The Long Shadow of Pandemic: Understanding the lingering effects of cause-specific mortality shocks

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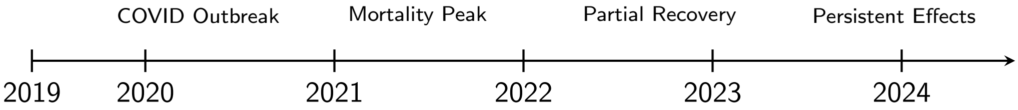
Motivation: A post-COVID perspective

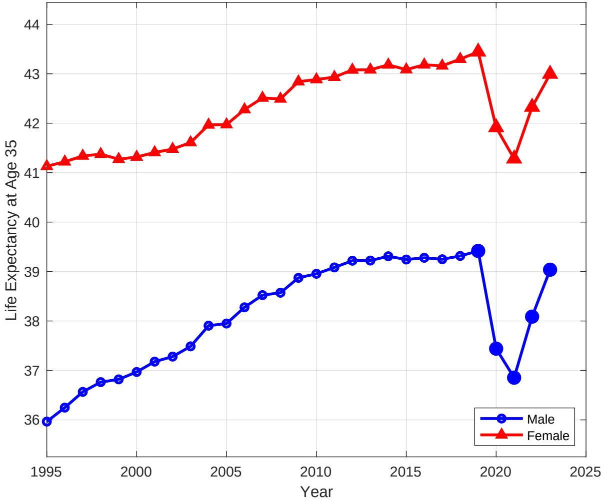


- Multiple years of post-COVID data available (2020-2023).
- Pandemic mortality shocks show *heterogeneous* impact and recovery:
  - Different by age groups
  - Different by causes-of-death.
  - Different by years-after-pandemic
- Need to analyze the long-lasting effects of pandemic mortality shocks to improve mortality forecasting and longevity risk management.

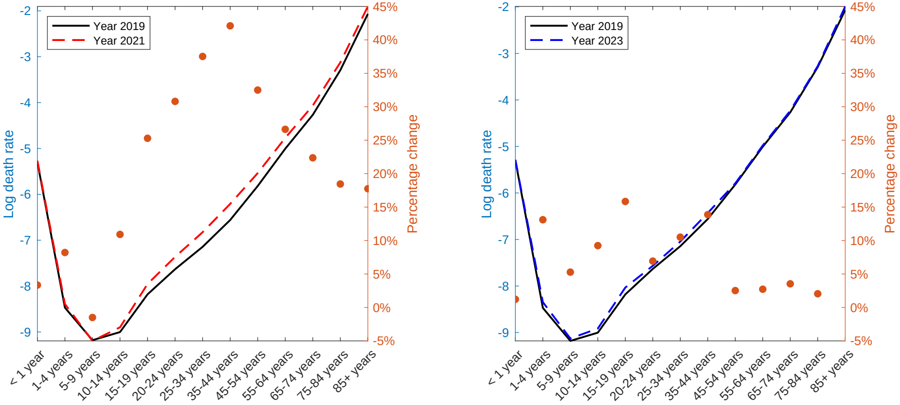
Empirical observations

Life expectancy at age 35 for years 1995-2023:

- Steady increase before 2010
- Slow down during 2010-2020
- Sharp drop in 2020-2021
- Partial recovery in 2022-2023



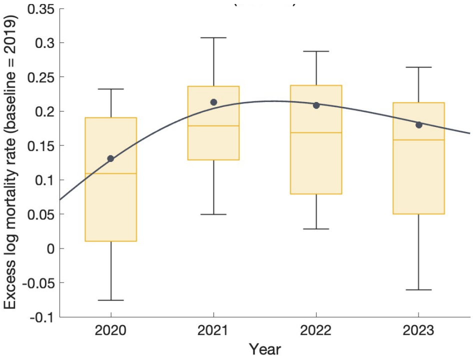
Empirical observations by age group



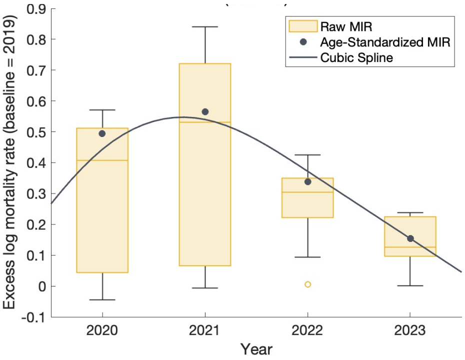
- Pandemic impact and recovery is *not uniform* across ages.

