Institut für Finanz- und Aktuarwissenschaften

A combined analysis of hedge effectiveness and capital efficiency in longevity hedging

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A combined analysis of hedge effectiveness and capital efficiency in longevity hedging

Agenda

Introduction
Motivation

Incentives for longevity hedging

- **Risk reduction**
  - Hedge effectiveness typically measured in terms of the achieved risk reduction (Coughlan et al. (2011), Cairns et al. (2014))
  - Success of insurance-based solutions (e.g. customized longevity swaps)
- **(Cost of) capital relief under modern solvency regimes**
  - Longevity hedges may provide a regulatory capital relief
  - Primary source of value creation from hedging (Börger (2010), Meyricke and Sherris (2014))
  - Capital efficient longevity hedging: (cost of) capital relief exceeds the hedging costs
  - Assessing the impact of hedging on regulatory capital is complex, in particular for index-based hedges due to population basis risk (Cairns and El Boukfaoui (2018))
  - At least five publicly announced tail-risk protection deals (Blake et al. (2018))

- So far, both aspects have only been examined independently of each other
- A separate analysis misses potential interrelations between the two aspects and therefore cannot provide a full picture of all implications of hedging

Model setup

Liability to be hedged

- **Model assumptions**
  - **Focus on longevity risk**
    - Constant risk-free interest rate r
    - Ignore counterparty default risk, investment risk and interest rate risk
  - **Simplified annuity provider**
    - Portfolio of immediate or deferred life annuities, closed to new business
    - Subset of the national population with differing mortality characteristics due to a specific sociodemographic structure
    - Focus on a single model point representing a cohort aged x_0 at time zero
- **Liability to be hedged**
  - **Unhedged liabilities**
    - Time-t random present value of liabilities: $\boldsymbol{L(t)}$
    - Time-t best estimate liabilities: $\boldsymbol{\tilde{L}\,(t)}$
  - **Hedged liabilities**
    - Time-t random present value of hedged liabilities: $\boldsymbol{L_H(t) \coloneqq L(t) - H(t)}$
    - Time-t best estimate hedged liabilities: $\boldsymbol{\tilde{L}_H(t) \coloneqq \tilde{L}(t) - \tilde{H}(t)}$

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Model setup

Basic stochastic mortality model framework

Multi-population extension of the actual/estimated mortality trend (AMT/EMT) framework of Börger et al. (2019)

- **AMT simulation model captures the following risk drivers**
  - Long-term mortality trend risk for a reference population
    - Stochastic trend process of Börger and Schupp (2018)
  - Mortality differentials of several subpopulations of different sociodemographic status
    - Common relative modeling approach (Villegas et al. (2017))
    - Random walk with drift (Villegas and Haberman (2014))
    - Characterization approach (Haberman et al. (2014))
  - Small sample risk due to a finite portfolio size
- **EMT valuation model**
  - Reference population
    - Estimating the prevailing mortality level and trend based on the observable mortality patterns
  - Subpopulations
    - Additional adjustment for differing mortality levels and trends relative to the reference populalation (in the spirit of Cairns and El Boukfaoui (2018))

Model setup

Solvency Capital Requirements (SCRs) under Solvency II

Internal model

99.5% quantile of the change in best estimate [hedged] liabilities over a one-year horizon:

$$\frac{\tilde{L}_{[H]}(T+1) + CF_{[H]}(T+1)}{1+r} - \tilde{L}_{[H]}(T)$$

Two components:

- More annuitants than anticipated might survive the year
- Longevity assumptions might change over the year in an unfavorable direction (typically the more relevant factor)

Standard formula

Change in best estimate [hedged] liabilities due to a sudden, permanent longevity shock:

$$SCR_{L_{[H]}}(T) := \tilde{L}_{[H]}(T|shock_T(20\%)) - \tilde{L}_{[H]}(T)$$

- Simple approximation for the 99.5% Value-at-Risk approach
- Many companies still rely on the standard formula

