CCR Stress Testing, WWR and Leverage

A Monte Carlo simulation based framework

Dr. Fabrizio Anfuso*

dr.fabrizio.anfuso@gmail.com

London, November 2024

*Any views expressed are solely those of the author and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This presentation should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.

Agenda

1. **CCR stress testing revival**
2. WWR and highly leveraged counterparties
3. Modelling WWR and leverage bottom-up
4. Modelling WWR and leverage top-down

CCR stress testing: reality check

Banks:

- SVB
- First Republic
- CS

NBFIs:

- LDI crisis
- LME Nickel
- Archegos

A large variety of distressed market dynamics at play: liquidity, concentration, WWR, IR environment, geopolitical events, poor disclosure, failed business models... On the back of this, regulators recommended to strengthen CCR stress testing frameworks... AKA we should model the pandora's box!

CCR stress testing: need of better frameworks

The guidelines for counterparty credit risk management issued by BCBS provides useful steering regarding CCR stress testing, including:

- It is called CCR stress testing (and not MR stress testing...)!
- Stressed exposure is a full-fledged metric to be monitored vs. risk limits.
- In different shapes and forms, WWR is a key driver of the stressed exposures.
- Banks should give due consideration to scenarios in which risk mitigation measures do not work as intended.
- A partial understanding of the cpty risk profile should be compensated by conservative assumptions.
- The severity of the applied shocks should be benchmarked vs. historically realised exposures.

CCR stress testing: need of better plumbing

- Stress testing methodologies are primarily based on single-point in time valuations, with multiple shortcuts:
  - ◇ At the level of the risk factors dynamics, often based on regression of few macro-scenarios.
  - ◇ At the level of the portfolio valuation, with no consistent way to deal with complex payoffs, margin requirements, portfolio term-structure.
- On the other hand, MC / full revaluation frameworks lack of systematic ways to generate stressed exposure distributions.

It would be handy to leverage the IMM/XVA MC scenarios and valuation capabilities to perform CCR stress testing...

Agenda

1. CCR stress testing revival
2. **WWR and highly leveraged counterparties**
3. Modelling WWR and leverage bottom-up
4. Modelling WWR and leverage top-down

What is so special about WWR?

In the wise words of my colleague Paul Filmore (featuring L.N. Tolstoy...):

...*Each unhappy default is unhappy in its own way.*

The epitaph of the credit officer:

Expected Loss = LGD x EAD x PD

LGD = Loss Given Default
EAD = Exposure At Default
PD = Probability of Default

- The gravestone formula hides one of the main complexities of CCR, i.e. the possibility of adverse correlations between the PD and the EAD. In other words, the very same market scenarios when the portfolio exposure is large can be the ones when the cpty default is more likely.
- Wrong Way Risk (WWR) is the most probable default dynamics for highly levered funds, whose solvency is basically measured by their portfolio performance.

Leveraging and deleveraging (Part I): Archegos, "bottle neck" WWR by single counterparty concentration

- Archegos Capital Management was a family office that in March 2021 defaulted on margin calls from several global investment banks. The aggregated loss topped \$10bn (\$5.5bn with CS alone)...
- The peculiarity of this default lies in the massive horizontal hoarding piled up by the fund; by replicating with multiple dealers its long equity bet on few Asian tech stocks, Archegos ended up with more than \$160bn gross exposure in such stocks, holding large fractions of their free floating amounts.
- ...Up until an idiosyncratic large gap move of Viacom, one of the main Archegos long exposures, burst this gigantic bubble...And triggered large margin calls that could not be met, since most of Archegos assets were by then tied up in WWR collateral (whose value was directly affected by the consequent disordered liquidation of Archegos positions across the street)...

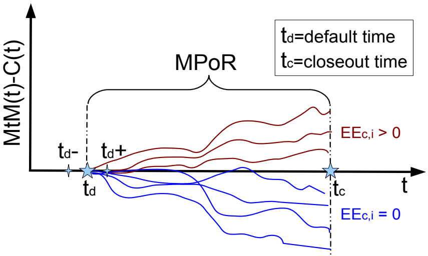
Leveraging and deleveraging (Part II): LDI, "bottle neck" WWR by industry-wide concentration

- LDI strategies enable definite benefits pension funds to use leverage to increase long exposure to long-term gilts.
- The turmoil in the gilts market induced by the September 2022 "mini budget" ignited a spiral of margin calls and gilts down-moves; with funds further depleting gilts prices (and therefore increasing their exposures) in the attempt of meeting cash margin calls by partially liquidating gilts positions.
- This was primarily a liquidity crisis, that showed how daily margining can be a double-edged sword; especially in the presence of:
  - ◇ Small or no collateral haircuts, i.e. a thin level of cushion between exposures and collateral.
  - ◇ WWR collateral, although on perceived safe assets like UK gilts.
  - ◇ Industry-wide concentration on specific assets across cptys with similar risk profiles and exit strategies.

1) MPOR: the "Magic Period Of Risk"

Only smooth defaults with risk dynamics taking place over the MPOR

Margin driven defaults with material collateral shortfalls cannot be modelled. The current exposure at default is 0 by construction; and the closeout loss is generated with "ordinary" dynamics over an horizon MPOR≈2w and few casual trimmings.



2) Over-collateralisation: when a man with the average meets a man with the 99% quantile...

