Modeling cause-of-death mortality with jump effects: Implications on risk management to life insurers

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Overview

Introduction
- Empirical analysis
- Existing methods

Modeling analysis
- Estimation results
- Forecasting results

Actuarial valuation
- Insurance products
- Scenario analysis

Conclusion

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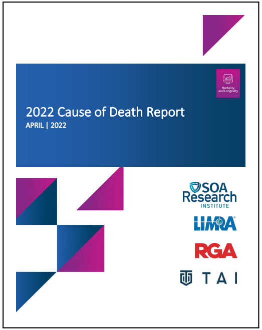
Findings from the SOA's 2022 Cause of Death Report



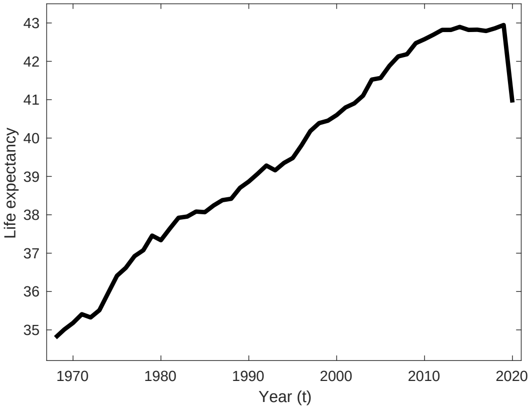
- Young ages' non-COVID excess deaths are much higher than their COVID deaths.
- Old ages' non-COVID excess deaths are lower than their COVID deaths.
- External deaths (e.g., by vehicle accidents) were noticeably increased for young ages.
- Respiratory deaths were roughly in line with expected.
- Cancer deaths were generally higher than expected.

Data

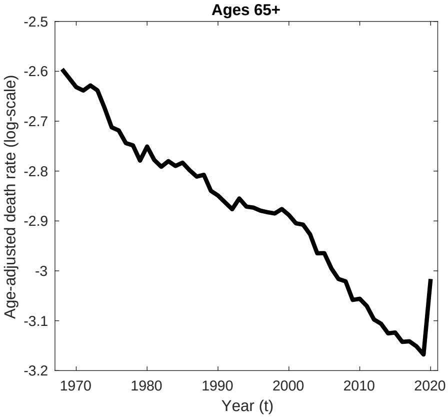
We consider the CoD-specific US male mortality data from the CDC's database:

ICD Code	**ICD-10**	**ICD-9**	**ICD-8**
Years	**1999-2020**	**1979-1998**	**1968-1978**
CoD 1 (Infectious)	A00-B99	001-139	001-136
CoD 2 (Cancer)	C00-D48	140-239	140-239
CoD 3 (Circulatory)	I00-I99	390-437	390-458
CoD 4 (Respiratory)	J00-J98	460-519	460-519
CoD 5 (External)	V00-Y89	E800-E999	E810-E999
CoD 6 (Other)	All other causes		

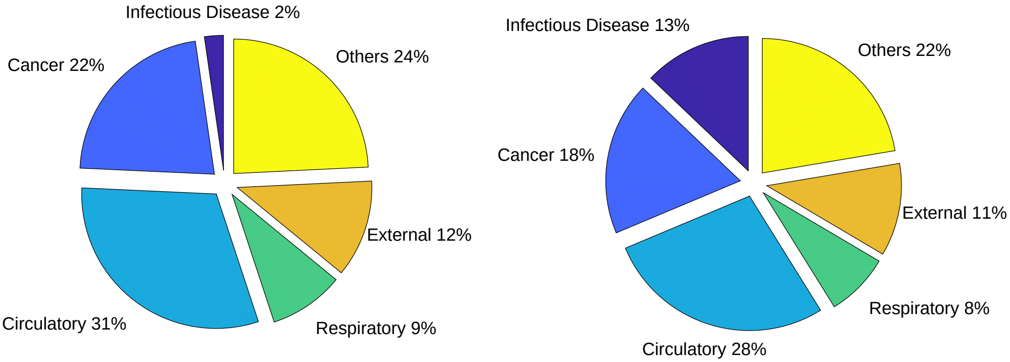
Life expectancy at age 35 from 1986 to 2020