- The SCR at time T is interpreted as an F_T-measurable random variable
- The SCRs the hedger would be required to hold over time **with or without a chosen longevity hedge** can be determined in each outer model path

➡ **Entire distributions for the company‘s SCRs over time**

Model setup
Hedging objectives

Quantify the impact of longevity hedging on the hedger's future cash flow profile

- **Adjusted [hedged] liabilities**

$$\Pi_{L_{[H]}} := L_{[H]}(0) + CoC_{L_{[H]}}, \qquad CoC_{L_{[H]}} := \sum_{t\geq 0} \frac{r_{CoC}\, SCR_{L_{[H]}}(t)}{(1+r)^{t+1}}$$

  - Benefit payments to surviving annuitants [minus hedge payments] plus
  - Cost of regulatory capital for the [hedged] liabilities
- **Capital efficiency**
  - *Net capital relief*: expected reduction in the company's future adjusted liabilities

$$NCR(H) := E(\Pi_L) - E(\Pi_{L_H})$$

  - *Capital efficiency:* proportionate reduction in the company's cost of regulatory capital

$$CE(H) := \frac{NCR(H)}{E(CoC_L)} \leq 1$$

- **Hedge effectiveness**
  - Achieved risk reduction in the centralized **adjusted liabilities** (under a risk measure ρ)

$$HE(H) := 1 - \frac{\rho(\overline{\Pi_{L_H}})}{\rho(\overline{\Pi_L})} \leq 1$$

Hedging instruments

Overview

- **Longevity swaps** $h(t) \coloneqq SI_{x_0+t,t} - K(t),\ 0 < t \leq \tau$
  - Exchange the realizations of a survivor index $SI_{x_0+t,t}$ against a set of fixed payments $K(t)$
  - Unlimited fully customized longevity swap provides a perfect hedge
- **Annuity forwards** $h(\tau) \coloneqq \widetilde{LI}(\tau) - K(\tau)$
  - Exchange the realization of a liability index $\widetilde{LI}(\tau)$ against a single pre-defined payment $K(\tau)$
  - $\widetilde{LI}(\tau)$ derived based on up-to-date mortality assumptions at maturity (EMT valuation model)
- **Q-forwards** $h(\tau) \coloneqq n\left(K(\tau) - q_{x_0+\tau,\tau}\right)$
  - Exchange realized mortality probabilities $q_{x_0+\tau,\tau}$ against a fixed forward rate $K(\tau)$
  - Simple portfolio of a single q-forward with reference age $x_0 + \tau$
- **Rolling portfolios of one-year call spread options**
  - At any point in time $t \geq 0,$ the hedger might enter into a one-year call spread option contract

$$h(t+1) = \left(EP(t) - AP(t)\right) max\left\{0, min\left\{\frac{\tilde{X}(t+1) - AP(t)}{EP(t) - AP(t)}\ ; 1\right\}\right\}$$

  - Hedge index $\tilde{X}(t+1),$ attachment point $AP(t)$ and exhaustion point $EP(t)$ are tailored to the company's SCR computation method at time t

Hedging instruments

Index populations and pricing

- **Different index population (IP)s**
  - **IP=*B*** (fully customized, linked to the ***B***ook population)
    - Linked directly to the realized survivors and the realized mortality in the book population
  - **IP=*S*** (index-based, linked to the ***S***ubpopulations)
    - Hedger bears small sample risk
  - **IP=*R*** (index-based, linked to the ***R***eference population)
    - Hedge exclusively covers the randomness originating from the reference population
    - Small sample risk and demographic basis risk remain with the hedger
    - Initial experience ratios are fixed to match the hedger‘s initial portfolio characteristics
- **Pricing**
  - Incomplete and illiquid market for longevity-linked securities
  - Market participants demand a risk premium for taking longevity risk
  - Key idea of **risk-adjusted pricing**: adjust the distribution of each risk driver to obtain a risk-adjusted version of the AMT simulation model (Boyer and Stentoft (2013), Freimann (2019))
  - Under Q, the price of any security is defined as expected value of its discounted future payoffs

