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Cass Business School
Faculty of Finance
106 Bunhill Row
London EC1Y 8TZ

Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data

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Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data

by

Niels Haldrup[*], Svend Hylleberg[*], Gabriel Pons[**], and Andreu Sansó[***]

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Abstract

We propose the multivariate representation of univariate and bivariate (possibly non-stationary) periodic models as a benchmark for the imposition of common periodic correlation (CPC) feature restrictions in order to obtain parameter parsimony. CPCs are short-run common dynamic features that co-vary across the different days of the week and possibly also across weeks and which can be common across different time series. It is also shown how periodic models can be used to describe interesting dynamic links in the interaction between stock and flow variables. The proposed modelling framework is applied to a data set of daily arrivals and departures in airport transit data.

Key Words: Periodic autoregression, seasonality, high frequency data, cointegration, common features.

JEL Codes: C12, C22, C32.

[*]Department of Economics, University of Aarhus, Building 322, DK-8000 Aarhus C, Denmark. E-mail: nhaldrup@econ.au.dk, shylleberg@econ.au.dk.[**]Institute of Economics, University of Copenhagen, and AKF, Institute of Local Government Studies, E-mail: gpr@akf.dk [***]Departament d'Economia Aplicada. Universitat de les Illes Balears. E-mail: andreu.sanso@uib.es.

1 Introduction

Periodic models are often considered a convenient and flexible framework to model seasonal variation in the data, see e.g. Ghysels and Osborn (2001) and Franses and Paap (2004). In particular, it was demonstrated by Franses (1994) how the multivariate representation of periodic models due to Gladyshev (1961) and Tiao and Grupe (1980) could be used as a basis for examining the non-stationary properties that frequently characterize economic time series. Because non-periodic models are nested within the periodic model, this is an attractive benchmark for testing various hypotheses. For instance, non-stationarity in the form of periodic integration can be challenged against seasonal integration, see Hylleberg *et al.* (1990).

A frequent criticism concerning the use of periodic models to describe seasonal phenomena is the fact that such models often require a huge number of parameters to be estimated, a problem which grows with the sampling frequency and periodicity of the observations, see Ghysels and Osborn (2001). The present paper makes two contributions. First, it suggests a method to alleviate the problems associated with the potential overparametrization of periodic models by appropriate imposition of periodic cointegration and common feature restrictions on the short run dynamics. Secondly, within a periodic setting the paper scrutinizes the dynamic interaction that may exist in a system with both flow and stock variables which potentially have non-stationary characteristics.

To obtain more parsimonious representations of periodic models, we suggest applying the concept of serial correlation common features, see Engle and Kozicki (1993), within a periodic framework. The notion of serial correlation common features was initially suggested as a convenient way to restrict the short-run dynamics of multivariate models. However, for a periodic model of a univariate (or possibly multivariate) time series (measured at a daily sampling frequency for illustration), it means that the periodic serial correlation features for the single days of the week appear to be common across the days and possibly also when linked to other series. (Of course, this generalizes to data of any periodicity). By imposing such restrictions on the dynamics, the full model can be greatly simplified. This kind of restrictions will be named *common periodic correlations.* The representation of Hecq *et al.* (2005), which discriminates between strong and weak form features, is adapted to periodic models.

Also, we extend the idea of non-synchronous features, see Cubbada and Hecq (2001), which allows common features to co-vary - possibly with a phase shift - across the different days of the week and also potentially across weeks. It is shown that the presence of multiple common periodic correlation features implies a nested reduced rank structure in a multivariate weekly model, which enables more efficient estimation of the highly parametrized model. Also, one may expect improved forecastability by imposing common feature restrictions.

In the present paper, our running example assumes data sampled daily for which a high degree of overparametrization is likely to occur in a periodic context. The sample consists of approximately 8 years of observations of daily arrivals and departures in the airport of Mallorca. The data exhibits a strong degree of periodic variation over the days of the week in addition to a strong seasonal variation over the year. When applying the common periodic correlation methodology to the airport data, it is found that common periodic correlation features is a distinct property of the data, and by imposing the resulting restrictions the number of estimated parameters can be reduced by 30-35 %. The fact that multiple common correlation features can be found in the data reflects that separate short-run dynamic features exist which are common across the different days of the week. Giving an exact economic interpretation of these features can be difficult. However, the common features across the week-days typically reflect travelling habits, the airport- and hotel infra-structure, and contractual agreements between travel agents and hotels.

The arrivals and departure series can be considered flow variables, and by looking at the difference between arrivals and departures on a given day, the net contribution to the stock of airline passengers visiting Mallorca can be calculated. The stock of visitors naturally follows as the accumulation of the net flow of passengers. Interestingly, the stock variable generated in this fashion tends to co-move with the individual arrivals and departures series. This is not a surprising finding because we would expect, based on economic reasoning, that airport transit activity is interrelated with e.g. the stock of hotel visitors. The analysis will show that the stock variable is very close to being *non-periodically* integrated, whereas the arrivals and departure series are clearly *periodically* integrated series. Hence, this opens up for a description of how different (periodic or non-periodic) cointegration possibilities may arise in a complicated dynamic system where flows interact with the stock. The phenomenon that a stock variable will cointegrate with the flow variables (from which it is generated) is called *multicointegration* and was initially defined by Granger and Lee (1989, 1991). The notion of multicointegration cannot be directly adopted to a periodic context, but in the paper we describe how periodic models can be formulated to account for similar features.

The characteristic features of the airport data set are described in Section 2, and special motivation for the stock-flow analysis is given. Section 3 contains a presentation and discussion of the periodic autoregressive model for daily data with focus on the representation of such a model for univariate as well as bivariate stock and flow series. Section 4 discusses the common periodic correlation features both within and across weeks. Section 5 contains the empirical application, and the final section concludes.

2 The data set

We will start by presenting some properties about the data to be examined later to motivate some of the theoretical analyses to be undertaken. The data set used in the paper consists of daily arrivals and departures in the Airport of Mallorca. The data spans the period from 1. January, 1994 to 28. February, 2002. This corresponds to 2981 daily observations (425 weeks). The Balearic Islands, and Mallorca in particular, are amongst the most important tourist destinations in the Mediterranean Sea. The annual volume of tourists is around 10 million people of whom over 95% travel by plane. More than 80% of these are tourists visiting Mallorca.

Since Mallorca is a "sun and sand" tourist destination, it is not surprising that passenger data exhibits a high degree of seasonal variation as can be seen from Figure 1, which displays the data for the full sample period. In addition to the arrivals and departures series, the figure shows the net flow of passengers to Mallorca, (arrivals minus departures), as well as the cumulation of the net flows denoted the stock. Note that the net flow variable indicates the contribution of the airline passengers to the stock of visitors in Mallorca on a given date. (The stock variable indicates the *level* of people staying in Mallorca and not the actual figure because the initial value of observations is unknown). The seasonal fluctuations and the changes over the longer period in the vacation patterns are of great importance for the economic and physical planning in the tourist industry and a proper and precise statistical description based on high frequency data will have a major impact on the planning process at multiple levels.

Figure 1 about here

Figure 2 about here

Figure 3 about here

From figure 1 the yearly variation of the transit data is most obvious. In particular, the very close co-movement of arrivals and departures is apparent suggesting the presence of common features at the low frequencies, e.g. the trend-cycle and the annual seasonal pattern. However, the day-of-week effect is also very apparent as can be seen from figure 2, where the daily variation for the year 2001 is shown. Whereas the arrivals and departures exhibit very strong weekly fluctuations, the net flow and the stock variable obviously have much less variation within the week. This seems to indicate that some kind of common seasonal feature exists amongst the arrivals and departures series.

To focus further on the weekly periodicity, figure 3 displays the individual weekday observations for each series for the year 2001. As can be seen, the arrivals and departures have strong day-of-week effects (especially for Saturdays and Sundays), and this feature seems to vary over the year, and thus suggesting dependency amongst the low-frequency and intra-day dynamics. Moreover, all days seem to co-move, which might indicate that the series potentially can be modelled as periodic processes. For the net flow series and the stock of visitors series, no significant periodic variation seems to be present.

A further aspect of the present data set concerns the possibility of a multicointegration like feature amongst the series, see Granger and Lee (1989, 1991). If we assume that the arrivals and departures series are cointegrated, it is of interest to look at the cumulated net flow series, i.e. the stock of visitors variable generated from the arrivals and departures. Interestingly, it appears from figures 1 and 2 that although the stock series has much less (if any) weekly variation, the level around some trend co-varies with both the arrivals and departures series. It is not surprising that the level of the arrivals and departures series co-vary with the level of the stock of visitors variable because airport transit activity necessarily has to be reflected in the number of visitors. Statistically, however, this is an interesting phenomenon because it allows for the possibility of more than a single cointegrating relationship existing between just two series (arrivals and departures). This is the property known as multicointegration. There are numerous examples of multicointegration in the literature (Granger and Lee (1989, 1991), Lee (1992), Engsted and Haldrup (1999), Leachman (1996), Leachman and Francis (2000)) even though less focus has been devoted to account for the implications when analyzing seasonal data. The notion of multicointegration cannot be generalized directly to seasonal data, but the challenge of the present paper includes an examination of how similar properties can occur in describing the interactions between stock and flow variables in a periodic model context.

