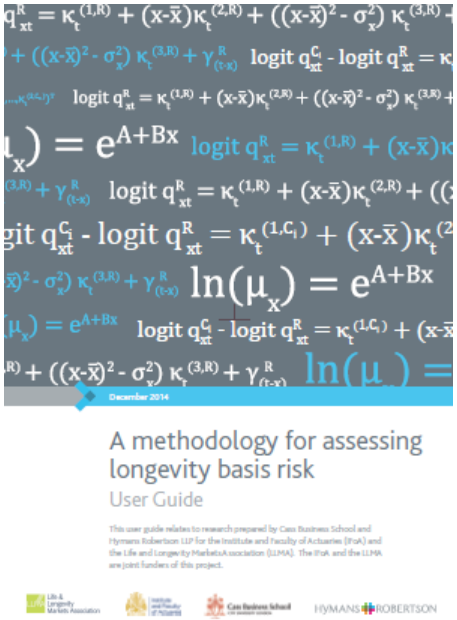
HYMANS ROBERTSON

A practical framework for assessing basis risk in index-based longevity hedges

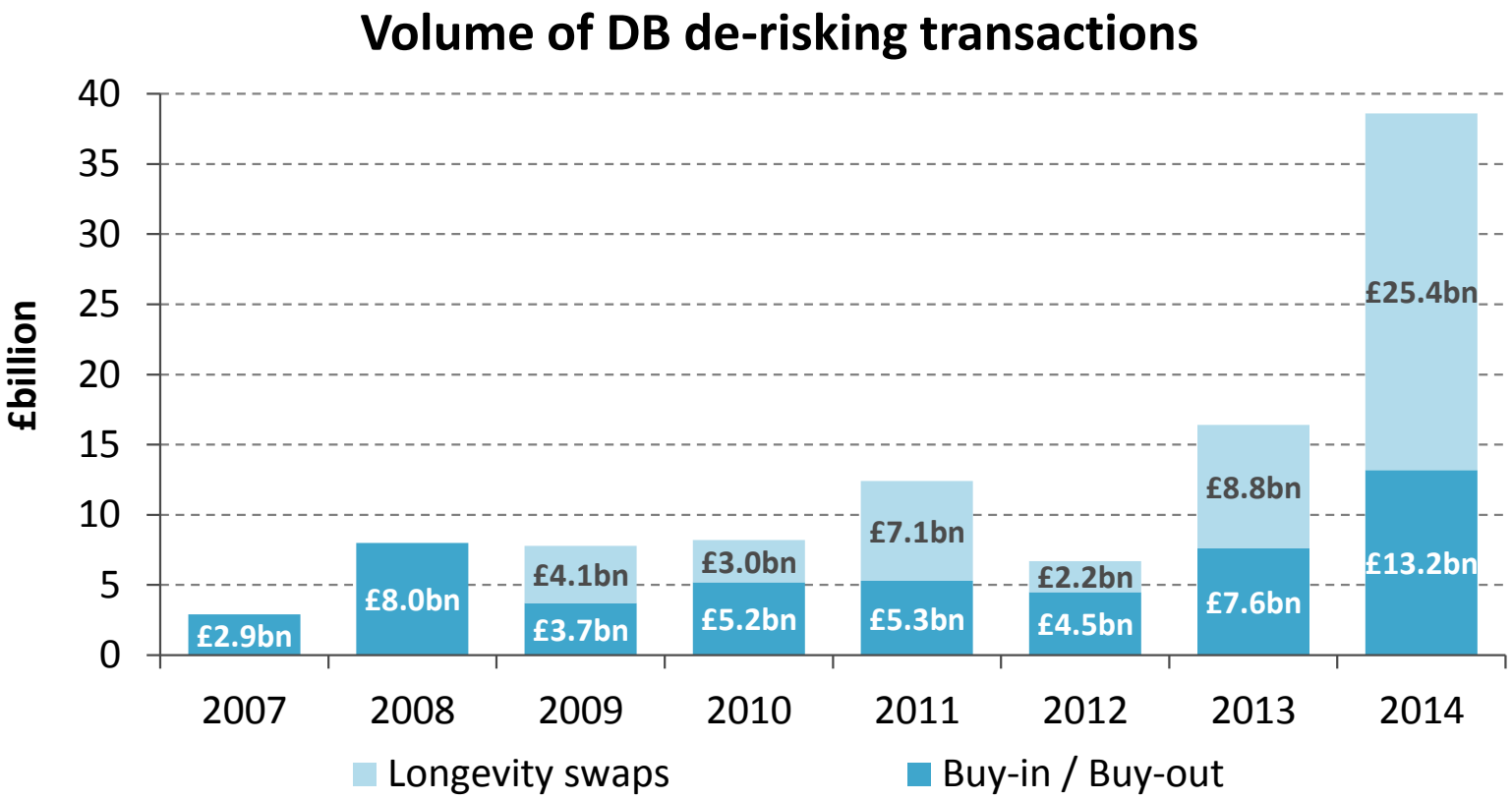
Longevity 11



- Steven Baxter
- 9th September 2015

steven.baxter@hymans.co.uk

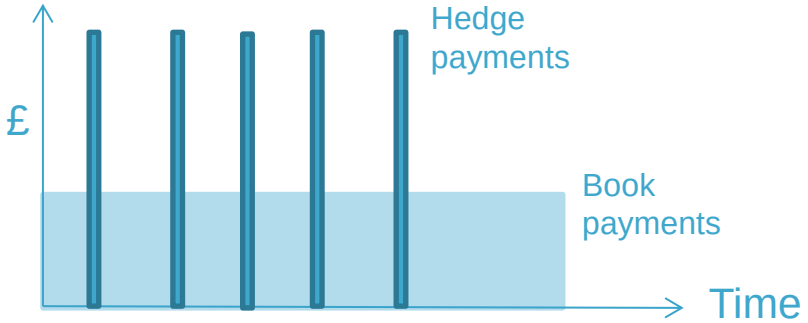