Death rates for ages 35-64 and 65+

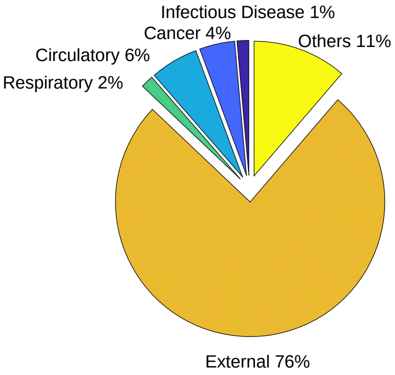


Composition of **total** deaths between 2019 and 2020

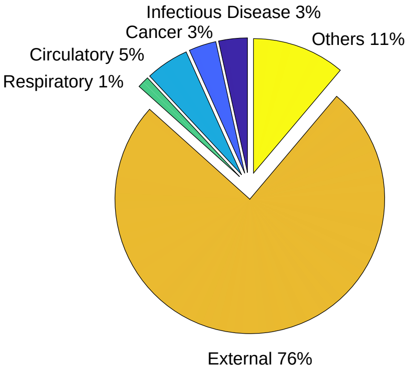


(a) Year 2019 (b) Year 2020

Composition of **young** ages' deaths between 2019 and 2020

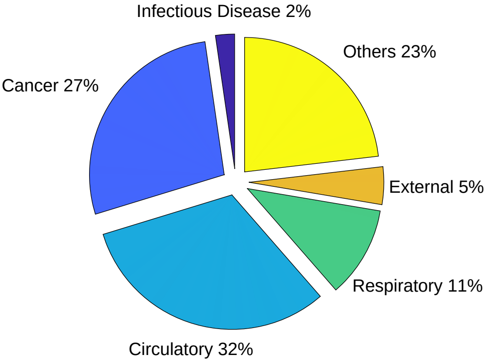


(a) Year 2019



(b) Year 2020

Composition of **old** ages' deaths between 2019 and 2020



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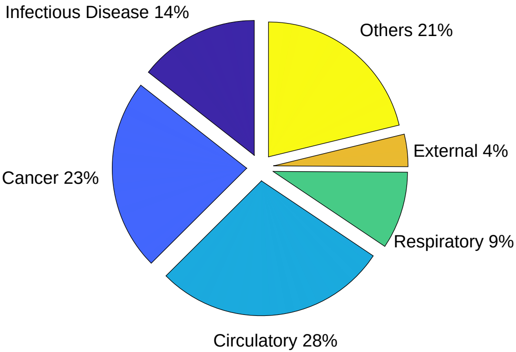
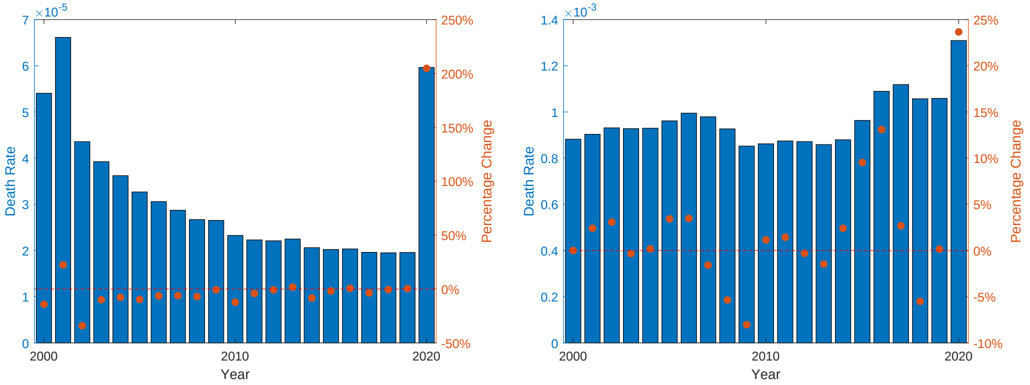