3 A Periodic Autoregressive Model for daily observations

As argued in section 2, it is likely that the arrivals and departures series follow periodic processes. Periodic models have frequently been criticized because such models require a lot of parameters to be estimated. Our aim is to develop a methodology to partially alleviate this problem. The periodic model formulation has a number of advantages. One implication is that models with fixed parameters and standard seasonal ARIMA processes, including seasonal unit root processes, are encompassed within the periodic model for certain restrictions on the parameters, and hence these restrictions can be tested. Hence the model framework is rather flexible. First we will review some known properties of periodic autoregressive models sampled at a daily periodicity.

3.1 The representation and properties of the model

Seasonal processes with a periodic correlation structure can be represented by periodic ARMA (henceforth PARMA) models, which allow for different parameters across the seasons. In practice, the estimation of pure PAR models has certain advantages over PARMA models, see Pagano (1978) and the review by McLeod (1994).

Let us describe the most relevant characteristics of the periodic model for a univariate time series where the periodicity is allowed for the day of the week. General comprehensive surveys of periodic models and required inferential tools can be found in e.g. Ghysels and Osborn (2001) and Franses and Paap (2004). In the following we abstract from deterministic components to simplify notation, but extensions to this case are straightforward.

The daily periodic autoregressive process of order p, PAR(p), reads:

$$y_t = \phi_{s,1} y_{t-1} + \phi_{s,2} y_{t-2} + \cdots + \phi_{s,p} y_{t-p} + \varepsilon_t, \; s = 1, .., 7 \tag{1}$$

where all the autoregressive parameters $\phi_{s,j}$ $(j = 1, ..., p)$ are allowed to vary with the season s, $(s = 1, ..., 7)$, i.e. the day of the week. Note that the season s is related to the observation number t through $s \equiv t (\text{mod}\, 7) + 1$. It should hold that at least one $\phi_{s,p} \neq 0$. ε_t is a white noise error term with periodic heteroskedasticity, $E\left(\varepsilon_t^2\right) = \sigma_s^2$. Note that, in this model, the parameters are allowed to be different for each day of the week, and therefore the PAR process is non-stationary since the autocorrelation function varies with the season.

Another interesting source of nonstationarity frequently observed for economic data is the presence of stochastic trends, which can be examined within a multivariate representation of the PAR process. We denote this the vector of days (VD) representation. This representation defines the 7-dimensional weekly multivariate process $Y_\tau \equiv (y_{1,\tau}, ..., y_{7,\tau})'$ with $\tau \equiv [(t-1)/7] + 1$ denoting the week and where we let $y_{s,\tau} \equiv y_{7(\tau-1)+s}$. The vector series has the following multivariate (nonperiodic) representation, (see Gladyshev, 1961, or Tiao and Grupe, 1980):

$$\Phi_0 Y_\tau = \Phi_1 Y_{\tau-1} + \cdots + \Phi_P Y_{\tau-P} + E_\tau, \tag{2}$$

where Φ_k $(k = 0, ..., P;\; P = [(p+6)/7])$ are appropriately defined 7×7 matrices of parameters, and $E_\tau \equiv (\varepsilon_{1,\tau}, ..., \varepsilon_{7,\tau})' \sim$N(0,$\Sigma$), where $\Sigma =$diag$(\sigma_1^2, \sigma_2^2, ..., \sigma_7^2)$.

The unit root properties of the multivariate process Y_τ determine those of the daily process y_t. Define the matrix lag polynomial

$$\Phi\left(L^7\right) = \Phi_0 - \Phi_1 L^7 - \cdots - \Phi_P L^{7P},$$

where $L y_{s,\tau} = y_{s-1,\tau}$ (with $L y_{1,\tau} = y_{7,\tau-1}$) and $L^7 y_{s,\tau} = y_{s,\tau-1}$. When all the roots of the characteristic equation $\left|\Phi\left(L^7\right)\right| = 0$ lie outside the unit circle, the process Y_τ is second order stationary, and y_t is PI(0). In the PAR(1) case for example, i.e. when $y_t = \phi_s y_{t-1} + \varepsilon_t$, the necessary and sufficient condition for second order stationarity of this model is $|\phi_1 \phi_2 \cdots \phi_7| < 1$. The multivariate process is integrated at the zero frequency if $|\Phi(L^7)| = 0$ has some roots equal to one.

A convenient way to represent the different possibilities of unit roots in daily processes is the error correction representation, see Franses (1994). Consider the VAR representation of (2) as:

$$\Pi(L^7) Y_\tau = U_\tau, \tag{3}$$

where $\Pi(L^7) = I_7 - \Pi_1 L^7 - ... - \Pi_P L^{7P}$, with $\Pi_k \equiv \Phi_0^{-1} \Phi_k$, and $U_\tau = \Phi_0^{-1} E_\tau \sim$N$(0, \Phi_0^{-1} \Sigma (\Phi_0^{-1})')$. Decompose the matrix lag polynomial as $\Pi(L^7) = -\Pi L^7 + \Gamma(L^7)(1 - L^7)$ where $\Pi = \Phi_0^{-1} \left(\sum_{j=1}^{P} \Phi_j \right) - I_7$, $\Gamma_0 = I_7$, and $\Gamma_k = \Phi_0^{-1} \Sigma_{j=k+1}^{P} \Phi_j$ $(k = 1, .., P-1)$ such that we obtain the VAR model on error correction form:

$$\Delta_7 Y_\tau = \Pi Y_{\tau-1} + \sum_{k=1}^{P-1} \Gamma_k \Delta_7 Y_{\tau-k} + U_\tau, \tag{4}$$

with $\Delta_7 = 1 - L^7$.

The different types of unit roots in the daily processes are associated with different properties of the impact matrix Π. In particular, when Π has rank 7, the process y_t is periodically integrated of order zero, PI(0), and when Π has rank 6, y_t is periodically integrated of order one, PI(1) with $\prod_{s=1}^{7} \phi_s = 1$. In this case we may distinguish two important cases. When the 6 cointegrating relations are given by $y_{2,\tau} - y_{1,\tau}$, $y_{3,\tau} - y_{2,\tau}$, ..., $y_{7,\tau} - y_{6,\tau}$, then y_t is a non-seasonally and non-periodically integrated process, i.e. an I(1) process. When the 6 cointegrating relations read $y_{2,\tau} - \phi_2 y_{1,\tau}$, $y_{3,\tau} - \phi_3 y_{2,\tau}$, ..., $y_{7,\tau} - \phi_7 y_{6,\tau}$ with at least one $\phi_s \neq 1$ $(s = 2, ..., 7)$, then y_t is PI(1) where ϕ_s are named the periodic integration coefficients. Hence the I(1) model appears as a special case of the PI(1) model. When $y_t \sim$PI(1), the difference operator Δ does not remove the stochastic trend from y_t. In this case it is necessary to apply a specific difference filter for every season, the quasi-difference filter, $\delta_s(L) \equiv 1 - \phi_s L$, such that $\delta_s(L) y_t \sim$PI(0).

When Π has rank $0 \leq r < 6$, and proper restrictions on the cointegration space apply (see Franses, 1994), y_t is a seasonally integrated process with $7 - r$ unit roots at seasonal frequencies, see Hylleberg *et al.* (1990), Franses (1994), and Ghysels and Osborn (2001).

Generally, under the reduced rank of Π $(0 < r < 7)$, the impact matrix can be decomposed as $\Pi = \alpha\beta'$, where α and β are $7 \times r$ matrices of full column rank that contain the adjustment vectors and the cointegrating vectors, respectively. Then we can rewrite (4) as

$$\Delta_7 Y_\tau = \alpha\beta' Y_{\tau-1} + \sum_{k=1}^{P-1} \Gamma_k \Delta_7 Y_{\tau-k} + U_\tau. \tag{5}$$

We denote the r-dimensional cointegrating disequilibrium process by $Z_\tau \equiv \beta' Y_\tau$.