A growing demand for longevity de-risking



Source: Buy-outs, buy-ins and longevity hedging Q1 2015, Hymans Robertson

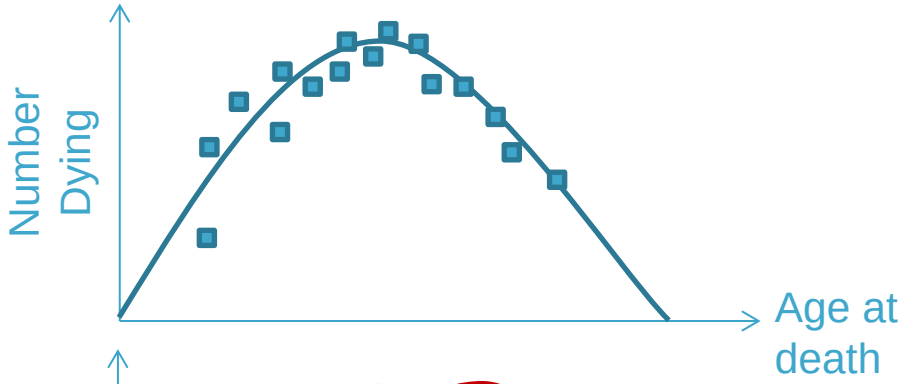
Structuring, Sampling & Demographic Risk

Structuring risk



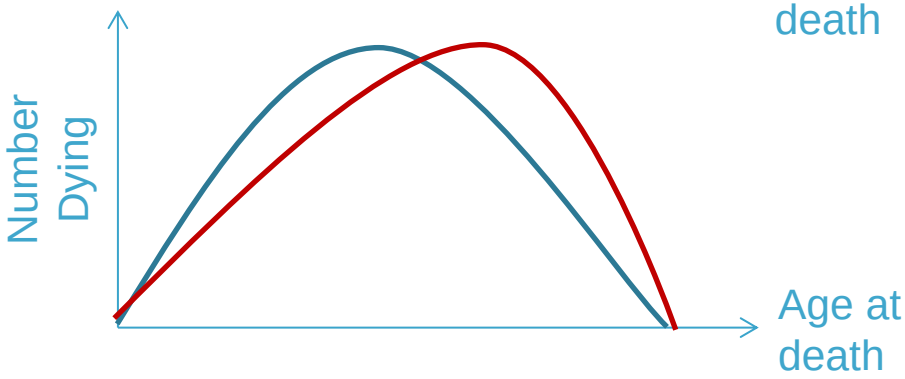
Risk that payoffs from hedging differs to that of portfolio