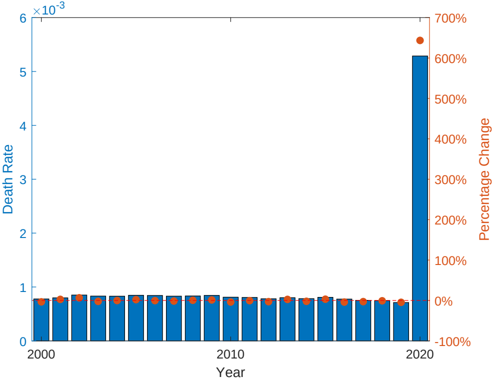
(b) Year 2020

The impact of COVID on young ages' mortality for different causes

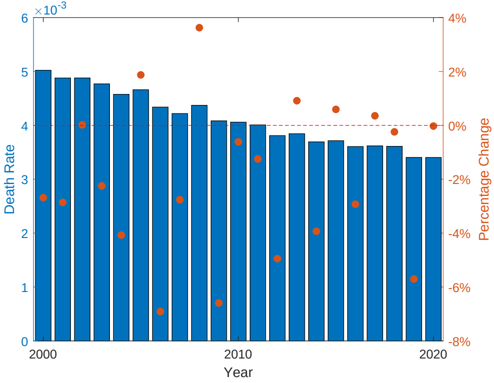


(a) Infectious (b) External

The impact of COVID on old ages' mortality for different causes

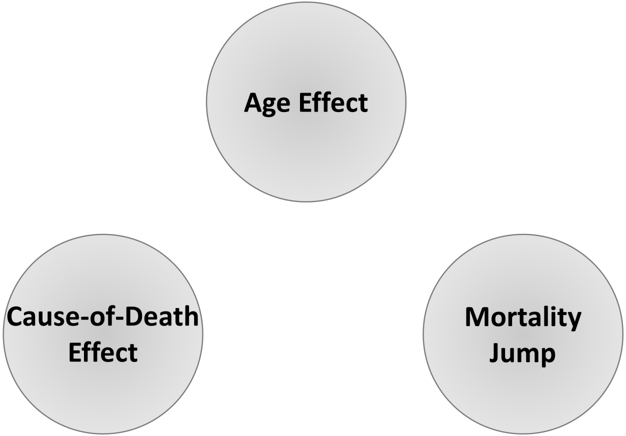


(a) Infectious

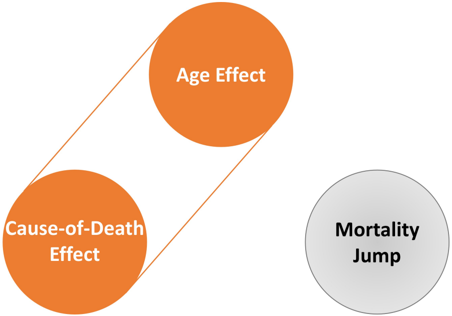


(b) Respiratory

Three components from the US mortality data

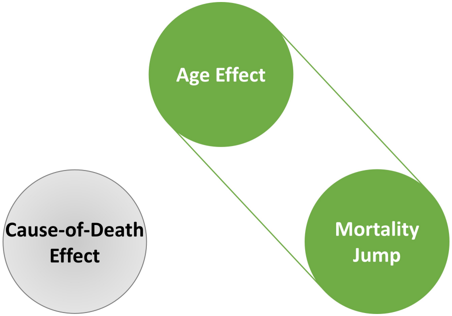


Existing studies on CoD effect modeling



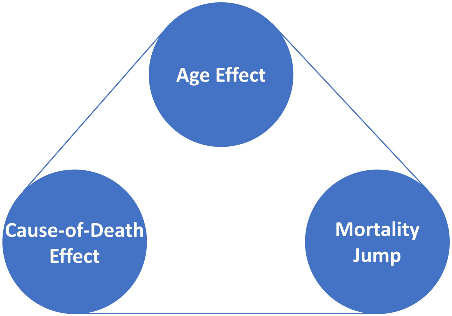
- Arnold & Glushko (2022, NAAJ)
  A cointegration analysis on CoD-specific mortality under a VECM setup.
- Li & Lu (2019, SAJ)
  Using hierarchical Archimedean copulas to model CoD-specific mortality.
- Li et al. (2019, IME)
  A forecast reconciliation approach to CoD-specific mortality.
- Many others ...

Existing studies on mortality jump modeling



- Zhou & Li (2022, AAS)
  Extending the Lee-Carter structure to capture age-specific excess mortality caused by COVID-19.
- Chen et al. (2022, IME)
  A threshold jump approach for modeling pandemic jumps, including the 1918 Spanish flu or COVID-19.
- Özen & Şahin (2020, JCAM)
  Using a renewal process to model transitory mortality jumps.
- Many others ...

Our approach



- Considers a range of existing modeling methods and combines their features to capture all three components.
- Proposes an age- and CoD-specific stochastic mortality model with transitory mortality jumps.
- Demonstrates the effect of modeling age- and CoD-specific mortality on life insurance products and portfolios.

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Model developments

- The Lee-Carter model (Lee & Carter, 1992, JASA):

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$$

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$$\ln m_{x,t,i} = \alpha_{x,i} + \beta_x \kappa_t + \beta_{x,i} \kappa_{t,i} + \epsilon_{x,t,i}$$

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2. The three-way extension (Russolillo et al., 2011, SAJ):

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3. The transitory jump extension (Liu and Li, 2015, IME):

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + N_t J_{x,t} + \epsilon_{x,t}$$

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- The proposed model:

$$\ln m_{x,t,c} = \alpha_{x,c} + \beta_x^{(1)} \kappa_t^{(1)} + \phi_c \beta_x^{(2)} \kappa_t^{(2)} + N_t J_{x,t,c} + \epsilon_{x,t,c}$$

Model Estimation

- Use the Route II approach (Haberman and Renshaw, 2012, Liu and Li, 2015, Li, et al., 2020), which focuses on the mortality improvement rates

$$Z_{x,t,c} = \ln m_{x,t,c} - \ln m_{x,t-1,c}$$

- Some simplifying assumptions:
  - $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ follow random walks with drift

$$\kappa_t^{(i)} = d^{(i)} + \kappa_{t-1}^{(i)} + \eta_t^{(i)},$$

    where $\eta_t^{(i)} \sim N(0, \sigma_i^2)$ for $i = 1, 2$.