For instance, the general PAR process is PI(1) when $\phi_1\phi_2 \cdots \phi_7 = 1$, (but not all $\phi_s = 1$), and the cointegrating matrix β will contain six of the coefficients ϕ_s:

$$\beta' = \begin{pmatrix} -\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi_6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\phi_7 & 1 \end{pmatrix}, \tag{6}$$

and the remaining coefficient is $\phi_1 = (\phi_2\phi_3\phi_4\phi_5\phi_6\phi_7)^{-1}$.

Note that the common stochastic trend can be found as $\beta'_\perp Y_\tau$ where $\beta_\perp$ is the orthogonal complement of β satisfying $\beta'_\perp \beta = 0$.

The multivariate representation can be used to select among the different types of unit roots by means of multivariate cointegration analysis (see Johansen, 1991). This procedure is proposed by Franses (1994) for quarterly PAR models. The same method can be used to test for periodic integration of the daily flows and stock series. Within the unifying framework of a periodic model we can test for a multitude of different types of order unit roots, which is somewhat more involved when considering the daily representation of the time series, see Ghysels and Osborn (2001).

3.2 Bivariate Periodic Models

3.2.1 Cointegration between Flow Variables

In this section we consider the multivariate representation of the daily flow variables y_t^1 and y_t^2. Consider the weekly representation of the daily bivariate process $\mathbf{y}_t = (y_t^1, y_t^2)'$, where now we define the 14−dimensional VD process $\mathbf{Y}_\tau \equiv (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$.

Assume that it can be represented by a VAR(P) model and an error correction model similar to (3) and (4) with $\mathbf{y}_t$ and $\mathbf{Y}_\tau$ in place of y_t and Y_τ with appropriately redefined dimensions.

Assume that the series in $\mathbf{y}_t$ are I(1) or PI(1), then under the presence of (periodic) cointegration between the daily series, the 14 weekly series have a common stochastic trend, and the impact matrix $\mathbf{\Pi}$ can be written $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are full column rank $14 \times 13-$matrices,

$$\boldsymbol{\beta}' = \begin{bmatrix} I_7 & \mathbf{K} \\ \mathbf{0} & \beta^{2\prime} \end{bmatrix}. \tag{7}$$

$\mathbf{I}_7$ is the 7-dimensional identity matrix, $\mathbf{0}$ is the 6×7-dimensional null matrix, $\mathbf{K}$ is a 7-dimensional matrix containing the cointegrating coefficients on the diagonal, $\mathbf{K}$ =diag$(-k_1, -k_2, ..., -k_7)$, and hence $y^1_{s,\tau} - k_s y^2_{s,\tau}$ ~PI(0) (s=1,...,7). β^2 is the 7×6-dimensional matrix containing the periodic integration coefficients associated with y^2_t. We define the general notation

$$\beta^{i\prime} = \begin{bmatrix} -\phi^i_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi^i_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi^i_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi^i_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi^i_6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\phi^i_7 & 1 \end{bmatrix}, \tag{8}$$

where ϕ^i_s are the periodic integration coefficients for the series y^i_t.

The model above assumes *rank* $\mathbf{\Pi} = r = 13$ and thus indicating that there is a single common stochastic trend driving the full system of 14 series (e.g. the arrivals and departures for each day of the week). An exactly identified structure can therefore be imposed by $r - 1 = 12$ independent zero restrictions on the rows of the $\boldsymbol{\beta}'$ matrix. Indeed, the structure defined in (7) satisfies this requirement. The implication of this is that *any* pair of variables in the 14 dimensional system will cointegrate. One advantage of the representation (7) is that the periodically varying cointegrating coefficients can be directly tested.

Osborn (2002) discusses how periodically and non-periodically integrated processes can potentially cointegrate in various cases. When both daily variables y^i_t $(i = 1, 2)$

are I(1), then $k_s = k$ $(s = 1, ..., 7)$, and $\phi_s^2 = 1$ $(s = 1, ..., 6)$, that is, in this case the daily variables are non-periodically cointegrated. When both daily variables y_t^i $(i = 1, 2)$ are PI(1), the cointegrating vectors may be different across the different days of the week, i.e. such that the series are fully periodically cointegrated $k_s \neq k$ (at least for some s), and $k_s \neq 0$ $(s = 1, ..., 7)$. However, the series could also be non-periodically cointegrated such that $k_s = k$, if $\phi_s^1 = \phi_s^2$, $(s = 1, ..., 7)$. When one daily variable is I(1), and the other one is PI(1), the variables may only be fully periodically cointegrated.

Under the absence of (periodic) cointegration between y_t^1 and y_t^2, the 14 weekly series have two stochastic trends. In particular, the impact matrix reads $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are full column rank $14 \times 12-$matrices, where e.g.

$$\boldsymbol{\beta}' = \begin{bmatrix} \beta^{1\prime} & \mathbf{0} \\ \mathbf{0} & \beta^{2\prime} \end{bmatrix},$$

and $\mathbf{0}$ is the 6×7-dimensional null matrix.

The multivariate representation of the flow variables is the basis for testing periodic cointegration between daily arrivals and daily departures. First, one should test for cointegration using e.g. the ML procedure of Johansen (1991). Then, one may test for non-periodic cointegration through the hypothesis $k_s = k$ $(s = 1, ..., 7)$. Given the relation $\phi_s^1 = \phi_s^2 k_s / k_{s-1}$, the test for non-periodic cointegration can be interpreted as a test of equivalent periodic integration coefficients $\phi_s^1 = \phi_s^2$ $(s = 1, ..., 7)$, such that under non-periodic cointegration, the periodic integration coefficients for the departures are also the periodic integration coefficients of the arrivals.

3.2.2 Cointegration between Stock and Flow Variables

Assume that the net flow $y_t^3 = y_t^1 - y_t^2$ is PI(0). Consequently there exists fully non-periodic cointegration between the daily arrivals y_t^1 and the daily departures y_t^2, and the stock variable $y_t^4 = y_0^4 + \sum_{j=1}^{t} y_j^3$ is I(1) by definition. (Note that the non-stationary feature following from the cumulation of a PI(0) process can be characterized as I(1). A PI(1) process is defined by using the periodic cumulation filter $(1 - \phi_s L)^{-1}$, with $\prod_s \phi_s = 1$, to a stationary sequence which can be PI(0) or I(0). In case the parameters are one across all seasons the PI(1) is in fact an I(1) process). It is then feasible that the daily stock y_t^4 cointegrates with the daily arrivals or with the daily departures much alike multicointegration. The property can be analyzed in the same way as the cointegration between the arrivals and departures by substituting for example the seven weekly arrivals (or departures) series by the seven stock series in $\mathbf{Y}_\tau$.

Potentially, the analysis can be undertaken in a smaller system when the stock variable is *non*-periodically integrated. Hence, if the flow variable is PI(1), and the stock variable is non-periodic I(1), then if these variables are cointegrated, cointegration between the stock and the flow variables can only be periodic (see Osborn, 2002)

$$y^2_{s,\tau} - k_s y^4_{s,\tau} \text{ with } k_s \neq k \text{ for some } k_s.$$

Because the stock variable $y^4_{s,\tau}$ is non-periodically integrated by assumption, it is natural to reduce the model from a 14-dimensional to a 8-dimensional system. Consider the following transformation matrix of dimension 8×14 :

$$\mathbf{\Xi} = \begin{pmatrix} I_7 & \mathbf{0} \\ \mathbf{0} & \frac{1}{7}\beta^{4\prime}_{\perp} \end{pmatrix},$$

where $\beta^4_\perp$ is the orthogonal complement associated with β^4 given in (8). In this particular case, *non*-periodic integration implies that $\phi^4_s = 1$, $(s = 1, ...7)$, where it can be shown that the orthogonal matrix simplifies to a vector of ones. This means that the reduced system can be defined for the variables $\mathbf{Y}^*_\tau = \mathbf{\Xi}\mathbf{Y}_\tau = (y^2_{1,\tau}, y^2_{2,\tau}, ..., y^2_{7,\tau}, \overline{y}^4_\tau)$ where $\overline{y}^4_\tau = \frac{1}{7}\sum_{s=1}^{7} y^4_{s,\tau}$ is the weekly average of the stock series. The new system reads

$$\Delta_7 \mathbf{Y}^*_\tau = \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*\prime} \mathbf{Y}^*_{\tau-1} + \sum_{k=1}^{P-1} \mathbf{\Gamma}^*_k \Delta_7 \mathbf{Y}^*_{\tau-k} + \mathbf{U}^*_\tau,$$

where $\boldsymbol{\beta}^*$ in this case takes the form

$$\boldsymbol{\beta}^{*\prime} = \begin{bmatrix} I_7 & -\mathbf{k} \end{bmatrix}. \tag{9}$$

where $\mathbf{k} \equiv (k_1, k_2, k_3, k_4, k_5, k_6, k_7)'$ is a 7×1 column vector containing the coefficients relating stock and flow variables such that $y^2_{s,\tau} - k_s\overline{y}^4_\tau$ is PI(0). Note that alternatively the stock variable of the single days represents the common I(1) stock trend.

