted as

$$I_x^{id}(t) = \sum_{j=1}^{x^*-x} P^d(t, t+j) S^d(x, t, t+j), \tag{11}$$

$$I_x^{if}(t) = \sum_{j=1}^{x^*-x} P^f(t, t+j) S^f(x, t, t+j), \tag{12}$$

with x^* being the maximum attainable age.

Value-Based Longevity Indexes cont...

- The longevity index is value-based, allowing for better quantification of risk.
- The index will include hedged components for both longevity risk and interest rate risk.
- As mortality is currently a non-tradable asset, the value-based longevity index will form the basis for new international markets and techniques for transferring longevity risk.
- The longevity indexes are designed to track the cost of lifetime annual income post-retirement.

Value-Based Longevity Indexes - Basis Risk Metrics

- We assume that there are liquid UK longevity indexes among cohorts born in 1950, 1945, 1940 and 1935 (whose members have already retired aged 65, 70, 75 and 80 respectively at the end of December 2015), with no such longevity indexes in Australia.
- We take the perspective of an insurer exposed to annuity payments to a pool consisting of an Australian cohort willing to mitigate the risk exposure by trading hybrid mortality swaps indexed to the UK mortality and interest rate experiences.
- As an example; for the Australian cohort born in 1950, we use UK cohorts born in 1940, 1945 and 1950 since the 1950 cohort is the youngest UK cohort available in retirement.

Basis Risk Metrics cont...

- Suppose that an Australian annuity provider is exposed to the risk of paying \$1 annually to each annuitant until the last remaining annuitant dies.
- Let

$$H^d = I_x^{id}(t) - E(I_x^{id}(t)), \tag{13}$$

and

$$H^f = I_x^{if}(t) - E(I_x^{if}(t)), \tag{14}$$

be the exceedances of $I_x^{id}(t)$ and $I_x^{if}(t)$ over their expected values respectively.
- We reference the graphical risk metric on H^d and H^f rather than $I_x^{id}(t)$ and $I_x^{if}(t)$, as practitioners are primary interested in the possible deviations from the expected outcomes.
- Additionally, H^d and H^f ensure all resulting risk metrics are centred at the origin, hence practitioners can readily compare the risk metrics for different foreign cohorts.

Basis Risk Metrics cont...

- Let h^f be the hedge ratio, i.e., units of foreign index used to hedge each unit of domestic index.
- Our aim is to minimize the uncertainty in $(H^d - h^f H^f)$.
- The optimization problem becomes that of finding the optimal h^f units of foreign index that minimizes the variance of $(H^d - h^f H^f)$, i.e.

$$\begin{aligned} \mathrm{Var}(H^d - h^f H^f) &= \mathrm{Var}(H^d) - 2h^f \mathrm{Cov}(H^d, H^f) + (h^f)^2 \mathrm{Var}(H^f) \\ &= \mathrm{Var}(H^f) \left[h^f - \frac{\mathrm{Cov}(H^d, H^f)}{\mathrm{Var}(H^f)} \right]^2 \\ &\quad + \mathrm{Var}(H^d) - \frac{\left[\mathrm{Cov}(H^d, H^f)\right]^2}{\mathrm{Var}(H^f)}. \end{aligned} \tag{15}$$

- This implies that we need to minimise

$$h^f = \frac{\mathrm{Cov}(H^d, H^f)}{\mathrm{Var}(H^f)}. \tag{16}$$

Basis Risk Metrics cont...

- The risk metric is illustrated with joint prediction regions at different confidence levels such that

$$Pr\left[\left(H^d, h^f H^f\right) \in \mathbf{J}_\alpha\right] = 1 - \alpha,$$

  where $\mathbf{J}_\alpha$ is the joint prediction region at the $1 - \alpha$ confidence level.
- Potential future paths of mortality rates as well as interest rates are obtained by simulation.
- The basis risk metric is constructed using the algorithm presented in the next slide.

Basis Risk Metrics Algorithm...

- Step 1: Simulate 20,000 best-estimate domestic and foreign cohort survival curves for respective cohorts.
- Step 2: Simulate 20,000 yield rates for Australia and UK and then calculate the corresponding discount bond prices.
- Step 3: Calculate values of domestic and foreign indexes using the cohort survival curves and zero-coupon bond prices obtained in previous steps, and then calculate realized H^d and H^f.
- Step 4: Calculate h^f such that $h^f = \frac{\mathrm{Cov}(H^d, H^f)}{\mathrm{Var}(H^f)}$.
- Step 5: For each realized $x = (H^d, h^f H^f)'$ calculate its Mahalanobis distance to the best estimate $\mu = (0, 0)'$.
- Step 6: Draw a convex hull that encloses $20,000(1-\alpha)$ realizations with the shortest Mahalanobis distances.
- The means of simulated survival curves in the first step are shown in the next figure.

Basis Risk Metrics cont...

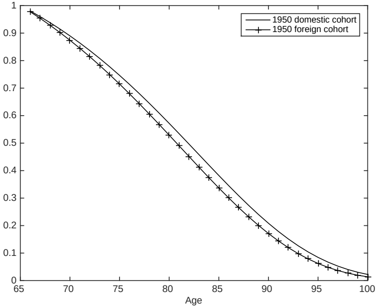

Figure: The means of simulated cohort survival curves for the 1950 cohorts. 20,000 simulations are performed using the three-factor joint ATSM.

Basis Risk Metrics cont...