Sampling risk



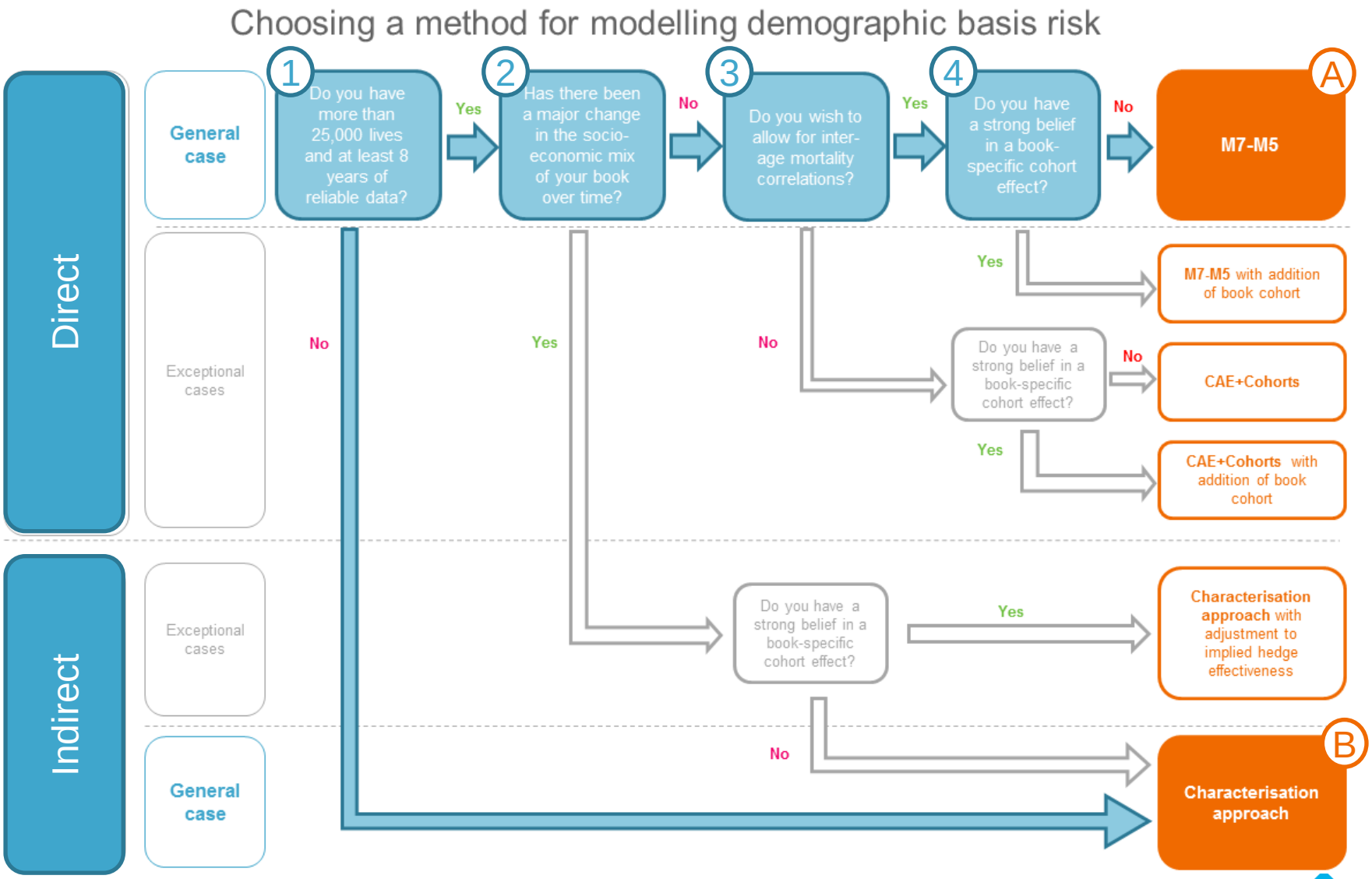
The random outcomes of the individual lives within the portfolio and the index population