When y^4_t and y^2_t are not cointegrated, then $\mathbf{\Pi}^* = \boldsymbol{\alpha}^*\boldsymbol{\beta}^{*\prime}$, where $\boldsymbol{\alpha}^*$ and $\boldsymbol{\beta}^*$ are full column rank 8×6 matrices, and the cointegration matrix can be written

$$\boldsymbol{\beta}^{*\prime} = \begin{bmatrix} \beta^{2\prime} & \mathbf{0} \end{bmatrix},$$

where $\mathbf{0}$ is a 6×1 column vector of zeros, and β^2 is given by (8).

4 Common Periodic Correlation Features

When building PAR models it is recommended to introduce restrictions on the periodicity of the parameters to increase the degrees of freedom and to gain parsimony, see Ghysels and Osborn (2001). We propose a new way to do this by imposing appropriately tested common feature restrictions on the model like common business cycles, common stationary annual seasonality, or common deterministic annual seasonality, see e.g. Engle and Hylleberg (1996). It is a very plausible assumption that the daily series y_t, in addition to the stochastic trend, will share common features across the days of the week. To our knowledge, these kinds of restrictions to describe the common short-run dynamics of different seasons have not yet been proposed in the literature.

4.1 Common Periodic Correlation features within the week

Engle and Kozicki (1993) introduced the notion of serial correlation common features to represent common cycles among different economic time series. For an n-dimensional system like (4) (e.g. with $n = 7, 8, 14$), we get that if there exists an $n \times q$ matrix $\widetilde{\beta}$ that annihilates both the short-run and the long-run dynamics

$$\begin{array}{llcll} \text{(i)} & \widetilde{\beta}'\Gamma_k & = & 0 & (k = 1, ..., P-1), \\ \text{(ii)} & \widetilde{\beta}'\Pi & = & -\widetilde{\beta}'\alpha\beta' = 0, & \end{array}$$

then Y_τ is said to have serial correlation common features.

Under these two conditions, the cofeature matrix $\widetilde{\beta}$ turns the differenced variables into a q-dimensional white noise process $\widetilde{\beta}'\Delta_7 Y_\tau = \widetilde{\beta}'U_\tau$, and the short-run dynamics of the n series is driven by $n - q$ dynamic factors. However, in this case the number of common features $n - q$ is bounded by the cointegration rank r, $r \leq n - q \leq n$. When the daily series ($n = 7$) is PI(1) or I(1), the number of common features cannot be smaller than 6. Hence, the serial correlation common features allow only little flexibility concerning the imposition of restrictions on the periodicity of the process.

Hecq *et al.* (2005) consider a less restrictive form of common cycles and introduce the idea of a Weak Form (WF) of serial correlation common features, which requires that the cofeature matrix removes the short-run component, but not the long-run. Hence, under i), but not ii), different common factors generate the long-run and the short-run dynamics of the variables. In the present case, under the WF structure there exists an $n \times q$ dimensional cofeature matrix $\widetilde{\beta}$ that turns the differenced variables adjusted for long-run effects into a q-dimensional white noise process $\widetilde{\beta}'(\Delta_7 Y_\tau - \alpha Z_{\tau-1}) = \widetilde{\beta}' U_\tau$. Then the cointegrated system can be expressed as

$$\Delta_7 Y_\tau = \alpha Z_{\tau-1} + \widetilde{\beta}_\perp \mathbf{W}_{\tau-1} + U_\tau,$$

where $Z_\tau = \beta' Y_\tau$, and $\mathbf{W}_\tau = \mathbf{\Upsilon X}_\tau$ contains the serial correlation common features. $\mathbf{X}_\tau = (\Delta_7 Y_\tau',...,\Delta_7 Y_{\tau-P+2}',)'$, $\mathbf{\Upsilon} \equiv [\Upsilon_1, ..., \Upsilon_{P-1}]$ is a $(n-q) \times (n(P-1))$ matrix, and $\widetilde{\beta}_\perp$ is an $n \times (n-q)$ full column rank matrix satisfying $\widetilde{\beta}'\widetilde{\beta}_\perp = 0$. In Hecq *et al.* (2005) inferential procedures for common serial correlation feature models are discussed in detail using canonical correlation techniques.

The notion of WF serial correlation common features is more flexible than the strong form in our setting because the number of common features $(n-q)$ is not bounded by the cointegrating rank. Concretely, the number of common features may take any value between 1 and n. For example, in the case of the multivariate representation of one of the daily series $(n = 7)$, $1 \leq 7 - q \leq 7$. When $7-q = 1$, the short-run dynamics of the seven day-of-week series are driven by the same factor. On the other extreme, when $7-q = 7$, the short-run dynamics of the single days is generated by different factors. In our framework, when $0 < q < n$, we name such common features *common periodic correlation (CPC) features*, and $n - q$ denotes the number of CPC features.

The CPC feature in the periodic correlation framework implies that the short-run dynamics of each day of the week (including the stationary annual seasonality as well as the business cycles) is driven by a reduced number of factors. To see this, consider the Beveridge-Nelson representation of the vector of days Y_τ, which can be written as $Y_\tau - C(1)\Sigma_{i=1}^{\tau} U_i = C^*(L^7)U_\tau$ (see Vahid and Engle, 1993). The CPC features imply the reduced rank of $C^*(L^7)$. As a consequence, the elements of the covariance matrix of $Y_\tau - C(1)\Sigma_{i=1}^{\tau} U_i$ are linearly dependent, and therefore the periodic autocovariance function of the short-run component of the daily periodic process is linearly dependent as well.

Strictly speaking, the common dynamic factors are asynchronous in terms of the daily model since they relate to different days of the week, but they are synchronous in terms of the weekly representation. Obviously, it is likely that asynchronous common cyclical components occur also in the weekly representation in the sense that the dynamics of consecutive days of different weeks could be as close as the dynamics of consecutive days of the same week.

Other types of restrictions regarding the short run matrices Γ_k in (5), including zero restrictions, could in principle be considered as well. However, it is not clear how such restrictions should be interpreted in terms of the underlying structural parameters Φ_j since $\Gamma_k = \Phi_0^{-1} \sum_{j=k+1}^{P} \Phi_j$.

4.2 Common Periodic Correlation features across the weeks

When considering the presence of common periodic cyclical features between high frequency variables, say the arrivals and the departures, it is likely that such variables will exhibit non-contemporaneous cyclical co-movements in the sense that cycles co-move with a phase shift of a particular number of days *exceeding* a week; a property that is not captured by the CPC features described in the preceding section. The notion of a polynomial serial correlation common feature (Cubadda and Hecq, 2001) can also be considered in its weak form (Hecq *et al.,* 2005), which in this periodic context we name *weak form polynomial CPC features.*

The PI(1) process Y_τ has weak form polynomial CPC of order m ($m < P-1$), denoted CPC(m), if there exists a $7 \times q_m$ polynomial matrix $\widetilde{\beta}_m(L) = \sum_{j=0}^{m} \widetilde{\beta}_{m,j} L^j$ such that $\widetilde{\beta}_{m,0}$ has full column rank, $\widetilde{\beta}_{m,m} \neq 0$, and

$$\widetilde{\beta}'_{m,0}\Gamma_k = \begin{cases} -\widetilde{\beta}'_{m,k} & \text{if } k = 1, ..., \ m, \\ 0 & \text{if } k > m. \end{cases}$$

Under CPC(m), the cofeature matrix reduces the order of the error correction model from $P-1$ to m, $\widetilde{\beta}'_{m,0}\left(\Delta_7 Y_\tau - \alpha Z_{\tau-1}\right) = -\widetilde{\beta}'_{m,1}\Delta_7 X_{\tau-1} - ... - \widetilde{\beta}'_{m,m}\Delta_7 X_{\tau-m} + \widetilde{\beta}'_{m,0}U_\tau$, such that under CPC($m$) the cointegrated system can be written as

$$\Delta_7 Y_\tau = \alpha Z_{\tau-1} + \sum_{k=1}^{m} \Gamma_k \Delta_7 Y_{\tau-k} + \widetilde{\beta}_{m,0\perp} \sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 Y_{\tau-j} + U_\tau,$$

where Υ_j are $(n-q_m) \times 7$ matrices, $\widetilde{\beta}_{m,0\perp}$ is a $7 \times (n-q_m)$ full column rank matrix satisfying $\widetilde{\beta}'_{m,0}\widetilde{\beta}_{m,0\perp} = 0$, and $\sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 Y_{\tau-j}$ contains the $(n-q_m)$-dimensional common dynamic factor. The presence of the CPC(m) implies restrictions on the periodic coefficients in a similar way as non-polynomical CPC (or CPC(0)), but now involving only more distant lags, and therefore implies complex restrictions among the autocorrelation coefficients of the cyclical component of Y_τ.