Empirical observations by cause-of-death (CoD)

- Define *excess log mortality* rate: $MI_{x,t,c}^{(Excess)} = \ln(m_{x,t,c}) - \ln(m_{x,2019,c})$



CoD A: Small jump and slow recovery



CoD B: Large jump and quick recovery

Key empirical observations

- COVID-19 introduced a major pandemic mortality shock with lingering effects on post-pandemic mortality dynamics.
- The magnitude and persistence of the pandemic mortality shocks vary across age groups and causes of death.
- These observations highlights the need for a flexible modeling framework to capture long-lasting and cause-specific jump effects.

Baseline mortality models

- **Lee & Carter (1992)**: The Lee-Carter model

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$$

- **Li & Lee (2005)**: Common factor extension

$$\ln m_{x,t,i} = \alpha_{x,i} + \beta_x \kappa_t + \beta_{x,i} \kappa_{t,i} + \epsilon_{x,t,i}$$

- **Russolillo et al. (2011)**: Three-way extension

$$\ln m_{x,t,i} = \alpha_{x,i} + \phi_i \beta_x \kappa_t + \epsilon_{x,t,i}$$

Jump mortality models (pre-COVID)

- **Cox et al. (2006)**: Proposed a permanent jump model where jump effects modify the dynamics of the period effect κ_t.
- **Lin & Cox (2008)** and **Chen & Cox (2009)**: Introduced a transitory jump term for catastrophic events:

$$\kappa_t = \tilde{\kappa}_t + N_t J_t$$

  where N_t is the jump occurrence (Bernoulli) and J_t is the jump severity (Normal).
- **Liu & Li (2015)**: Extended LC by adding *age-specific* jump effects:

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + N_t J_{x,t} + \epsilon_{x,t}$$

  where $J_{x,t}$ is an age-specific jump severity term (Normal).

Jump mortality models (post-COVID)

- Several new approaches for modeling mortality jumps since 2020:
  - **Özen & Şahin (2020)** - Renewal process for transitory jump modeling
  - **Zhou & Li (2022)** - Age-specific COVID-19 excess mortality extension
  - **Chen et al. (2022)** - Threshold jump model for pandemic shocks
- **Goes et al. (2025)**: Introduced *serial dependence* and *auto-regression* in jump severity:

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \beta_x^{(J)} J_t + \epsilon_{x,t}$$

$$J_t = a J_{t-1} + N_t Y_t$$

where $a \in [0, 1)$ is an AR coefficient.

The proposed model

- Three-Way Parallel Factors with Cause-Specific Long-Lasting Jumps Model
- Decomposes mortality rate $m_{x,t,c}$ by

$$\ln(m_{x,t,c}) = a_{x,c} + B_x K_t + \varphi_c b_x k_t + N_t J_{x,c}^{(\tau)} + e_{x,t,c}$$

where

- $a_{x,c}$ captures the baseline level by age and cause,
- $B_x K_t$ captures the general mortality trend of all causes,
- $\varphi_c b_x k_t$ captures cause-specific deviation from general trend, and
- $N_t J_{x,c}^{(\tau)}$ captures the long-lasting jump effect.

The long-lasting jump effects

- Jump occurs at time T_J,

$$N_t = \mathbb{1}_{\{t \geq T_J\}}$$

- Long-lasting jump effects modeled as

$$J_{x,c}^{(\tau)} = \begin{cases} 0 & \tau < 0 \\ J_{x,c} & \tau = 0 \\ J_{x,c} \cdot \pi_c(\tau) & \tau > 0 \end{cases}$$

where $\tau = t - T_J$,

  - $J_{x,c}$ is the jump severity at age x for cause c, capturing the initial impact, and
  - $\pi_c(\tau)$ is a parametric decay function, controlling the persistence.