Demographic risk

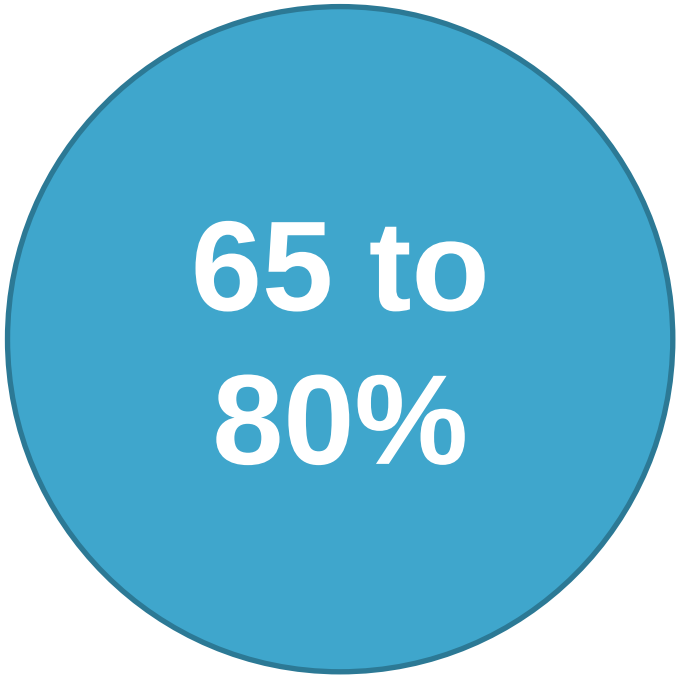


Demographic differences in the composition of the portfolio

Choosing a method



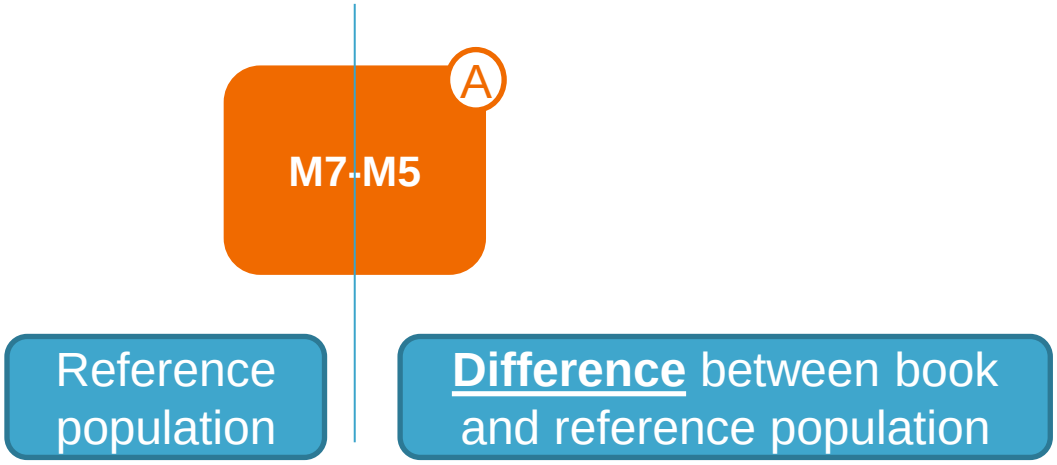
How effective are index-based hedges?



What is direct modelling?

Summary

- Relies on historical experience of
  - Book
  - Reference population
- Calibrates times series models
- Uses results to project future mortality rates for book and reference population



A model for the reference population…

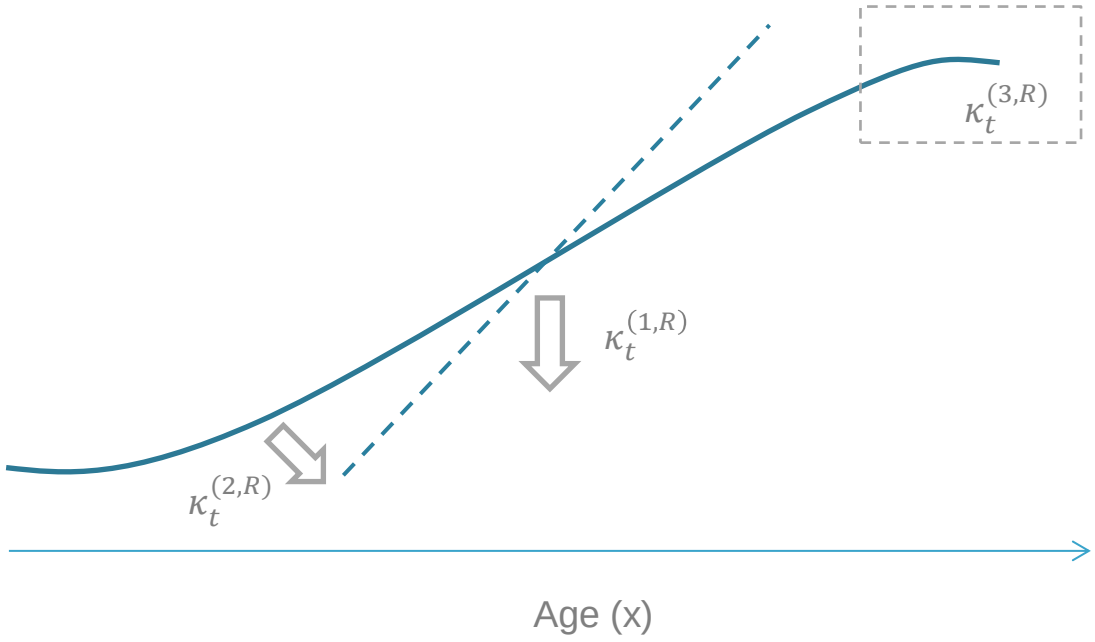
A

M7-M5

Reference population (M7)

$$\text{logit}\, q_{xt}^{R} = \kappa_t^{(1,R)} + (x - \bar{x})\kappa_t^{(2,R)} + \left((x - \bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3,R)} + \gamma_{t-x}^{R}$$