Notice that CPC(m) of different orders may cohabit in the error correction model and thus imply different restrictions on the autocorrelation structure. To illustrate this consider the 7-dimensional error correction model $\Delta_7 Y_\tau = \alpha\beta' Y_{\tau-1} + \Gamma_1 \Delta_7 Y_{\tau-1} + \Gamma_2 \Delta_7 Y_{\tau-2} + U_\tau$. In the unrestricted case without any CPC features, the short-run matrices Γ_1 and Γ_2 each contain 49 parameters, that is, we have 98 parameters to estimate. Now, consider the case where we have a contemporaneous common feature CPC(0) of $q_0 = 3$. In this case, $\Gamma_1 = \widetilde{\beta}_\perp \Upsilon_1$, and $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$, where $\widetilde{\beta}_\perp$ is $n \times (n - q_0) = 7 \times 4$, and Υ_1 and Υ_2 are both $(n - q_0) \times n = 4 \times 7$ matrices, which implies a total of 84 parameters. Next, consider the case with polynomial common feature CPC(1) of $q_1 = 4$. Under this property there are no synchronous common dynamic factors, and Γ_1 is $n \times n = 7 \times 7$, with 49 parameters. There are three common dynamic factors among the day-of-week series, where at least one of these elements pertains to the preceding week, and $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$ with $\widetilde{\beta}_\perp$ being $n \times (n - q_1) = 7 \times 3$ and Υ_2 being 3×7 matrices. Hence, the number of parameters is $49 + 42 = 91$. Alternatively, consider the case where we have CPC(1) with $q_1 = 6$. Here Γ_1 is $n \times n = 7 \times 7$ as above, while $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$ with $\widetilde{\beta}_\perp$ being 7×1 and Υ_2 being 1×7 matrices, such that the number of short-run parameters is given by $49 + 14 = 63$.

Some limitations of the existing methods to detect the presence of common features (see Cubadda and Hecq, 2001, and Hecq *et al.*, 2005) are that the statistical tests for q_0 CPC(0) and for q_1 CPC(1), for instance, are not independent, i.e. the test for non-polynomial common features imposes the same rank for all the short-run matrices, while the test for polynomial common features tells nothing about the first short-run matrices. All in all, we can test for the presence of different CPC(m), but we cannot impose the structure implied by all of them. A solution is thus to select the CPC(m) that is the most parsimonious representation of the short-run dynamics. Returning to our example, we may distinguish two VAR models, one with CPC(0) of $q_0 = 3$ and CPC(1) of $q_1 = 4$, and another one with CPC(0) of $q_0 = 3$ and CPC(1) of $q_1 = 1$. The CPC(0) implies a more parsimonious representation of the short-run dynamics in the first model, while the CPC(1) gets the biggest reduction of parameters in the second model. We suggest selecting the most parsimonious model. Again the estimation procedure of Cubbada and Hecq (2001) can be used to estimate the models using canonical correlation analysis where the canonical correlation problem to be solved (abstracting from deterministics) reads $CanCor(\Delta Y_\tau, (\Delta Y_{\tau-j}, ..., \Delta Y_{\tau-p})|(\widehat{\beta}' Y_{\tau-1}, \Delta Y_{\tau-1}, ...\Delta Y_{\tau-j+1}))$. As a particular case when $j = 1$ the Hecq *et al.* (2005) procedure follows for the weak form case. The imposition of a CPC(m) structure does not only allow more efficient estimation of the model, which in a periodic model context is very important, but improves also upon forecastability, see Vahid *et al.* (2004).

5 Empirical Application

To perform both the univariate and the bivariate analyses of the airport passenger data described in section 2, we specify the following model

$$\Delta_7 \mathbf{Y}_\tau = \boldsymbol{\mu} + \mathbf{\Psi d}_\tau + \mathbf{\Theta c}_\tau + \mathbf{\Pi Y}_{\tau-1} + \sum_{k=1}^{P-1} \mathbf{\Gamma}_k \Delta_7 \mathbf{Y}_{\tau-k} + \mathbf{U}_\tau, \tag{10}$$

where $\boldsymbol{\mu}$ is a $(n \times 1)$ vector of unrestricted intercepts (and linear trends when $\mathbf{Y}_\tau$ includes stock series), $\mathbf{\Psi}$ is a $n \times 12$ matrix of unrestricted parameters associated with $\mathbf{d}_\tau$, which is a matrix of 12 trigonometric variables $\cos(j\pi/26 \times \tau)$ and $\sin(j\pi/26 \times \tau)$ $(j = 1, ..., 6)$ to account for the deterministic annual seasonality in a parsimonious way, $\mathbf{\Theta}$ is a $n \times 5$ matrix of unrestricted parameters corresponding to calendar effects. (Concretely, we introduce five calendar type dummy variables accounting for Easter, Christmas, end-of-the-year, May-the-first and the All-Saints week of festivals. These variables take value one the week corresponding to the calendar effect, and the value zero otherwise). The reason why the the intercepts in (10) enter unrestrictedly, is that we want to allow the trend slopes to be periodic, see Paap and Franses (1999).

As previously, $\mathbf{\Pi}$ and $\mathbf{\Gamma}_k$ are $n \times n$ matrices possibly of reduced rank. The $\mathbf{Y}_t$ vector consists of various combinations of the passenger series, i.e. arrivals y_t^1, departures y_t^2, net flow $y_t^3 = y_t^2 - y_t^1$, and the stock of visitors variable $y_t^4 = y_0^4 + \sum_{j=1}^t y_t^3$. For the periodic integration and CPC(0) analyses, $n = 7$, and $\mathbf{Y}_\tau = (y_{1,\tau}^i, ..., y_{7,\tau}^i)'$ $(i = 1, 2, 3, 4)$; whereas for the periodic cointegration and multiple CPC(m) analyses $n = 14$, and $\mathbf{Y}_\tau = (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$. Based upon the univariate empirical findings it appears useful for the analysis of stock-flow interactions to consider $n = 8$ and $\mathbf{Y}_\tau = (y^2_{1,\tau}, ..., y^2_{7,\tau}, \overline{y}^4_\tau)'$ where $\overline{y}^4_\tau$ is the weekly average of the stock series.

The daily series are filtered from additive outliers to prevent the potential distortionary effect of such outliers on the cointegration analysis (see Haldrup *et al.*, 2005 whose method can be adopted to periodic models).

The order P of the VAR model has been chosen according to the AIC criterion, which performs reasonably well within high dimensional systems (see Gonzalo and Pitarakis, 2002). In general, $P = 7$ or $P = 8$ was selected for the different cases. The choice of P was supported by a test that the highest order matrix of VAR coefficients employed was indeed significant.

5.1 Testing for periodic integration and cointegration amongst stocks and flows

5.1.1 The univariate series

The rank of the matrix $\mathbf{\Pi}$ has been determined according to the Johansen procedure. Table 1 reports the Johansen trace test statistic (LR_r) and the estimated coefficients ϕ_s of the quasi-difference operators $\delta_s(L) \equiv 1 - \phi_s L$ associated with the cointegrating vectors of the four series. The cointegration analysis of the VD series corresponding to the daily arrivals and departures provides strong evidence favouring the PI(1)/I(1) characteristic of such series by detecting 6 cointegrating relations. The fact that 6 cointegrating relations are present means that the 7 daily series exhibit the same stochastic trend which further implies that the series cannot have any weekly seasonal unit roots.

The cointegration analysis of the net flow series detects 7 stationary relations and hence the daily series is a PI(0) series. The cointegration analysis of the stock series again detects a cointegration rank of 6 and hence suggests the series to be PI(1) or I(1). I(1) against PI(1) of the arrivals, departures and the stock series can be tested by restricting the value of the cointegrating vectors. More specifically, when $\phi^i_s = 1$ for all $s = 1, 2, ..., 7$, a (1, -1) cointegrating relation exists across the single days of the VD representation, and hence in this case non-stationarity is non-periodic I(1). This test, denoted $LR_{\phi_s=1}$ which is asymptotically distributed as $\chi^2(6)$, is reported in Table 1 for each of the series. As seen from the tests and the estimates and their corresponding standard errors, both the arrivals and departures series are PI(1), i.e. the periodic coefficients are significantly different. Hence, the series are potentially non-periodically cointegrated with vector (1,-1), given that the net flow series is PI(0). The stock series is seen to have most coefficients almost exactly equal to one as one might expect given the way this series is constructed from the net flow series. Also note, that the stock variable does not have strong periodic properties relative to those exhibited by the arrivals and departures series as is clear from the estimated periodic coefficients, see also figure 3. Despite of this ϕ_7^4 is actually estimated to be significantly different from 1 and a formal test of all coefficients equal to one rejects that the series is non-periodically integrated I(1).