Numerical results: model calibration

Overview of model parameters

Model calibrated to the historical mortality experience of English and Welsh males

- National population serves as the reference population (Human Mortality Database (2018))
- Five subpopulations of different sociodemographic status based on the Index of Multiple Deprivation (IMD) for England (Office for National Statistics (2018))

Description	Parameter
Initial model point age	$x_0 = 65$
Retirement age	65
Initial portfolio size	$10{,}000$
Sociodemographic book composition	$(0, 0, 0.3, 0.3, 0.4)$
Interest rate	$r = 2\%$
Cost of capital rate	$r_{CoC} = 6\%$
Risk premium of the hedge provider	$\lambda = 0.275$
Risk measure for assessing hedge effectiveness	$\rho = TVaR_{0.90}$

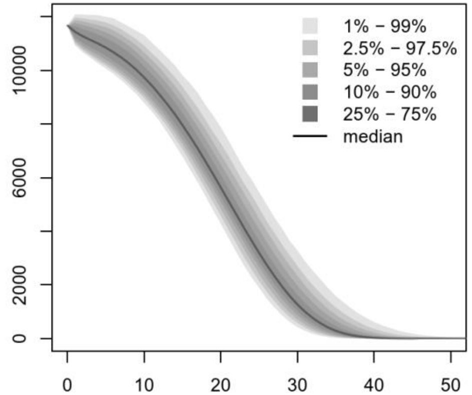
Numerical results: unhedged (adjusted) liabilities

The impact of stochastic cost of regulatory capital

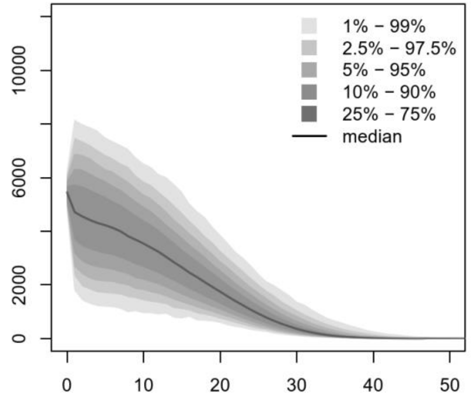
- **Initial situation without hedging**

	$L(0)$	$\Pi_L(SF)$	$\Pi_L(IM)$
$E(X)$	165,800	176,900	169,900
$TVaR_{0.90}(\bar{X})$	10,100	11,300	11,500

- **Quantile plots of the company's SCRs over time**



(a) Standard formula



(b) Internal model

Adjustment for future cost of regulatory capital increases both, the

- Expected future liabilities (more pronounced under the standard formula)
- Overall risk exposure (more pronounced under the internal model)

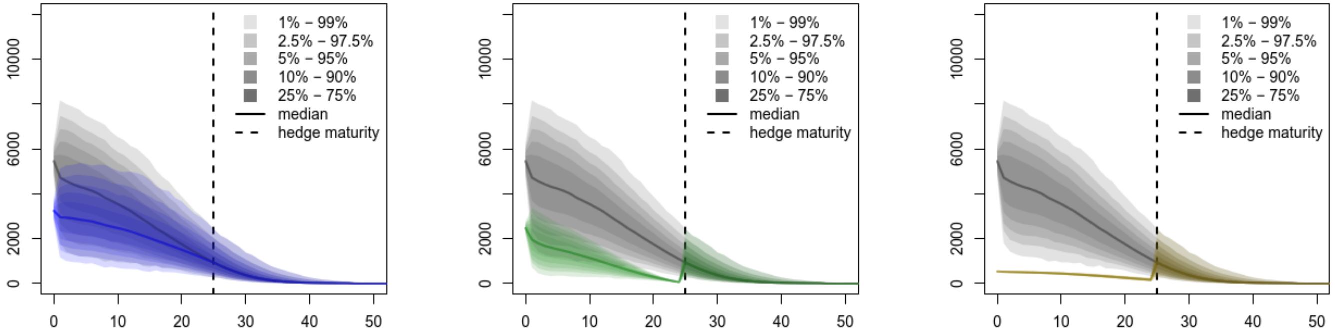
Numerical results: the separate effects of hedging