Parametric function for persistence

- Long-lasting effect function:

$$\pi_c(\tau) = \gamma_c \cdot \beta_c^{\alpha_c} \cdot \tau^{\alpha_c - 1} \cdot e^{-\beta_c \tau}$$

  where
  - α_c is a cause-specific shape parameter, controlling timing of peak.
  - β_c is a cause-specific rate parameter, controlling rate of decay.
  - γ_c is a cause-specific impact parameter, controlling magnitude.

- $\pi_c(\tau)$ has a Gamma-like form.

Model summary

$$\ln(m_{x,t,c}) = a_{x,c} + B_x K_t + \varphi_c b_x k_t + N_t J_{x,c}^{(\tau)} + e_{x,t,c} \tag{1}$$

$$J_{x,c}^{(\tau)} = \begin{cases} 0 & \tau < 0 \\ J_{x,c} & \tau = 0 \\ J_{x,c} \cdot \pi_c(\tau) & \tau > 0 \end{cases} \tag{2}$$

$$\pi_c(\tau) = \gamma_c \cdot \beta_c^{\alpha_c} \cdot \tau^{\alpha_c - 1} \cdot e^{-\beta_c \tau} \tag{3}$$

- $B_x K_t$: Common factor from the Li-Lee model for general mortality.
- $\varphi_c b_x k_t$: Cause-specific factor based on the three-way LC model.
- $N_t J_{x,c}^{(\tau)}$: Generalized long-lasting jump effect from the Liu-Li model.

Distributional assumptions

- Common period factor: $K_t = D + K_{t-1} + \eta_t,\ \eta_t \sim N(0, \sigma_\eta^2)$
- Cause-specific period factor: $k_t = d + k_{t-1} + \xi_t,\ \xi_t \sim N(0, \sigma_\xi^2)$
- Jump occurrence: $N_t \sim \mathrm{Ber}(p)$
- Jump severity: $J_{x,c} \sim N(\mu_{x,c}, \sigma_J^2)$, where $\mu_{x,c}$ varies by age and cause.
- Jump persistence: $J_{x,c}^{(\tau)} \sim N(\mu_{x,c}\,\pi_c(\tau), \sigma_J^2\,\pi_c^2(\tau))$
- Error term: $e_{x,t,c} \sim N(0, \sigma_e^2)$

Likelihood estimation

- Use *Route II* approach (Haberman & Renshaw, 2012) for efficient estimation of Lee-Carter-type models.
- Applied to log mortality improvement rates:

$$Z_{x,t,c} = \ln(m_{x,t,c}) - \ln(m_{x,t-1,c})$$

- Conditional distribution of $Z_{x,t,c}$ depends on jump occurrence $\vec{N}_t = (N_{t-1}, N_t)'$:
  1. $\vec{N}_t = (0,0)'$ for no jump
  2. $\vec{N}_t = (0,1)'$ for jump at t
  3. $\vec{N}_t = (1,1)'$ for jump persists

Likelihood construction and algorithm

- The log-likelihood is a *Gaussian mixture* over possible jump scenarios:

$$l(\vec{\theta}) = \ln \left(\sum_{\vec{N}} f\left(\vec{Z} \mid \vec{N};\ \vec{\theta}\right) P\left(\vec{N}\right) \right)$$

  where the mixture weights $P(\vec{N})$ is determined by jump probability p.
- The parameter set to be estimated jointly is

$$\vec{\theta} = (\vec{B}, D, \sigma_\eta, \vec{b}, d, \sigma_\xi, \vec{\mu}, \sigma_J, \vec{\alpha}, \vec{\beta}, \vec{\gamma}, p)$$