It has to be mentioned that interpretation of the single periodic coefficients should be made with care. If a number of periodic coefficients are below one some necessarily have to be above one to satisfy the restriction that $\prod_{k=1}^{7} \phi_k = 1$. Shocks occuring on a particular day of the week are transmitted to the other days of the week. That is if we only focus on the periodic coefficients in the PAR(1) model a shock in day i will have the effect $\frac{dy_j}{d\varepsilon_i} = \prod_{k=i \pmod 7 + 1}^{j} \phi_k$ on day j.

Table 1 about here

5.1.2 Bivariate analyses of the flow variables and their interaction with the stocks

The univariate properties displayed in the preceding section have several implications for the bivariate analysis. Because the daily arrivals y_t^1 and daily departures y_t^2 are PI(1), and the net flow y_t^3 is PI(0), the (flow) arrivals and departures series are potentially non-periodically cointegrated with cointegrating vector $(1, -1)'$, that is, $y_{s,\tau}^1 - y_{s,\tau}^2 \sim I(0)$. However, the analysis may also be considered for a full (14-dimensional) system where the simultaneous analysis of the arrivals and departures series is conducted. This analysis may show whether periodically cointegrating relations amongst the flow variables exist. Secondly, from the properties of the stock variable y_t^4 and the flow variables, possible cointegration amongst the stocks and the flows is most likely to be periodic, whereby e.g. $y_{s,\tau}^1 - k_s \overline{y}_\tau^4$ is stationary and $k_s \neq k$ for at least one $s = 1, ..., 7$.

Consider the analysis of the weekly flow series $\mathbf{Y}_\tau \equiv (y^1_{1,\tau}, ..., y^1_{7,\tau}, y^2_{1,\tau}, ..., y^2_{7,\tau})'$. For the extended system we follow the same procedure as for the univariate analysis in section 5.1.1, that is, we first test for the cointegration rank, and next hypotheses regarding the cointegrating space are tested.

Table 2 about here

The test results are reported in Table 2. The trace test does not reject the null hypothesis for $r = 12, 13$, but the likelihood ratio test ($LR_{12} = 9.40$) is rather close to the 10% critical value 9.67, which leads us to conclude that the rank equals 13. Hence the daily arrivals and departures are cointegrated and thus share the same stochastic trend, which confirms the results of the previous section where the net flow series was found to be PI(0). The analysis of PI(1)-ness of the flow variables can also be undertaken from the multivariate model of the flows $\mathbf{Y}_\tau$ for $r = 13$ in this highly parametrized model. Note from Table 2 that the estimates of the $\widehat{\phi}^i_s$ coefficients when estimated jointly are very similar to those obtained in the smaller system reported in Table 1. Note that a similar test (and standard errors) for the arrivals series y^1_t are not reported because the estimates are derived from the $\widehat{\phi}^2_i$ and $\widehat{k}_s$ series. Again we test for non PI(1)-ness of the departures variable through the linear hypothesis H$_0$: $\phi^2_1 = ... = \phi^2_6 = 1$, and obtain $LR_{\phi_s=1} = 22.38$, which is asymptotically distributed as $\chi^2(6)$ under the null and hence rejects at the 1% level. This reinforces the evidence about PI(1)-ness of the daily departures series found from the univariate analysis.

The next step is to test for nonperiodic cointegration between the arrivals and departures. The estimates of the cointegrating coefficients $\widehat{k}_s$ associated with the arrivals series are also displayed in Table 2. We want to test the hypothesis H$_0$: $k_s = 1$ for all $s = 1, 2, ..., 6$, and reject the null at 1% level with a test value $LR_{k_s=1} = 21.21$. From the estimated k_ss and their standard errors we recognize that especially the k_5 cointegrating coefficient is significantly different from 1.

Finally, we test for periodic cointegration amongst the flow and stock series by considering the cointegration analysis of the 8-dimensional process $\mathbf{Y}^*_\tau \equiv (y^2_{1,\tau}, ..., y^2_{7,\tau}, \overline{y}^4_\tau)'$, which includes the departures series (for illustration) and the weekly average stock series (for the reasons previously given). Conducting the Johansen ML-procedure on this system it is found, see Table 3, that the cointegration rank is 7, implying that the stock (derived from arrivals and departures) itself cointegrates with the departure (and arrivals) series. Hence the transit-activity measured by departures co-vary with the stock of visitors in a periodic fashion. Similarities to the notion of multicointegration therefore seem apparent. It was tested whether the cointegrating relationship between the stock and the flow variables is periodic or non periodic. The likelihood ratio test firmly rejects the null ($LR_{k_s=k} = 38.81$) and hence the cointegration relationship is clearly periodic which is also what we would expect given the univariate properties of the stock and flow series.

Table 3 about here

5.2 Testing for common periodic cyclical features

Given the evidence of the cointegration analysis in the previous section, we test for the presence of common periodic correlation features within the week and across the weeks of the individual variables y_t^1, y_t^2, and y_t^4 and the bivariate time series (y_t^1, y_t^2). We use the likelihood ratio test ($\xi_{m=0}(q)$) given by Hecq *et al.* (2005) for the case of contemporaneous cycles and the likelihood ratio test given by Cubadda and Hecq (2001) for the case of common periodic features across different weeks ($\xi_{m>0}(q)$). Because the estimated cointegrating rank $r = 6$ for all the cases, we can safely concentrate out the cointegrating vectors without affecting the limiting distribution (see the limiting result of Paruolo (2002), and the finite sample results of Hecq *et al.* (2005)).

Table 4 about here

Table 5 about here

Table 4 shows the results for the arrivals variable. The hypothesis of CPC(0) with $q_0 = 1$ is not rejected at the 10% level. Therefore, we do not reject $n - q_0 = 7 - 1 = 6$ dynamic factors driving the short-run component of the arrivals system. We do not reject at the 10% level, $n - q_1 = 7 - 2 = 5$ CPC(1) factors, $n - q_3 = 7 - 4 = 3$ CPC(3) factors, $n - q_5 = 7 - 5 = 2$ CPC(5) factors, and finally for the last lagged matrix Γ_7 we do not reject $n - q_6 = 7 - 6 = 1$ CPC(6) scalar factor. These results suggest that the more remote the past is, the less influence it has on the present of the short-run dynamics of the arrivals.

The most parsimonious representation of the short-run dynamics is given by CPC(3) of $q_3 = 4$ with 252 parameters (compared to 343 free parameters), while, for example, the CPC(0) of $q_0 = 1$ implies 336 short-run parameters. Similar results are obtained for the departures variable (see table 5). In this case the most parsimonious representation is obtained with the CPC(2) with $q_2 = 3$ with 266 parameters. These two cases illustrate that in our setting there is a relevant efficiency gain by imposing CPC(m) type restrictions.

Table 6 shows the results for the stock of visitors. The test statistics do not reject CPC(0), or CPC(1) with $q_i = 6$ ($i = 0, 1, 2$) which imply that the short-run dynamics of the day-of-week stocks are generated by just one dynamic factor. This greatly simplifies the model and reflects the non-periodic behavior of the stock of visitors. The results for the stock series contrast the results for the arrivals and departure series where more than a single factor characterizes the short run dynamics. In other words, for the arrivals and departures data only has *some* common periodic correlation features across the week. Interpreting the single factors in economic terms can be difficult, but typically the cross week common features will capture things like travelling habits, the airport- and hotel infra-structure, and contractual agreements between travel agents and hotels.

Table 6 about here

Table 7 about here

Finally, table 7 presents the likelihood ratio tests for the 14-dimensional system including arrivals and departures. As seen in the table, we do not reject $n - q_0 = 14 - 5 = 9$ CPC(0) common factors between arrivals and departures. This suggests that the flow variables do have idiosyncratic and common dynamic factors. In this case, the more parsimonious representation is obtained with CPC(1) of $q_1 = 6$ or CPC(2) with $q_2 = 7$ with a 30% reduction of the number of estimated parameters.

6 Conclusion

Periodic models are often criticized for being too flexible in the sense that they require too many parameters to be estimated. In the present paper, we have suggested restricting the correlation structure of periodic models by identifying common periodic correlation features that can be imposed upon the model. An application to arrivals and departures data for passenger trafic in the airport of Mallorca demonstrated that a significant reduction in the number of estimated parameters can be obtained by such common feature restrictions. We have also suggested a way to model stock and flow data with a daily periodicity of observations, and in so doing we have generalized the notion of multicointegration to a periodic context. It is our belief that the suggested advances are quite promising avenues for future research and in particular for the way of making periodic models parsimonious and operational.