Overview of hedging instruments

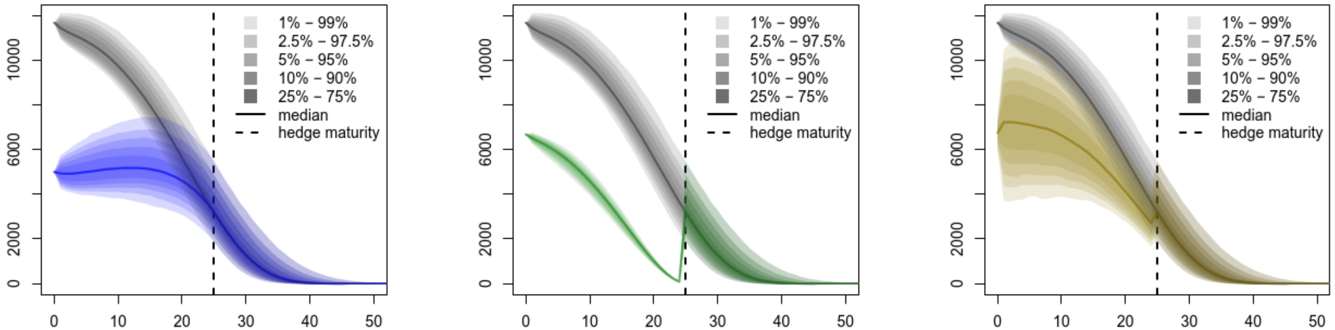
Instrument	Parametrization	Value
Longevity swaps	index population maturity	$\in \{\boldsymbol{Book}, \boldsymbol{Subpops}, \boldsymbol{Refpop}\}$ ≥ 25
Annuity forwards	index population maturity	$\in \{\boldsymbol{Book}, \boldsymbol{Subpops}, \boldsymbol{Refpop}\}$ ≤ 25
Q-forwards	index population maturity	$\in \{\boldsymbol{Subpops}, \boldsymbol{Refpop}\}$ ≤ 25
Rolling portfolios of one-year call spread options (standard formula design)	index population maturity attachment point exhaustion point	$\in \{\boldsymbol{Book}, \boldsymbol{Subpops}, \boldsymbol{Refpop}\}$ ≤ 25 $ap = 0.05$ $ep = 0.20$
Rolling portfolios of one-year call spread options (internal model design)	index population maturity attachment point exhaustion point	$\in \{\boldsymbol{Book}, \boldsymbol{Subpops}, \boldsymbol{Refpop}\}$ ≤ 25 $ap = 0.01$ $\alpha^{EP} = 99.5\%$

Numerical results: the separate effects of hedging

Capital relief for selected fully customized hedges over 25 years



(a) Longevity swap (b) Annuity forward (c) Rolling call spreads



(d) Longevity swap (e) Annuity forward (f) Rolling call spreads

The structures of the remaining SCRs differ in terms of both, **level and variability**

- Among the instruments
- Between the internal model (upper row) and the standard formula (lower row)