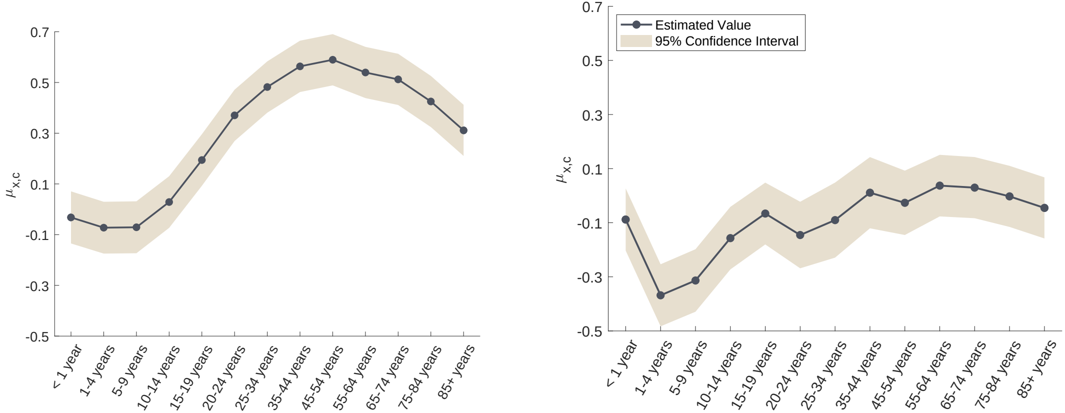
  and is estimated via *BFGS quasi-Newton algorithm*.

Data

Coding System	ICD-10	ICD-9	ICD-8
Time Period	1999-2023	1979-1998	1968-1978
CoD 1: Infectious Diseases	A00-B99	001-139	001-136
CoD 2: Cancer	C00-D48	140-239	140-239
CoD 3: Circulatory Diseases	I00-I99	390-437	390-458
CoD 4: Respiratory Diseases	J00-J98	460-519	460-519
CoD 5: External Causes	V00-Y89	E800-E999	E810-E999
CoD 6: Other Causes	All other causes (incl. COVID-19)		

Note: Data from U.S. CDC WONDER dataset. The coverage is 13 age groups, 56 years and 6 causes of death (CoD). ICD codes are mapped across three revisions.

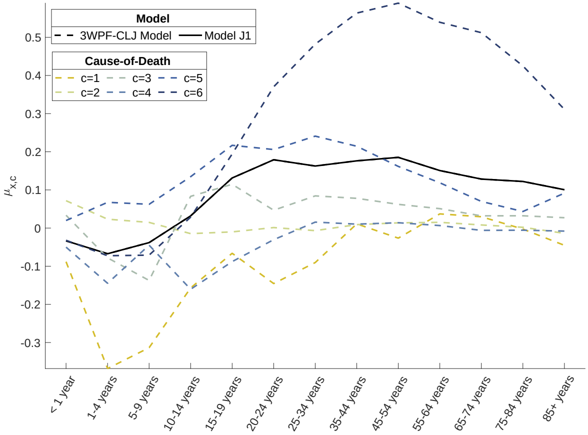
Jump severity



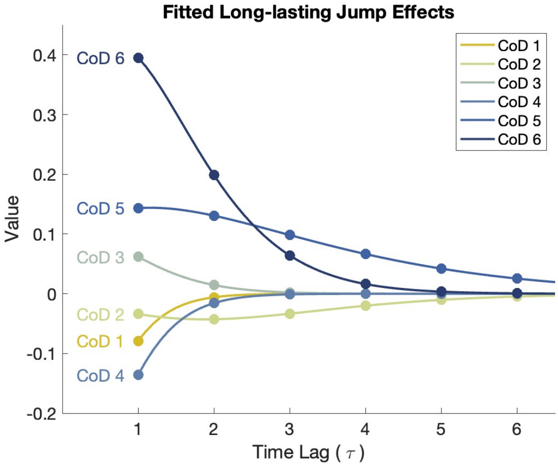
(a) CoD 6: COVID-19

(b) CoD 1: Infectious

Cause-specific jumps severity



Long-lasting jump effects



Conclusion

- Empirical analysis shows heterogeneous and long-lasting pandemic effects on cause-specific mortality dynamics.
- Proposes a new stochastic mortality model that integrates age, cause, and persistent jump effects.
- Provides flexible modeling framework for pandemic scenario analysis and risk management applications.