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Figures

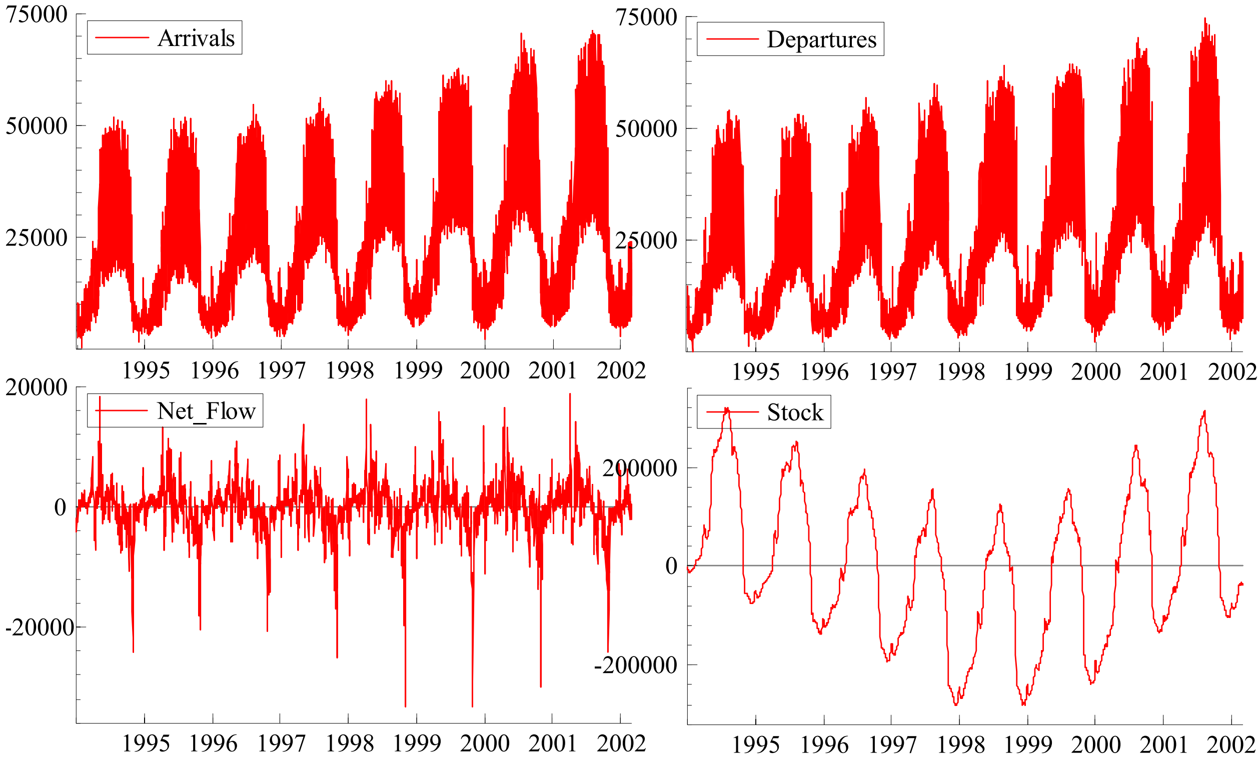

Figure 1: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, 1 January, 1994 - 28 February, 2002.

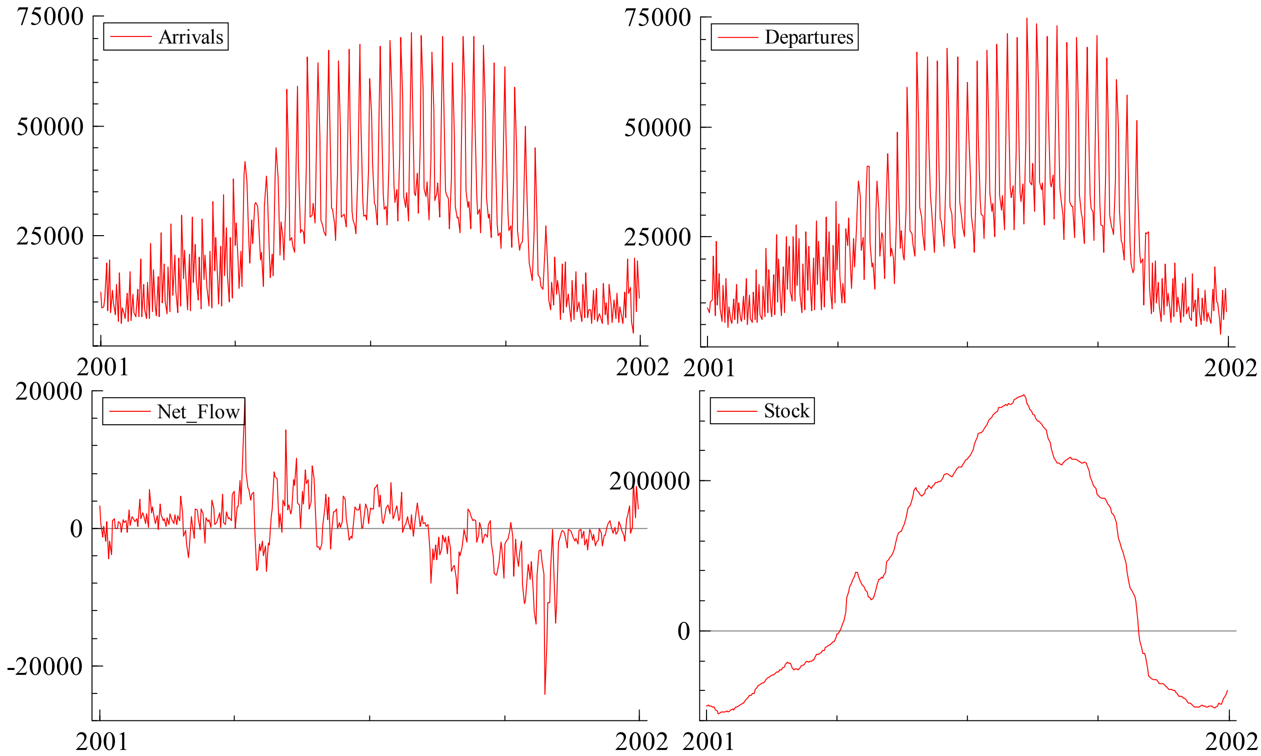

Figure 2: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, 1 January, 2001 - 31 December, 2001.

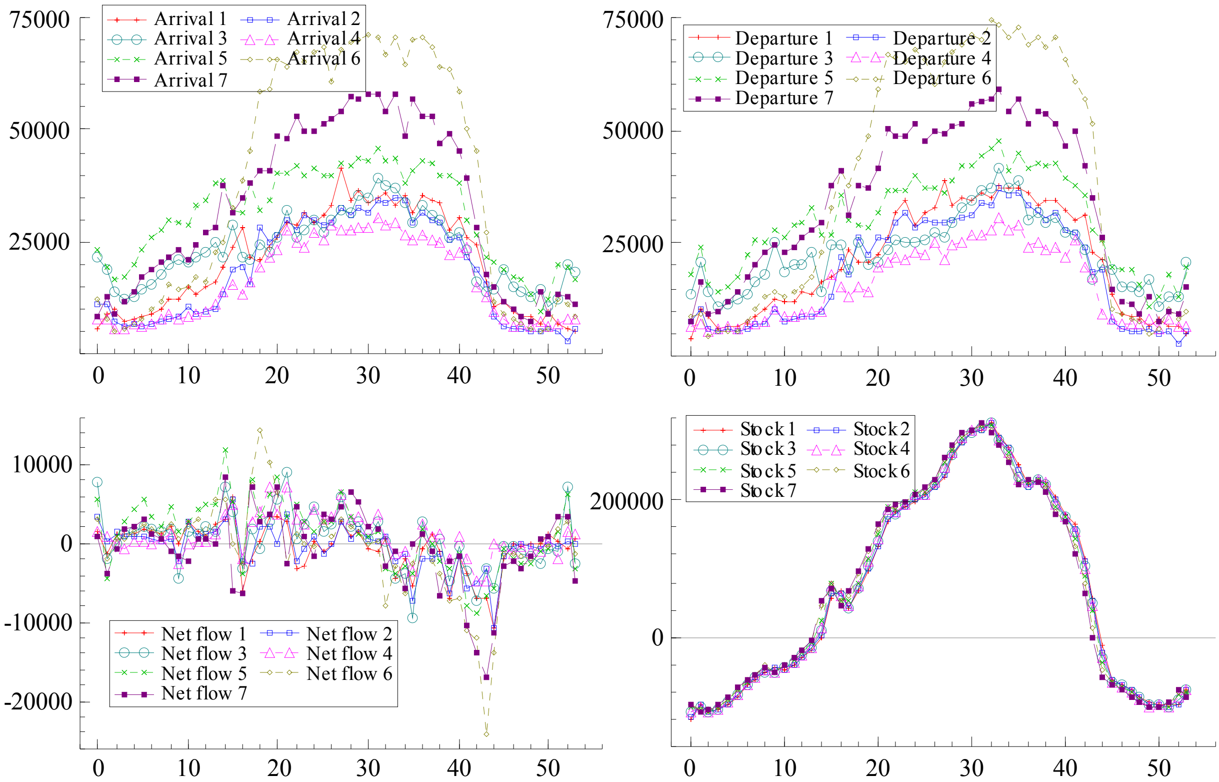

Figure 3: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, for each weekday (Monday 1, Tuesday 2,..., Sunday 7) of 2001.