- Transform to a scale in which broadly linear
- Linear term (intercept and slope change over time)
- 'Curl' term (either top or bottom of ages, strength of 'curl' can change over time)
- Cohort term (captures birth year specific impacts)



…and for the book population

A

M7-M5

Book population (M5)

$$\text{logit}\, q^B_{xt} - \text{logit}\, q^R_{xt} = \kappa_t^{(1,\text{B})} + (x - \bar{x})\kappa_t^{(2,B)}$$

- Model difference between book and reference population
- We have explored lots of models and identify that in general
  - A book-specific 'curl' can not be supported
  - A book-specific cohort is not required*

Time series

- To project need to fit a time series to each of the κ_t and γ^R_{t-x}
- Conventionally these would be:
  - $\kappa_t^{(*,R)}$: Multivariate Random Walk with Drift
  - $\kappa_t^{(*,\text{B})}$: Vector Autoregressive of order 1 (VAR(1))
  - γ^R_{t-x} : Autoregressive Integrated Moving Average (ARIMA), typically ARIMA(1,1,0)

* We return to this later. In general a non-parametric cohort effect can not be supported but there may be cases where a parametric one can be justified.

Modifying the method for some cases

2 Has there been a major change in the socio–economic mix of your book over time?

Usual answer: ***No***

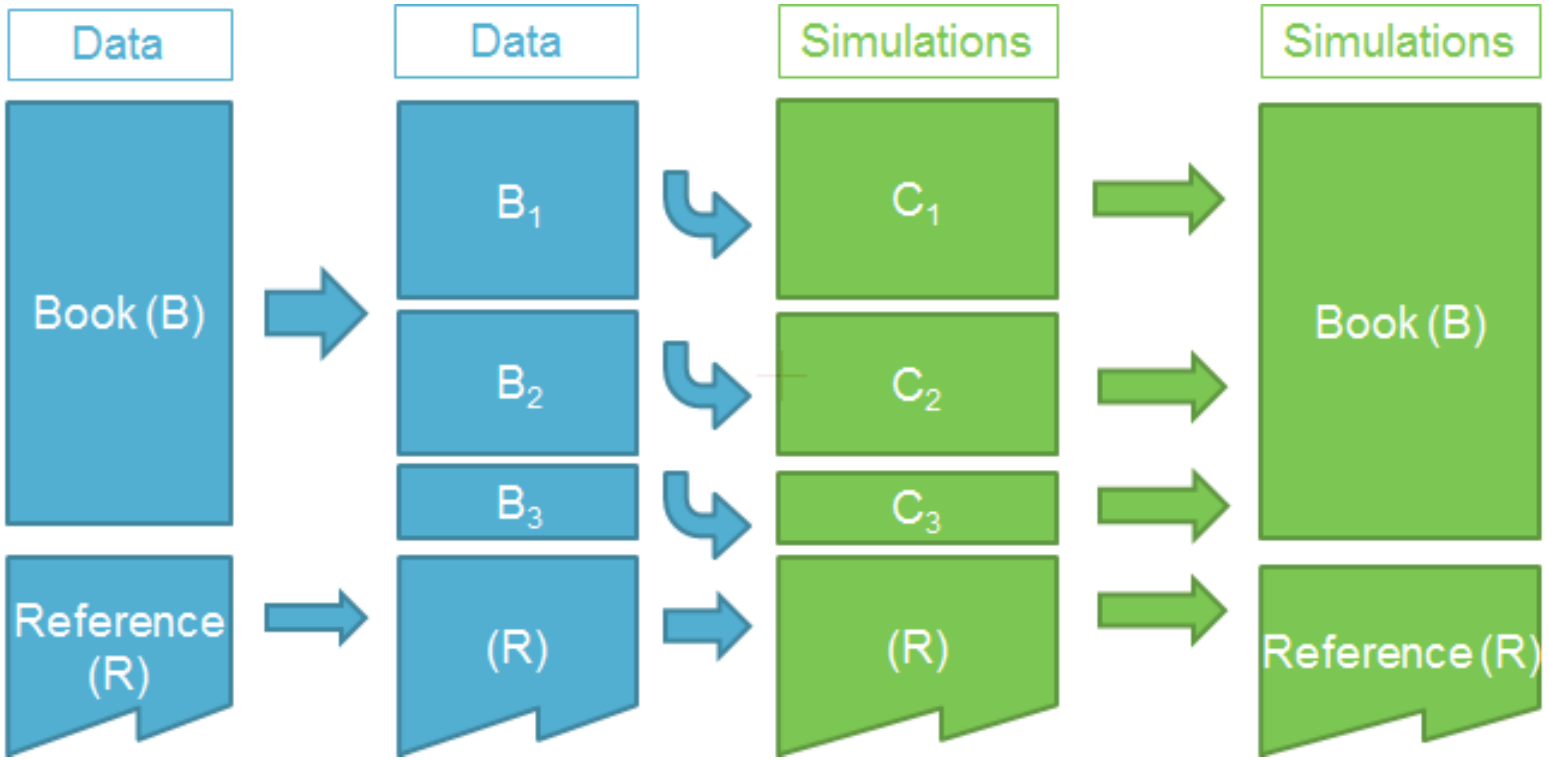
Example ***Yes****:* Back-books for UK individual annuity market

