Impact of hedging strategy on capital charges for variable annuities

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November 2, 2021

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Before jumping in...

- Published in Risk (available on SSRN)
- This work was done in collaboration with Vivek Shah
- EDHEC Scientific Analytics: **#Fintech #NewVenture #Startup #NoProductYet**
- This institute is doing some really interesting research: https://risk.edhec.edu/retirement-investing

Variable Annuities

Definition

- Variable annuities (VA) are insurance contracts
- Premiums are invested in risky investment portfolio
- For a fee, an insurer guarantees a minimum growth rate over a (long) period of time
- The guaranteed amount can be transformed into an annuity to provide income
- Used primarily in the context of financing retirement.

Guaranteed Minimum Withdrawal Benefit

VAs involves market risk for the insurer. Hedging this risk requires financial engineering.

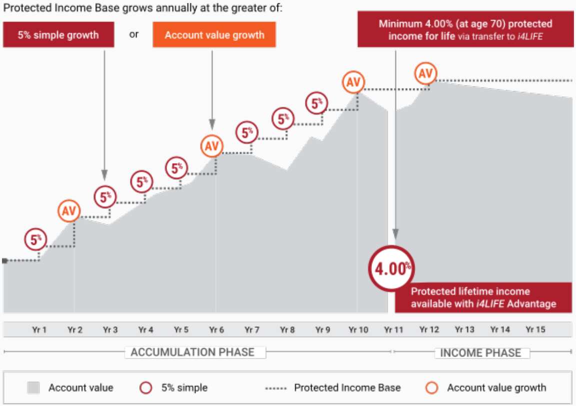


Image: Lincoln Financial group

More risk needed to guarantee income

Variable annuity contracts are expensive and yet unavoidable.

Demography

People live longer, healthier

Retirement System

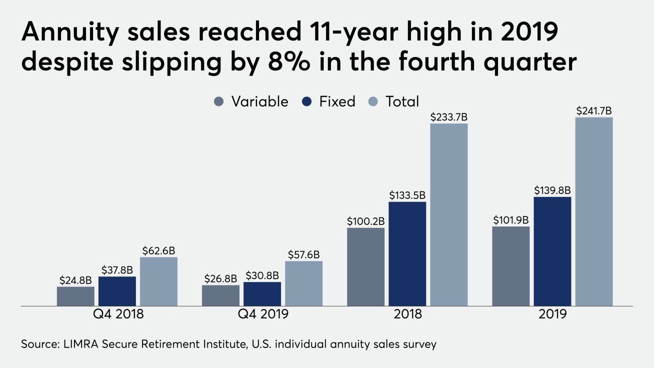
Defined benefit is long gone

Markets

Interest rates rock-bottom low

A big market

Beyond mere profitability for insures, the life savings of many people are invested in those products. Research is important to **reduces costs and improve safety** when hedging these products.



Hedging Variable Annuities

A long put option

- GMWB are essentially long-dated **put options**
- Long time horizon **amplifies the effect of market incompleteness**: transaction costs, availability of instruments, etc.
- Continuous delta hedging not realistic here.

Good bye delta hedging

> *Given that, in an incomplete market, the intrinsic risk of an option cannot generally be fully hedged, one idea for computing an optimal hedging strategy is to minimize a particular measure of this intrinsic risk.*

Coleman, Li, Patron, Insurance: Mathematics and Economics
Volume 38, Issue 2, 7 April 2006, Pages 215-228

Local hedging

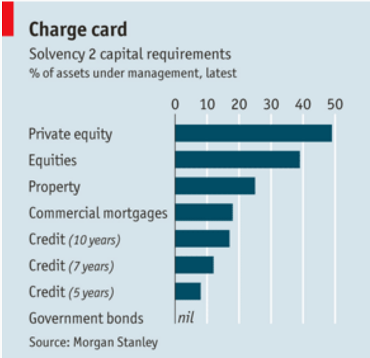
Delta hedging is replaced by locally **reducing the cost of rebalancing the hedging portfolio**. This is a typical dynamic programming problem that must be solved backward.

$$(\overset{\textit{underlying}}{\pi^\star}, \overset{\textit{cash}}{\alpha^\star}, \overset{\textit{put}}{\xi^\star}) = \arg\min E(\overset{\textit{cost}}{C^2_{t+1}} | S_t)$$

This technique was proven to be empirically superior to delta hedging (ibid). It also makes it easy to integrate any instrument in the hedging portfolio.