Tables

Table 1: Periodic integration analysis of the arrivals y_t^1, departures y_t^2, net flow y_t^3 and stock series y_t^4.

	Trace Test							
	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6	
y_t^1	168.51***	114.49***	76.70***	50.11***	28.54***	11.41**	1.28	
y_t^2	181.64***	115.55***	80.39***	52.99***	32.09***	14.65***	2.31	
y_t^3	985.82***	733.18***	545.96***	397.70***	272.08***	168.67***	70.55***	
y_t^4	850.82***	634.46***	438.52***	271.06***	167.71***	73.15***	2.91	
	Periodic Integration Coefficients							
	$\widehat{\phi}_1^i$	$\widehat{\phi}_2^i$	$\widehat{\phi}_3^i$	$\widehat{\phi}_4^i$	$\widehat{\phi}_5^i$	$\widehat{\phi}_6^i$	$\widehat{\phi}_7^i$	$LR_{\phi_s=1}$
y_t^1	0.910 (–)	0.561 (.065)	3.039 (.178)	0.683 (.043)	1.100 (.067)	1.390 (.107)	0.617 (.047)	33.91***
y_t^2	0.889 (–)	0.551 (.052)	3.087 (.178)	0.602 (..034)	1.044 (.061)	1.583 (.112)	0.665 (.088)	37.44***
y_t^3	-	-	-	-	-	-	-	-
y_t^4	0.999 (–)	0.999 (.001)	1.000 (.001)	0.998 (.001)	0.997 (.001)	1.001 (.001)	1.006 (.001)	17.76**

Note: LR_r signifies the trace statistic of Johansen. The $LR_{\phi_s=1}$ is the Likelihood ratio test that all periodic coefficients equal unity. This test is asymptotically distributed as $\chi^2(6)$. Note that the figures in parenthesis are standard errors of the estimated coefficients. In case no standard error is reported it is because the estimated parameter was obtained from other estimates through the identities relating parameters, see section 3 for details. Also note that in the lower panel no periodic integration coefficients are reported for the net flow series y_t^3 because the upper panel suggests that this variable is not periodically integrated. *,** ,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

Table 2: Periodic cointegration analysis of the arrivals y_t^1, and departures y_t^2 series.

Trace Test								
	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6	
	769.77***	594.74***	475.16***	376.21***	300.01***	232.61***	175.08***	
	LR_7	LR_8	LR_9	LR_{10}	LR_{11}	LR_{12}	LR_{13}	
	124.88***	90.72***	61.25***	40.81***	22.20***	9.40	1.35	
Periodic Cointegration Coefficients								
	$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$	$LR_{k_s=1}$
	1.003 (.018)	1.043 (.043)	1.013 (.013)	1.105 (.023)	1.237 (.025)	1.030 (.016)	0.977 (.037)	21.21***
Periodic Integration Coefficients								
	$\widehat{\phi}_1^i$	$\widehat{\phi}_2^i$	$\widehat{\phi}_3^i$	$\widehat{\phi}_4^i$	$\widehat{\phi}_5^i$	$\widehat{\phi}_6^i$	$\widehat{\phi}_7^i$	$LR_{\phi_s=1}$
y_t^1	0.895	0.592	3.125	0.646	1.156	1.331	0.609	-
y_t^2	0.872 (–)	0.569 (.036)	3.219 (.170)	0.592 (.030)	1.032 (.047)	1.598 (.105)	0.642 (.080)	22.38***

Note: $\hat{k}_i$ are the estimated periodic cointegration parameters relating arrivals and departures, whereas $\hat{\phi}_j^i$ are the estimated periodic coefficients of the single series. LR_r signifies the trace statistic of Johansen. The $LR_{\phi_s=1}$ and $LR_{\kappa_s=1}$ are Likelihood ratio tests testing whether all periodic coefficients equal unity. These tests are asymptotically distributed as $\chi^2(6)$. Note that the figures in parenthesis are standard errors of the estimated coefficients. In case no standard error is reported it is because the estimated parameter was obtained from other estimates through the identities relating parameters, see section 3 for details. *,**,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

Table 3: Periodic cointegration analysis of the departures, y_t^2, and stock, $\bar{y}_t^4$, series.

Trace Test								
	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6	LR_7
	305.62***	200.04***	130.20***	70.32***	46.51***	29.12***	13.64**	3.39*
Periodic Cointegration Coefficients								
	$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$	$LR_{k_s=k}$
y_t^2	0.017 (.008)	0.005 (.003)	0.014 (.0113)	0.014 (.006)	0.012 (.009)	0.014 (.012)	0.025 (.004)	38.81***

Note: LR_r signifies the trace statistic of Johansen. $LR_{\kappa_s=k}$ is the Likelihood ratio test that all periodic cointegration coefficients are equal. This test is asymptotically distributed as $\chi^2(6)$. *,**,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

Table 4: Common periodic correlation feature analysis of the arrivals series y_t^1.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	50.39	33.38	28.34	15.34	10.71	4.37	0.02
2	107.22*	77.47	65.00	35.78	27.82	14.28	0.54
3	196.99***	139.77*	113.24*	62.61	47.59	26.05	1.82
4	306.10***	210.00***	167.96**	107.96	78.75	43.57	10.00
5	432.96***	303.89***	228.06***	154.76*	122.33**	64.56	23.61
6	581.74***	412.46***	307.06***	228.13***	185.20***	108.84**	44.82
7	862.96***	637.36***	503.77***	337.59***	277.53***	168.61***	75.92***

Note: $\xi_m(q)$ signifies the likelihood ratio test. *,**,*** indicates rejection of the null hypothesis at 10% , 5%, 1%.

Table 5: Common periodic correlation feature analysis of the departures series y_t^2.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	49.12	36.53	24.49	11.65	9.31	2.56	0.01
2	120.19**	81.42	58.56	40.49	30.80	13.84	1.14
3	211.65***	137.26*	107.46	81.31	58.17	31.55	4.67
4	307.36***	212.90***	173.78***	141.49***	102.82***	50.98	15.22
5	441.22***	306.19***	254.94***	209.20***	160.48***	88.58***	29.24
6	581.41***	422.86***	345.65***	292.86***	229.93***	138.92***	58.77***

Note: See table 4.

Table 6: Common periodic correlation feature analysis of the stock series y_t^4.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$
1	4.90	2.10
2	17.61	8.54
3	37.09	18.73
4	59.75	32.11
5	92.30	48.72
6	135.96	72.61

Note: See table 4.

Table 7: Common periodic correlation feature analysis of the arrivals and departures series y_t^1 and y_t^2.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	52.27	42.61	36.66	22.69	17.88	5.83	0.02
2	136.33	102.35	75.61	52.08	40.38	13.55	1.17
3	236.20	173.72	136.88	87.64	67.47	24.49	3.30
4	343.12	257.12	202.66	138.22	99.97	42.25	6.44
5	467.96	347.37	276.21	192.96	140.66	64.49	15.88
6	600.17**	457.92	360.89	260.51	193.04	96.02	26.92
7	772.26***	581.78*	457.75	341.69	253.23	135.95	40.17
8	951.53***	719.11***	566.19**	440.51*	331.96**	181.30	63.61
9	1151.53***	890.87***	705.88***	563.89***	429.98***	228.21	91.21
10	1359.52***	1081.58***	858.38***	706.25***	533.55***	299.35***	124.66**
11	1615.92***	1282.73***	1027.67***	858.45***	658.08***	392.23***	168.34***
12	1924.45***	1493.79***	1207.28***	1018.86***	786.36***	490.94***	222.04***
13	2252.74***	1733.56***	1414.33***	1188.78***	939.01***	607.36***	299.13***

Note: See table 4.