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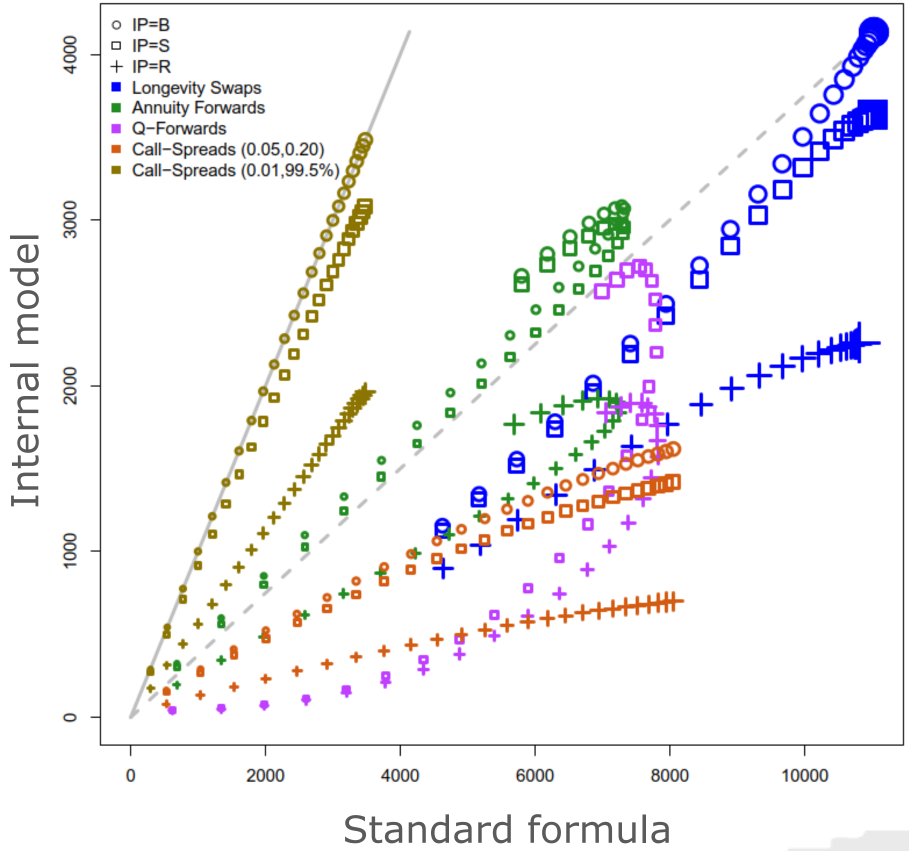
Numerical results: the separate effects of hedging

Cost of capital relief: standard formula vs. internal model



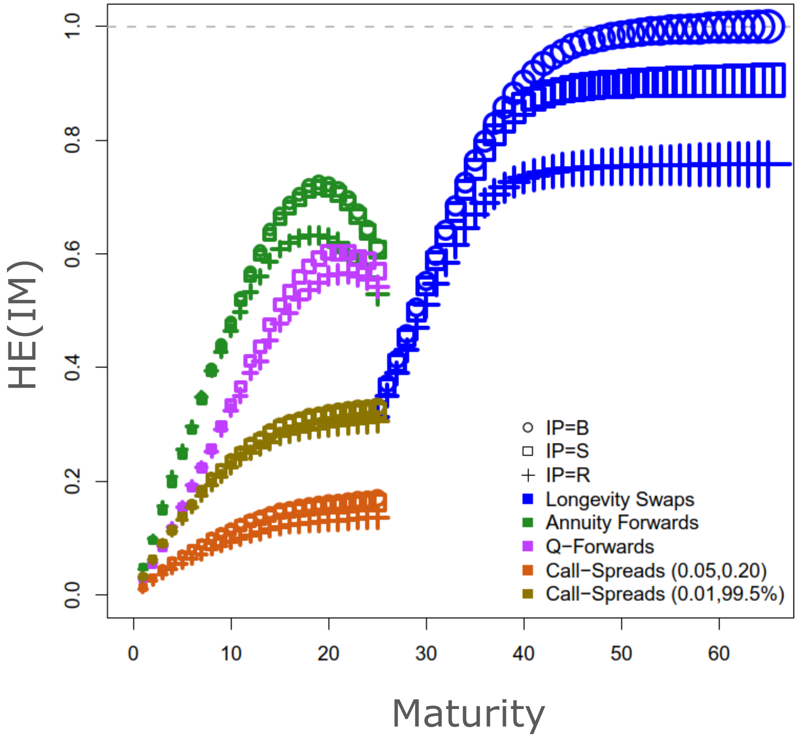
- Higher CoC relief under the standard formula
  - Limited longevity swaps
  - Q-forwards
  - Rolling call-spreads (0.05,0.20)
- Comparable CoC relief
  - Annuity forwards
- Higher CoC relief under the internal model
  - Rolling call-spreads (0.01,99.5%)