Solvency

The solvency charge is an opportunity cost

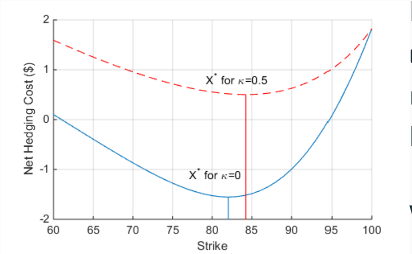


For each dollar invested in a risky instrument, **insurers must keep a cash reserve**.

The solvency charge is the **opportunity cost** that this **un-invested** cash represents.

It depends on the insurer's investments rate of return (**R**)

Holding put options reduces solvency charges



Holding put options **mitigates risk** for insurers and hence reduces the cash they have to keep un-invested.

What if the cash thus liberated produces more returns that the cost of the put option?

Vaucher, "Optimal Equity Protection of Solvency II Regulated Portfolios", Journal of Risk, Vol. 20, No. 3, 2018

Solvency charge reduction

Holding options liberates cash for the insurer to invest. The option has thus an positive externality.

$$SCR = R \times \Delta t \times \underbrace{D \times \alpha_k \times S_t}_{\text{notional liberated}} \tag{1}$$

Here D represents the percentage of capital liberated. For ATM options, D=39%, for 5% OTM D=34%, etc.

Our paper

What we tried to figure out

When using protective puts in hedging portfolios, are the insurers profits significantly affected by the reduction in solvency charges they entail?

Can we optimize hedging strategies to maximize reduction in solvency charges?

Are these profits robust when we use more realist return distributions?

Our main findings

The extra return coming from capital charges reductions significantly improve the total risk-adjusted profits for insurers.

The extra profits due to the use of protective puts are robust to tailed return distributions

Optimising the hedging portfolio to maximise SCR, hum, is basically useless... (but I will show you why)

Methodology

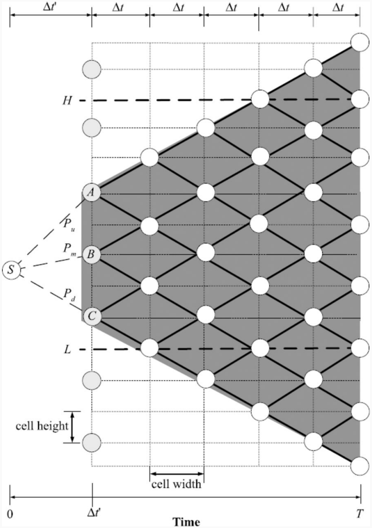
Our setup is similar to that of the excellent paper by Bernard and Kwuk (2016). We implemented a tree-based (similar conceptually to binomial trees) to recursively solve the hedging portfolio at every t

$$(\overset{\textit{underlying}}{\pi^\star}, \overset{\textit{cash}}{\alpha^\star}, \overset{\textit{put}}{\xi^\star}) = \arg\min E(C_{t+1}^2 | \overset{\text{price}}{S_t})$$

$$C_{t+1} = \begin{cases} X_{t+1} - X_t^+, & \text{Gross cost} \\ X_{t+1} - (X_t^+ + \varepsilon F_{t+1}) & \text{Net cost} \\ X_{t+1} - (X_t^+ + \varepsilon F_{t+1} + SCR) & \text{Net cost incl. SCR} \end{cases} \tag{2}$$

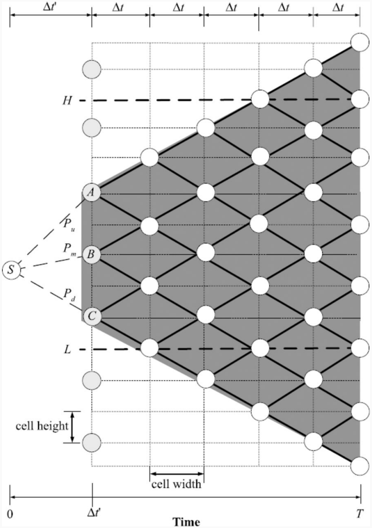
Carole Bernard and Minsuk Kwak in Insurance: Mathematics and Economics, 2016, vol. 67, issue C, 173-186.

Backward induction



The hedging portfolio is estimated at every node.

Backward induction



A price trajectory is simulated using a given probability distribution and the PnL of the hedge is evaluated.

Parameters

We did not try to be original anyhow. Most papers use these parameters so our results are easily comparable.