- From the figure above, we note that survival probabilities of an Australian cohort are consistently higher than that of the UK cohort born in the same year.
- This is consistent with the trend in historical data where the average force of mortality of an Australian cohort is generally lower than that of the UK cohort born in the same year.
- The lower average force of mortality suggests the higher survival probabilities in the Australian population.
- In the numerical illustrations that follow, we consider three existing foreign indexes around similar ages specified as follows:

Example	Australian Cohort	UK Cohort		
I	1950	1940	1945	1950
II	1945	1940	1945	1950
III	1940	1935	1940	1945

Numerical Illustrations - Hedging Australian Exposure with UK Indexes

- Table 3 reports the optimal hedge ratio h^f calculated using Eq (16).
- For each domestic cohort, we consider three existing foreign indexes around similar ages.

Example	Domestic Cohort i	Foreign Index	$h^f = \frac{\mathrm{Cov}(H^d, H^f)}{\mathrm{Var}(H^f)}$
I	1950	$I_{75}^{1940f}(t)$	0.9997
		$I_{70}^{1945f}(t)$	0.8540
		$I_{65}^{1950f}(t)$	0.7507
II	1945	$I_{75}^{1940f}(t)$	0.8423
		$I_{70}^{1945f}(t)$	0.7188
		$I_{65}^{1950f}(t)$	0.4815
III	1940	$I_{75}^{1935f}(t)$	0.8423
		$I_{75}^{1940f}(t)$	0.6882
		$I_{70}^{1945f}(t)$	0.4359

Table: The calculated values of h^f, where h^f refers to the optimal hedge ratio.

Illustrations cont...

- The figure below shows an example of the basis risk metric with α being 10%, 20%, 30%, 40% and 50%.
- The red dots are the realized $(H^d, h^f H^f)$. If a perfect hedge occurs $(H^d = h^f H^f)$, the red dot will lie on the 45-degree line;
- If an under hedge occurs $(H^d > h^f H^f)$, the red dot will lie below the 45-degree line;
- If an over hedge occurs $(H^d < h^f H^f)$, the red dot will lie above the 45-degree line.

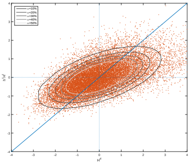

Figure: Basis risk metric using $I_{65}^{1950f}(t)$ to hedge risks associated with the domestic 1950 cohort. The optimal hedge ratio is 0.7507.

Illustrations cont...

- The 45-degree line indicates the perfect hedge, and the deviation from this line indicates the extent of over- or under-hedge.

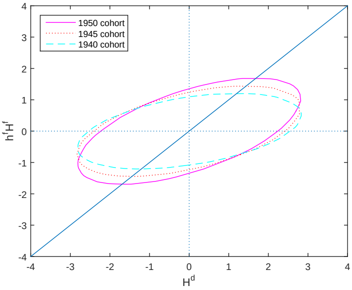

Figure: Example I: Comparison of basis risk metrics where a 1950 domestic cohort is hedged with three different foreign indexes

Illustrations - Common Interest Rate environment

- We perform similar analysis using France and Netherlands data assuming a constant interest rate of 5% as they are both in the Eurozone.
- Thus the basis risk only arises from different mortality experiences.
- We take Netherlands to be the domestic country and France is the foreign country.
- French longevity indexes are used to hedge the risk exposures of an annuity provider in Netherlands.
- We use male mortality data aged 65 to 100 for cohorts born from 1857 to 1911 for both countries.

Common Interest Rate environment cont...

- We assume that French longevity indexes among cohorts 1940, 1945 and 1950 are available.
- As the two countries are in the same interest rate regime, the correlation between H^d and H^f is much higher as compared to that in the Australia and UK case.

Table: The calculated values of h^f, where h^f refers to the optimal hedge ratio.

Example	Domestic Cohort i	Foreign Index	$h^f = \frac{\mathrm{Cov}(H^d, H^f)}{\mathrm{Var}(H^f)}$
IV	1945	$I_{75}^{1940f}(t)$	1.0585
		$I_{70}^{1945f}(t)$	0.9428
		$I_{65}^{1950f}(t)$	0.6298

Common Interest Rate environment: Basis Risk Metrics cont...

- With properly chosen h^f all foreign indexes provide satisfactory hedges to domestic cohorts, the best hedge is the foreign index of the same cohort since it best matches the 45-degree line.

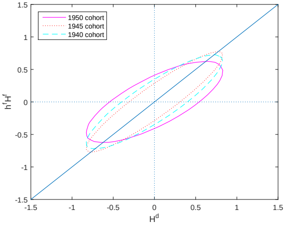

Figure: Comparison of basis risk metrics, $\alpha = 10\%$. The risks associated with 1945 domestic cohort are hedged with the 1940, 1945 and 1950 foreign indexes.

Common Interest Rate environment: Basis Risk Metrics cont...

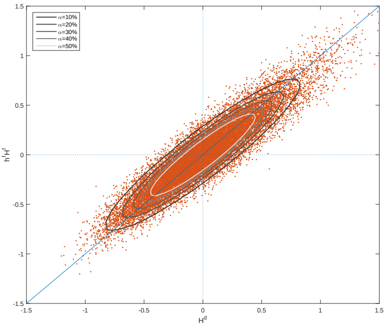

Figure: Basis risk metric using $I_{70}^{1945f}(t)$ to hedge risks associated with the domestic 1945 cohort. The optimal hedge ratio is 0.9428.

Conclusion

- We have developed a cohort-based affine term structure model for multi-country mortality developments.
- Adopted an arbitrage-free multi-country Nelson-Siegel model for the dynamics of interest rates.
- Devised longevity indexes to track the cost of lifetime annual income post-retirement taking into account the major sources of risks impacting life insurance portfolios.
- Devised a graphical basis risk metrics that provide valuable visual demonstrations of the relationship between an insurer portfolio and hedging strategies.
- Illustrations performed for the pairs: Australia vs UK and Netherlands vs France.

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Questions and Comments?