  - $N_t \sim Ber(p)$, $J_{x,t,c} \sim N(\mu_{x,c}^{(J)}, \sigma_J^2)$, and $\epsilon_{x,t,c} \sim N(0, \sigma_\epsilon^2)$

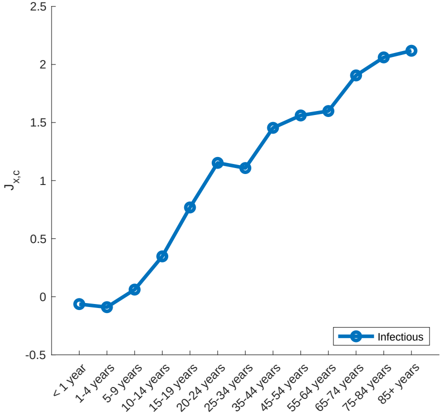
Model Estimation

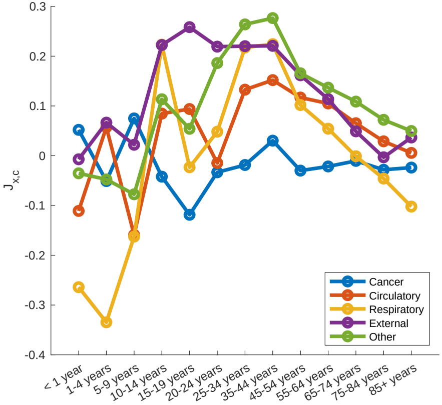
- The log-likelihood function can be derived using conditional maximum likelihood:

$$l(\vec{\theta}) = \sum_{t=2}^{T-1} \left(\ln \left(f \left(\vec{Z}_t, \vec{Z}_{t+1} \; ; \; \vec{\theta} \right) \right) \right) - \sum_{t=3}^{T-1} \left(\ln \left(f \left(\vec{Z}_t \; ; \; \vec{\theta} \right) \right) \right)$$

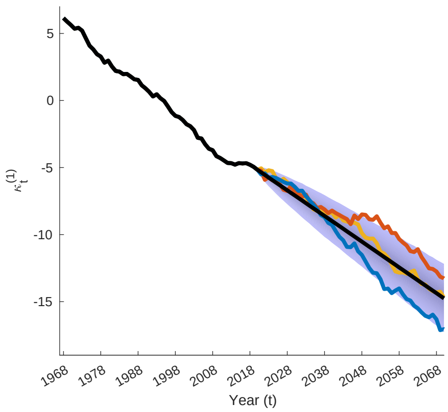
  where $\vec{\theta}$ is the parameter vector containing $\vec{\phi}, \vec{\beta}, \vec{d}, \vec{\mu}, \vec{\sigma}$ and p.
- We use a quasi-Newton method called the Broyden–Fletcher–Goldfarb–Shanno (B-F-G-S) algorithm to conduct the numerical optimization.

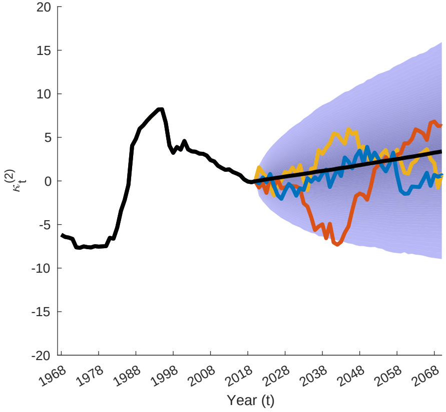
Estimation results: $\mu_{x,c}^{(J)}$



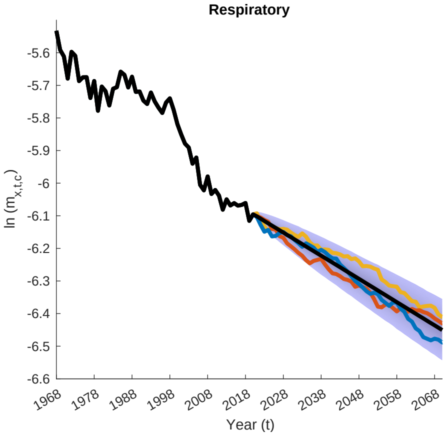


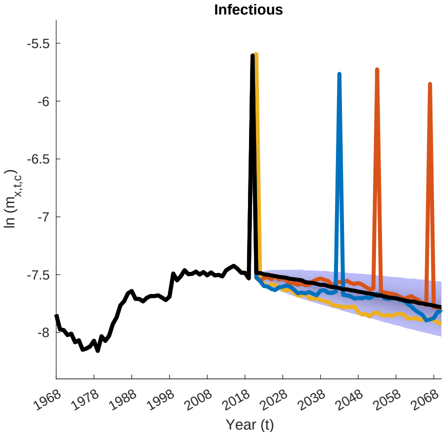
Estimation results: $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$