4 Do you wish to allow for a book-specific cohort effect?

Usual answer: ***No***

Example ***Yes****:* Smoker book

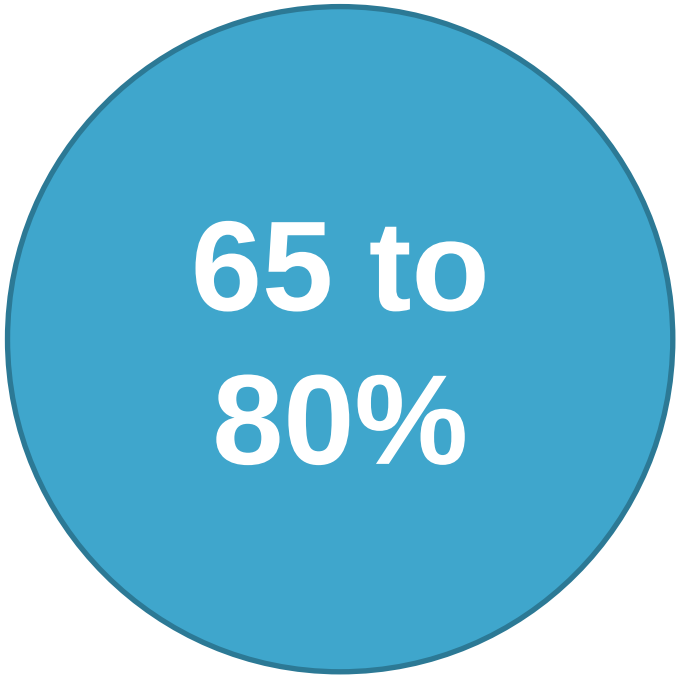
What is indirect modelling?