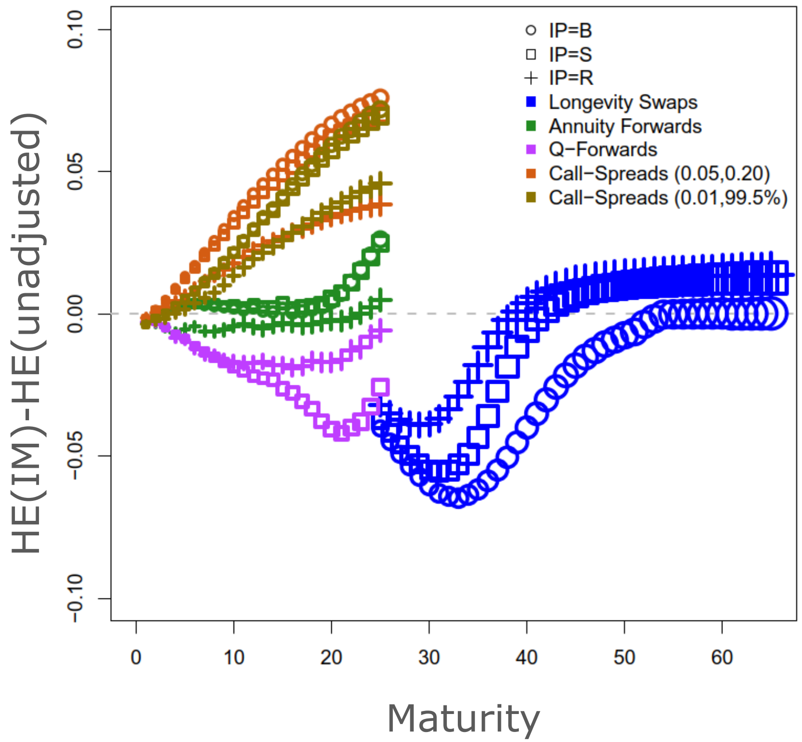
- Standard formula does not detect population basis risk
- Internal model: haircut for small sample risk (*IP=S*, □) and additionally for population basis risk (*IP=R*, ✚)