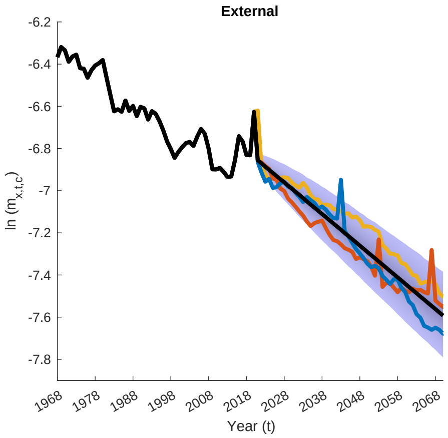


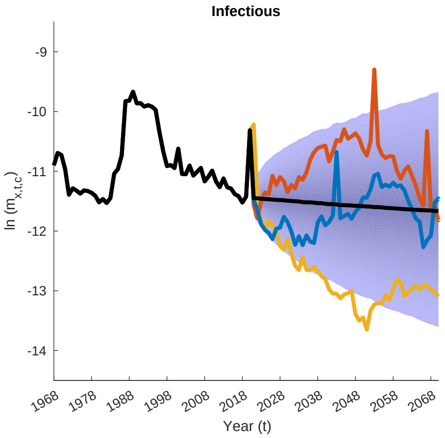
Projected mortality rates for ages 65-74



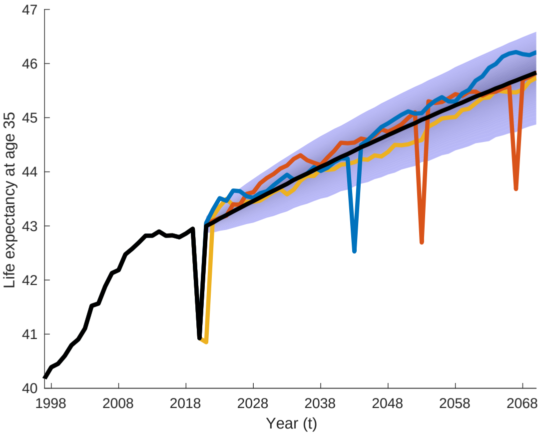


Projected mortality rates for ages 20-24





Projected life expectancy at age 35



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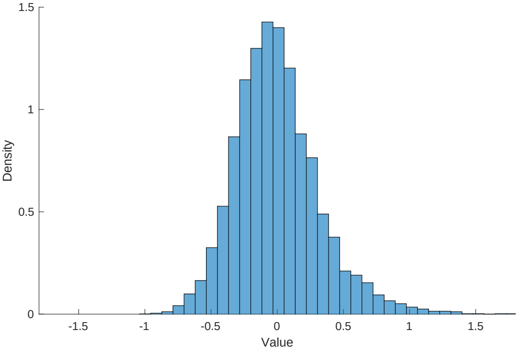
Life insurance

Consider a 5-year term life insurance issued to an individual aged 35 at year 2021:

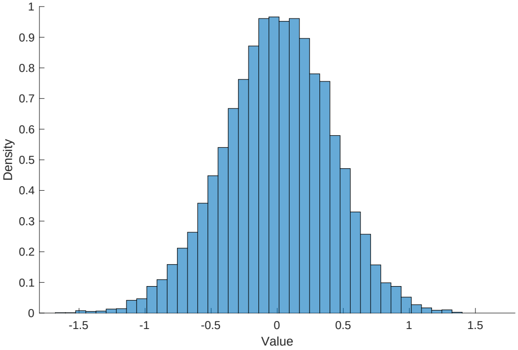
$$\mathcal{I} = \sum_{s=1}^{5} (1+r)^{-s} \times (1 - e^{-m_{35+s,2020+s}})$$



Life annuity

Consider a 30-year term life annuity issued to an individual aged 65 at year 2021:

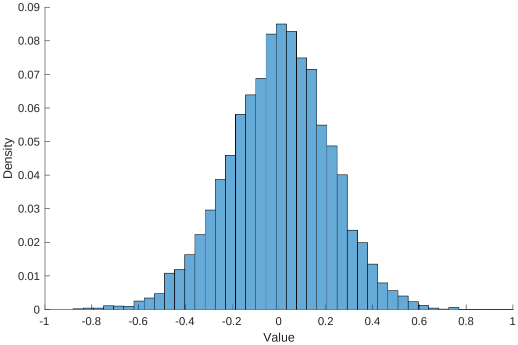
$$\mathcal{A} = \sum_{s=1}^{30} (1+r)^{-s} \times e^{-\sum_{n=1}^{s} m_{65+n,2020+n}}$$



Insurance portfolio

Consider a portfolio of the life insurance ($\mathcal{I}$) and life annuity ($\mathcal{A}$) products:

$$\mathcal{P} = \omega\mathcal{A} + (1-\omega)\mathcal{I}$$



Modeling scenarios

To analyze the modeling results, we consider the following modeling scenarios:

- Baseline: The proposed model
- Scenario A: The proposed model without the jump term ($N_t J_{x,c}$).
- Scenario B: The Chen and Cox model (Chen and Cox, 2009, JRI).
- Scenario C: The J1 model (Liu and Li, 2015, IME).