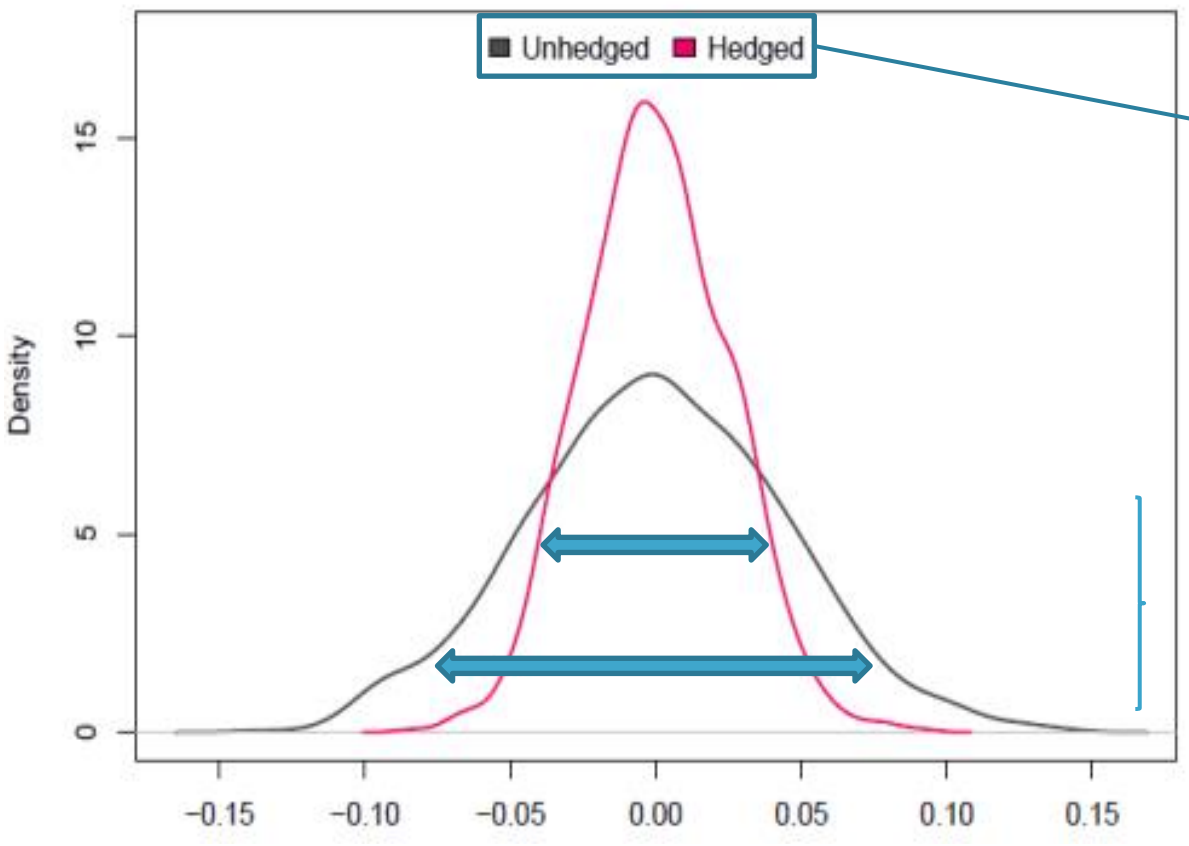
B

Characterisation Approach

How effective are index-based hedges?



A simple measure of hedge effectiveness



Survival probabilities **relative** to average value

$_{20}p^{B}_{70,10}$

1 — 20 year survival probability at time horizon of 10 years

2 — Compare outcomes from book ('unhedged') and book net of reference population ('hedged')
Note: Both presented relative to average

3 — Compare spread of outcomes under 'hedged' to 'unhedged. Reduction in spread is a measure of hedge effectiveness

$$1 - \frac{variance\ of\ hedged}{variance\ of\ unhedged}$$

How effective are index-based hedges?

Portfolio	Direct Modelling
A	78%
B	80%
C	65%
D	77%

65 to 80%

Reference population: England & Wales.

Indirect approach a robust alternative

Portfolio	Direct Modelling	Indirect Modelling
A	78%	84%
B	80%	79%
C	65%	77%
D	77%	80%

Will often give slightly higher hedge effectiveness

Similar results

Reference population: England & Wales.

Indirect modelling approach based upon Club Vita characterising data split by socio-economic groups.

Key model choices

Indirect modelling – which external data to use

		Indirect modelling 'Characterising' dataset	
Portfolio	**Direct modelling**	**Club Vita Socio-economics**	**England & Wales IMD data**
A	78%	84%	88%
B	80%	79%	85%
C	65%	77%	84%
D	77%	80%	85%

5-10% spread

Very granular, highly relevant, licensed access

Less granular, less relevant, publically available

Reference population: England & Wales.

Based upon indirect modelling approach and two different datasets to create characterising groups. Both datasets have applied a vector-autoregressive times series to ensure comparability.