- Constant volatility (20%) and rates (2%)
- 10 years rebalanced bi-annually
- Insurer equity rate of returns (5%)
- Insurer's fees (4.0%)
- Initial amount \$100 and strike \$120
- Moneyness of the option (solvency optimal 10% OTM)
- Maturity of protective puts: 1Y

Results

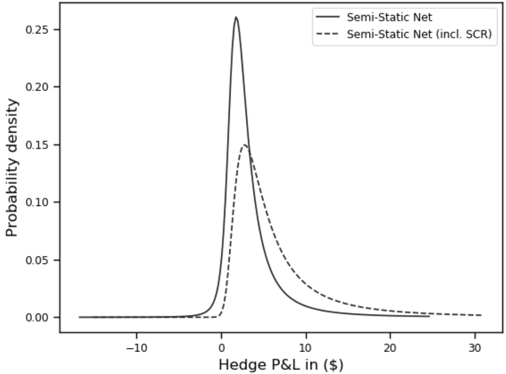
Contenders

Benchmarking is important in this case, so we compared our results with:

- No hedging!
- Simple delta hedging
- Delta-gamma hedging
- Semi-static hedging (gross)
- Semi-static hedging (net)
- All of the above for geometric (standard) and geometric with jumps price processes.

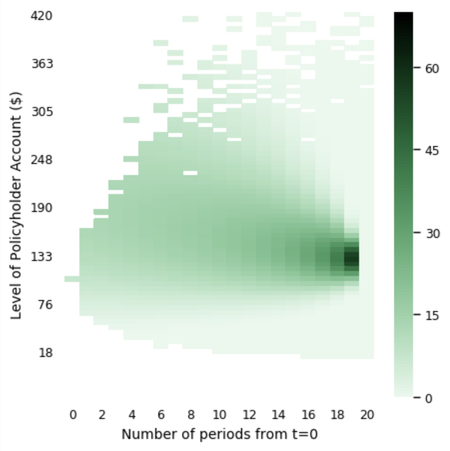
Profit distribution for the insurer

A typical profit distribution. Here we see the effect of adding the SCR into the optimisation process. The aggregate statistics are not so different (Why on the next slide)



Why SCR optimisation is useless

The use of protective puts is concentrated at the end of the life of the hedging around the strike. Not much to optimise...



The color scale represent the position in the protective put.

Insurer's profits

The semi-static hedging methodology produces the largest risk adjusted returns for the insurer. The real impact really comes from taking **fees** into account (see Bernard&Kwuk). Here prices follow a jump diffusion model.

	No Hedge	Delta	Delta Gamma	Semi-Static	Semi-Static Net
Profit: $E[G_T]$	\$14.74	\$7.49	\$9.23	\$6.19	\$2.95
$\sigma[G_T]$	\$59.30	\$29.89	\$29.30	\$29.02	\$2.99
$\mathrm{VaR}_{10\%}[G_T]$	\$52.52	\$24.80	\$22.80	\$25.06	\$0.01
$E[G_T^{full}]$	\$14.74	\$7.49	\$12.36	\$7.98	\$4.75
$E[G_T]/\sigma$	\$0.25	\$0.25	\$0.32	\$0.21	\$0.99
$E[G_T^{full}]/E[G_T]$	1.00	1.00	1.34	1.29	1.61

Profits are robust

The price diffusion process impacts the results. The profits from semi-static hedging are quite robust to the addition of jumps.

Jump diffusion	No Hedge	Delta	Delta Gamma	Semi-Static Net
Profit: $E[G_T]$	\$14.74	\$7.49	\$9.23	\$2.95
$\mathrm{VaR}_{10\%}[G_T]$	\$52.52	\$24.80	\$22.80	\$0.01
$E[G_T^{full}]/\sigma$	\$0.25	\$0.25	\$0.42	\$1.59
Geometric				
Profit: $E[G_T]$	\$31.20	\$15.62	\$16.62	\$3.39
$\mathrm{VaR}_{10\%}[G_T]$	\$35.39	\$15.75	\$15.61	\$0.00
$E[G_T^{full}]/\sigma$	\$0.54	\$0.52	\$0.61	\$1.75

Outlook

Topics of interest (IMHO)

- Can we make hedging portfolios more robust by produce return distributions with realistic expectations, rates, and volatility dynamics?
- Reduce fees by making fees proportional to price of option and portfolio level?

Final word

Thank you so much for attending, I hope you found it interesting!