Portfolio weight calculation

Consider a portfolio of life insurance ($\mathcal{I}$) and life annuity ($\mathcal{A}$) products:

$$\mathcal{P} = \omega\mathcal{A} + (1-\omega)\mathcal{I}$$

The portfolio weight ω can be determined by

$$\omega = \frac{1}{1+w},$$

where

$$w = -\sqrt{\frac{\text{Var}(\mathcal{A})}{\text{Var}(\mathcal{I})}} \times \text{Corr}(\mathcal{A},\mathcal{I})$$

Modeling scenarios' portfolio weights

Scenarios	**Weight** ω
Baseline	51.67%
Scenario A	21.36%
Scenario B	30.03%
Scenario C	33.91%

Compared to the Baseline case,

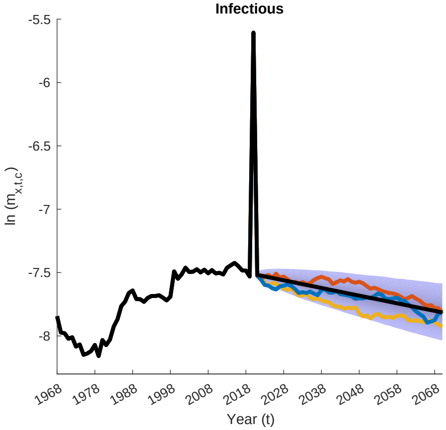
- Scenario A does not incorporate any jump effect, and provides a 30% lower weight.
- Scenario B does not incorporate age- or cause-specific jump effect, and provides a 22% lower weight.
- Scenario C does not incorporate cause-specific jump effect, and provides a 18% lower weight.

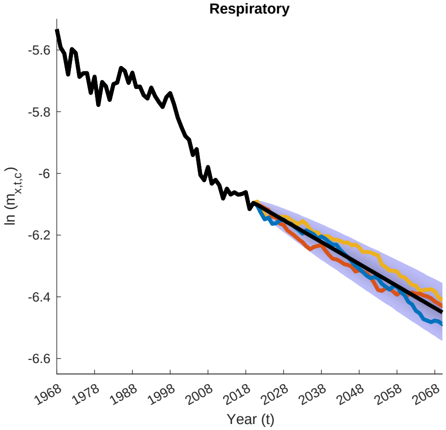
What-if scenarios

We can modify the distribution of $J_{x,t,c}$ (e.g., the jump size $\mu_{x,c}^{(J)}$) to create the following possible future scenarios:

- Scenario 1: The proposed model with no more jumps in the future.
- Scenario 2: The proposed model with 50% less severe but 50% more likely than the COVID-19 pandemic.
- Scenario 3: The proposed model with a possibility of cancer cure for ages 65+.
- Scenario 4: The proposed model with a possibility of a catastrophic event for ages 20-64.

Scenario 1 - No more jumps





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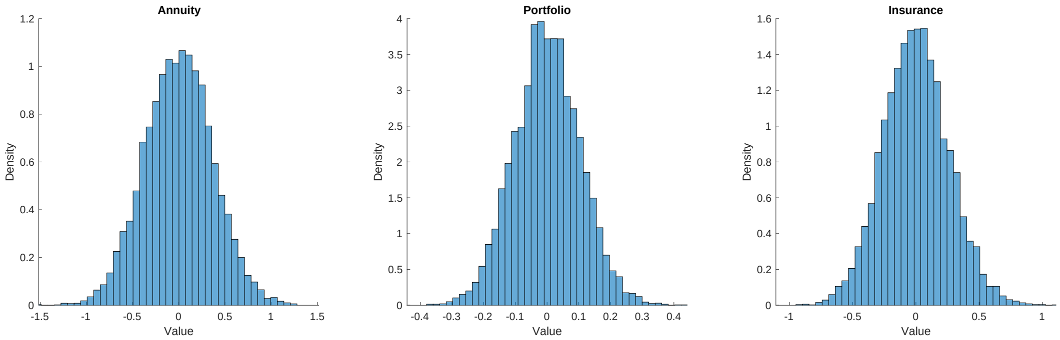
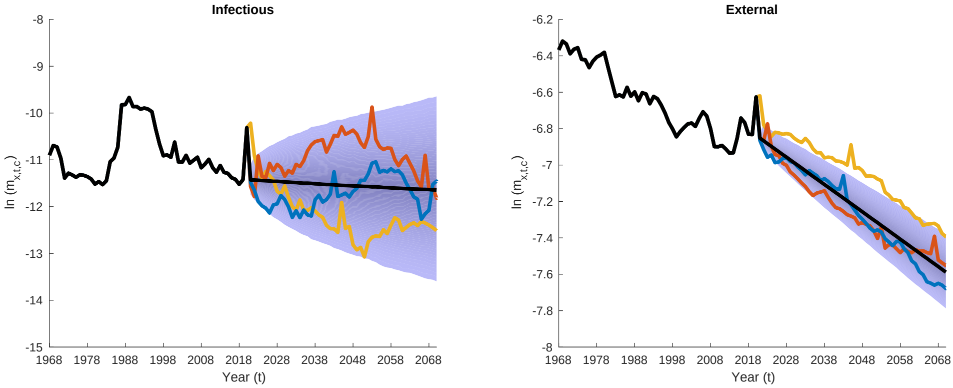