Key model choices

Time series

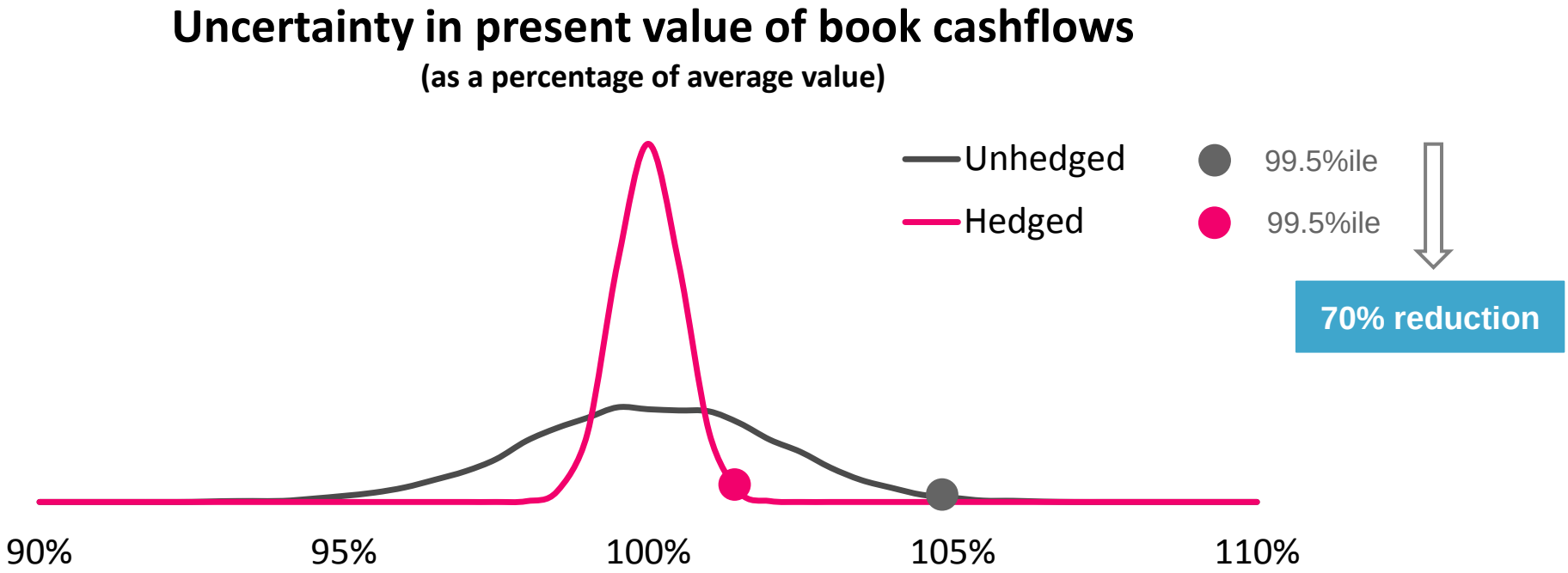
	Time series for $\kappa_t^{(1,C_i)}$ and $\kappa_t^{(2,C_i)}$ in 'M5'	
Portfolio	**E&W IMD data**	**MRWD**
A	88%	77%
B	85%	73%
C	84%	73%
D	85%	75%
	Trending to stable relative mortality	Unbounded divergence

10% spread

Reference population: England & Wales.

Indirect modelling approach based on ONS data split by IMD into three characterising groups C_1,C_2 and C_3. Each has been modelled as an M5 model with correlated times series for the $\kappa_t^{(1,C_i)}$ and $\kappa_t^{(2,C_i)}$ terms.

Alternative metrics



- Initial analysis suggests meaningful (trend) risk reduction under alternative metrics e.g. percentiles of present value of run-off cashflows
- Index-based swaps offer potential for capital relief (*provided* price is right)

Notes on calculation:
Distribution of present values of payments from aportfolio of 60 to 90 year olds. Payments restricted to ages 60 to 90 and 20 calendar years. A net discount rate of 1% has been used at all durations. Modelling assumes a simplified 'buy and hold' strategy on derivatives at outset with derivatives spanning ages 60 to 90 and durations 1 to 20, with strategy based on PV expectations at outset. Risk reduction relates to 'trend risk' (i.e. process risk). Model risk (parameter uncertainty), sampling risk and structuring risk would all need to added on to the numbers shown here. Overall risk reduction will depend on size of book and structuring.

Summary

- Index-based hedges offer **material risk reduction**
- Modelling framework **works** for all sizes of portfolio
- Need to give thought to:
  - **Time series** *(opportunity for user judgement)*
  - **Dataset** when indirect modelling
  - Choice of **index**

References & acknowledgements

References:

- Longevity Basis Risk: A methodology for assessing basis risk

  Available from:
  http://www.actuaries.org.uk/research-and-resources/documents/longevity-basis-risk-methodology-assessing-basis-risk

- A methodology for assessing longevity basis risk: User Guide

  Available from:
  http://www.actuaries.org.uk/research-and-resources/documents/longevity-basis-risk-user-guide

Acknowledgements:

The methodology described in this presentation resulted from research carried out by Cass Business School and Hymans Robertson LLP in response to a call for research from the Life & Longevity Markets Association and the Institute & Faculty of Actuaries.

The research team was:

Hymans Robertson LLP

- Steven Baxter
- Sveinn Gunnlaugsson
- Andrew Gaches
- Mario Sison

Cass Business School

- Prof Steven Haberman
- Prof Vladimir Kaishev
- Dr Pietro Millosovich
- Andres Villegas

HYMANS ROBERTSON

Thank you

Any questions?