Discordant cost of capital reliefs

Numerical results: the separate effects of hedging

Risk reduction

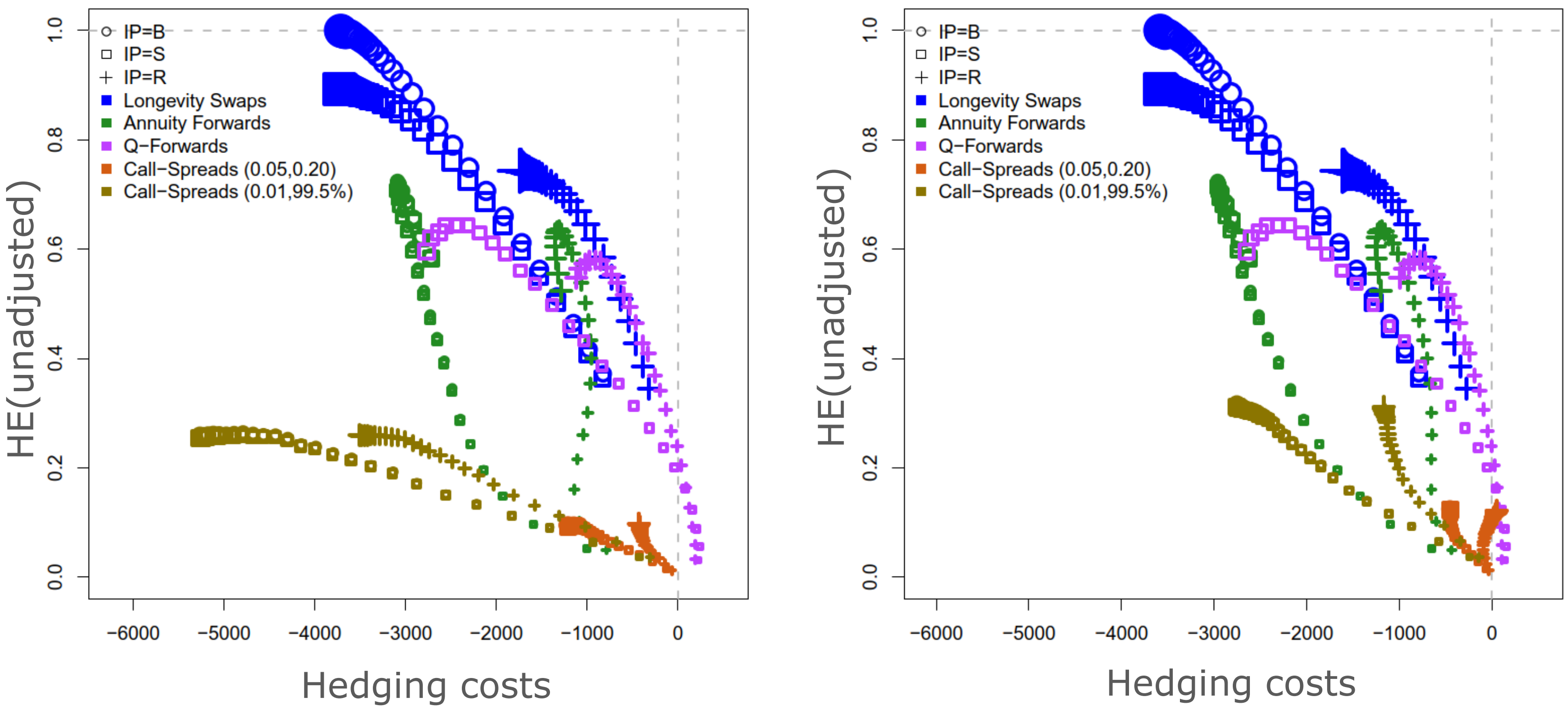




- For a selected hedge structure, clear ranking among the index populations:
  HE(*IP=B*,○)>HE(*IP=S*,□)>HE(*IP=R*,+)
- Hedge effectiveness is sometimes underestimated, sometimes overestimated when ignoring the uncertainty in future cost of capital (relief)