Scenario 2 - Less severe but more frequent jumps



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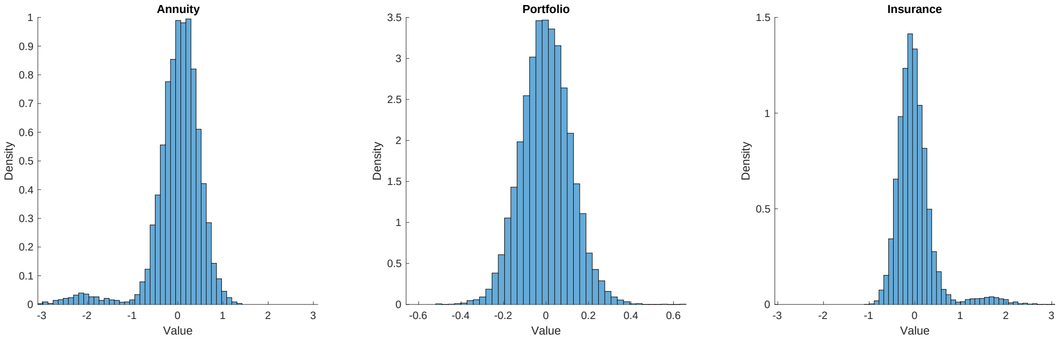
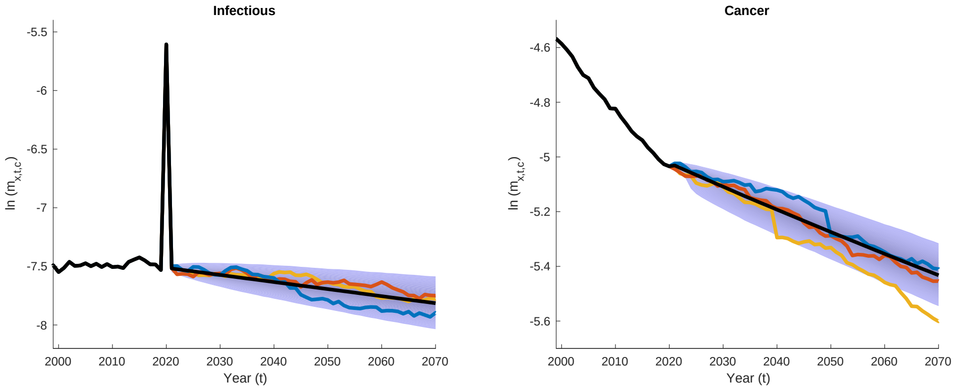


Scenario 3 - Cancer cure for ages 65+



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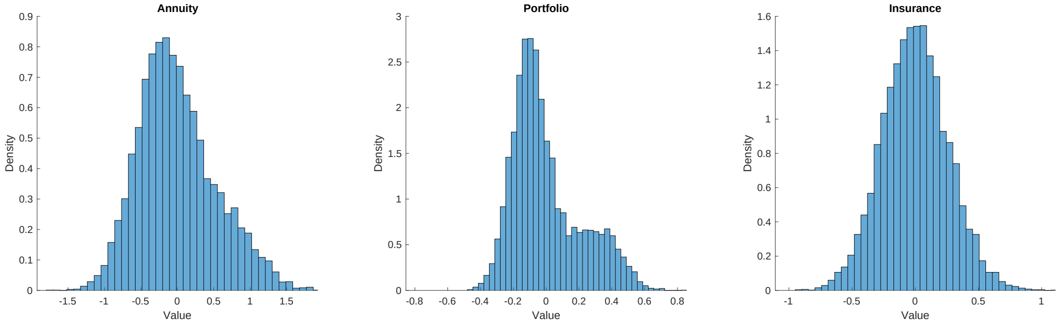
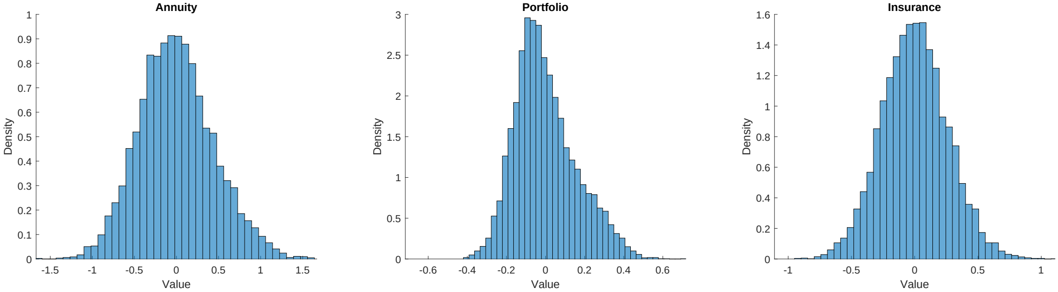
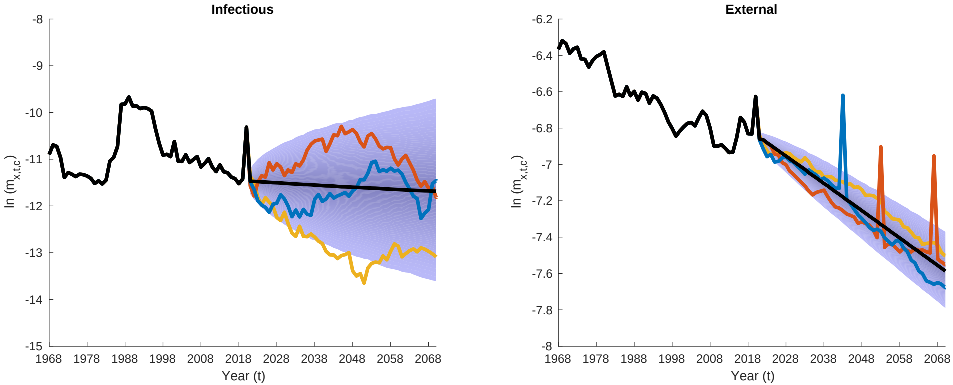


Scenario 3 - Cancer cure for ages 65+ (with competing risk)



Scenario 4 - Catastrophic event for ages 20-64



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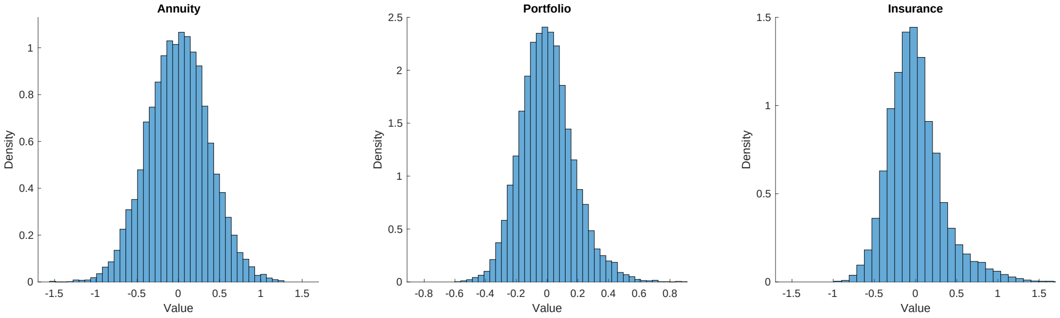

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- This paper proposes a three-way parallel factors (PARAFAC) model with age- and cause-specific mortality jumps.
    1. It preserves the general trend shared by all causes of death and allows cause-specific deviation from the general trend.
    2. It captures the asymmetric age and cause effects resulted from catastrophic mortality events.
- We have conducted two scenario studies:
    1. On multiple modeling settings to analyze the impact on the calculation of portfolio weights.
    2. On various future possibilities to visualize the implications on an life insurer's portfolio.

Limitation and Future work

- Limitation:
  Extra caution should be taken when interpreting the jump parameters:
  - Only one year of COVID data used: transitory, low variance of jump size.
  - Only one meaningful jump observed: probability of jump associated with the length of data.
- Future Work:
  - Apply the regime switching framework (Hardy, 2001) to characterize mortality evolution in post-pandemic era.
  - Consider other mortality-related features such as socioeconomic statues and geographical effects.
  - Include additional terms (e.g., $\phi_c^{(3)}\beta_x^{(3)}\kappa_t^{(3)}$) to improve the model performance.

Thank You!

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