Numerical results: combined analysis of HE and CE

The costs of effective hedging



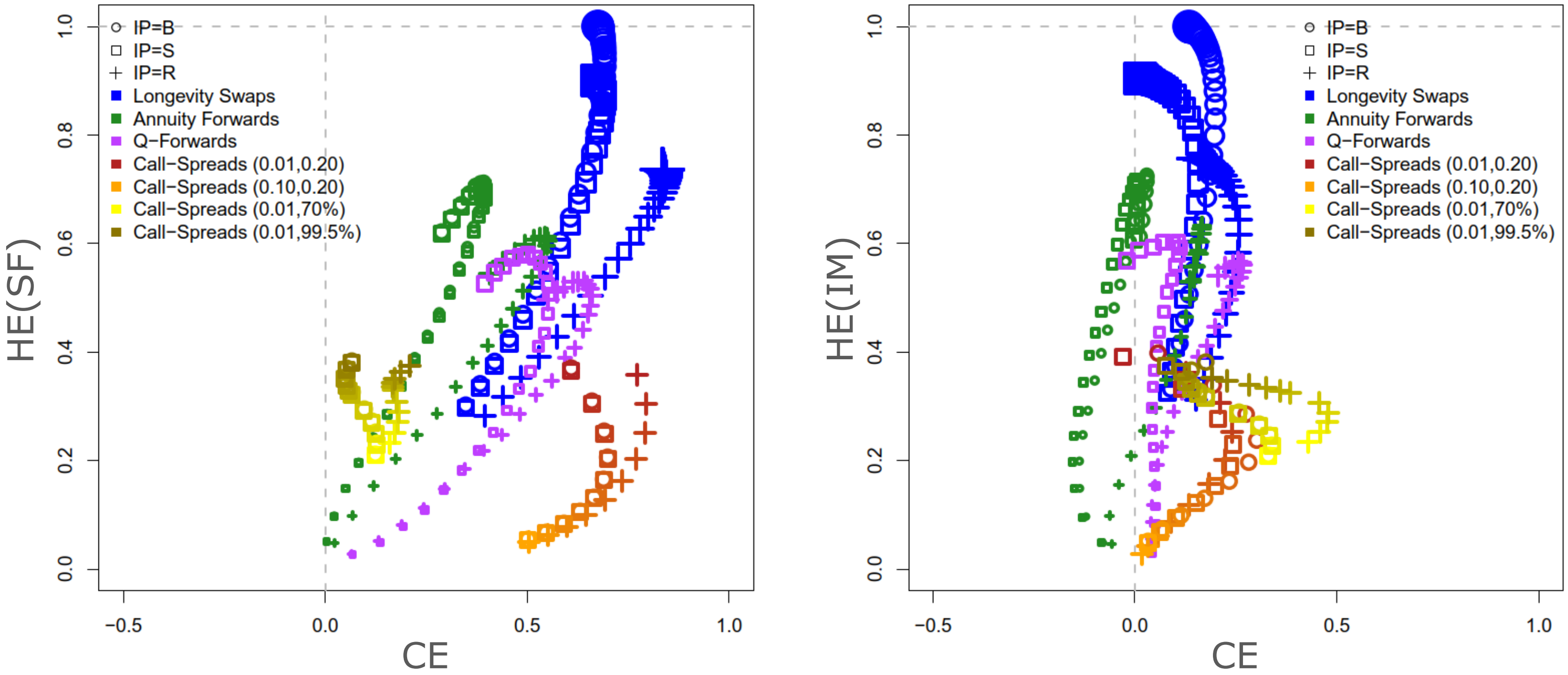
(a) $\lambda = 0.275$ for all risk drivers

(b) reduced $\lambda_\epsilon = 0.15$ for yearly noise

- Higher hedge effectiveness typically comes along with higher hedging costs
- Index-based designs (*IP=R*,✚) are offered at a better price
- The costs for rolling portfolios of one-year call spreads are disproportionately high if the hedge provider does not reduce the risk loading for yearly noise

Numerical results: combined analysis of HE and CE

The trade-off between HE and CE



(a) standard formula (b) internal model

- The most effective hedge does not provide the highest capital efficiency
- Customized designs (*IP=B*,O) dominate in terms of hedge effectiveness
- Index-based designs (*IP=R*,+) dominate in terms of capital efficiency

➡ **Trade-off between hedge effectiveness and capital efficiency**

Summary

Framework for a joint analysis of hedge effectiveness and capital efficiency

- Uncertainty in future cost of regulatory capital is incorporated into the assessment of hedge effectiveness
- Applied in the context of an economic capital model under Solvency II to a variety of different customized and index-based instruments taking into account population basis risk

Key findings

- The standard formula�shocks prescribed longevity shock provides different and less consistent capital reliefs than a risk-based internal model
- Hedge effectiveness might, depending on the underlying hedge structure, rise or fall when allowing for uncertain future cost of regulatory capital and appropriate capital reliefs
- Rolling portfolios of one-year contracts are only competetive if market participants lower the risk premium for random noise around the prevailing mortality trend
- Index-based solutions have the potential to outperfrom fully customized hedges in terms of capital efficiency
- Hedgers might face a trade-off between hedge effectiveness and capital efficiency when hedge providers demand a risk premium

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