In the memorable words of Clint Eastwood (*A Fistful of Dollars*, 1964):

"*When a man with .45 meets a man with a rifle, you said, the man with a pistol's a dead man. Let's see if that's true...*"

IM calibration	0	q=95%	q=99%
EAD (IMM)	\$1	\$0.05	\$0.01
PFE (99%)	\$2.32	\$0.68	\$0

The EAD/PFE ≈ 0 but be mindful of "quant" complacency!

The EAD/PFE ≈ 0 since a model average/quantile is offset by a model quantile of (more or less) the same distribution in a nicely factorised wonderland! Reality can be painfully different...(*and btw the man with the rifle dies in the end*)

3) Leverage: all you can eat

- The exposure calculation for standard B3/IMM/XVA modelling frameworks is agnostic to leverage; that is instead (generally) accounted for at level of the the credit rating.
- For counterparties like HFs, whose solvency is basically the portfolio performance, leverage is akin to WWR; since it amplifies the impact of adverse market moves on the credit worthiness of the counterparty.
- In addition and very much related to the above, high leverage can be combined with illiquid / concentrated positions...With all the non-linear troubles that come with it!

Recent work on leveraged and WWR exposures

Bottom-up models:	**Top-down models:**

What is common?

- Can predict material exposures for levered cptys, even if IM is present
- Can model concentrated porfolios with uncertainty regarding the cpty's true risk profile...No "Market Implied" calibration and more akin to stress testing

What is different?

Bottom-up models	Top-down models
Model the WWR dynamics microscopically, at the level of the underlying drivers	Model the WWR dynamics macroscopically, with some exogenous conditioning
Complex to implement / integrate in existing CCR modelling workflow	Easier to implement / integrate in existing CCR modelling workflow
Do not require exposure multipliers	Require exposure multipliers

Recent work on leveraged and WWR exposures

Bottom-up models:

- Mind the gap, A. Dickinson, *Risk*, 2022
- Collateralised Exposure Modelling: Bridging the Gap Risk, FAn, *Risk*, 2023
- Bridging the gap risk reloaded: modelling wrong-way risk and leverage, FAn, *Risk*, 2024
- Beyond Value-at-Risk: Wrong-Way-Risk Add-ons and Reverse Stress Testing, C. Albanese, SSRN, 2022

Top-down models:

- Leveraged Wrong-Way Risk, M. Arnsdorf, *Risk*, 2023
- Credit exposure to leveraged counterparties, M. Pykhtin, SSRN, 2024
- A MC framework for CCR stress-testing, FAn and D. Karyampas, in preparation

Agenda

1. CCR stress testing revival
2. WWR and highly leveraged counterparties
3. **Modelling WWR and leverage bottom-up**
4. Modelling WWR and leverage top-down

A WWR and leveraged portfolio

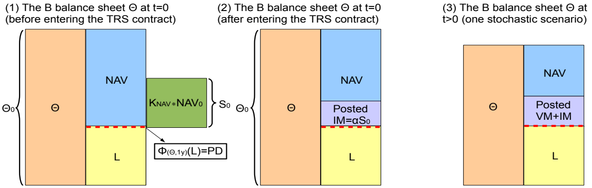
- Two CPTYs (A and B) enter a derivatives portfolio; where B is the defaulting CPTY in the following.
- The portfolio MtM(t)=$g(S(t), \sigma_S(t)) + V(t)$ is made of two components:
  - ◇ $g(S(t), \sigma_S(t))$, denoting the fair value of the derivatives with WWR underlying S (implied volatility σ_S).
  - ◇ $V(t)$, denoting the fair value of the derivatives whose underlyings and prices are not WWR (and uncorrelated with S).
- CPTY B is long S and the contractual terms comprehend daily bilateral VM to fully offset the current MtM of the trade.

The non-factorised model

To account for the WWR, we introduce the simplest model that can couple the tail dynamics of the B balance sheet and of the A-B portfolio.

Defaulter balance sheet: the non-factorised view

- The B balance sheet size Θ is stochastic with constant liabilities L (i.e. margin calls cannot be financed with additional debt!). Hence, *before entering the portfolio*, L marks the the default threshold for B. *After entering the portfolio*, the balance sheet includes collateral obligations.
- The available capital, after accounting for posted (VM/IM) and received (VM) collateral, is referred as Net Asset Value (NAV). The *going concern* condition is NAV$(t) > 0$.
- The leverage $K_{\text{NAV}} = \text{MaxLoss}/\text{NAV}$ measures the risk of the position in units of the available capital (see example below for a long TRS position).



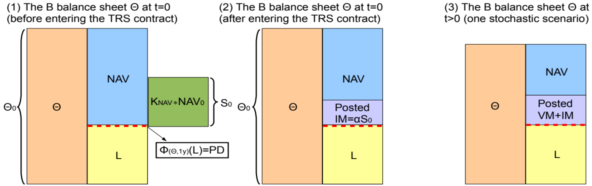
Non-factorised model vs. standard MPOR framework

Non-factorised model:

- The balance sheet Θ and the exposure are simulated jointly
- Leverage is a key risk input
- The WWR S and Θ can experience large simultaneous down-moves
- The default occurs in simulation when NAV falls below 0. And Margin calls can trigger it

B3/XVA/IMM MPOR model:

- The exposure is simulated stand-alone and Θ plays no role
- Leverage plays no role
- The WWR S dynamics is based on continuous market risk models
- The default is superimposed. Hence, even very large margin calls play no role



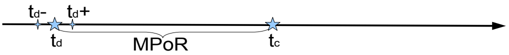
A minimal model for coupled $S - \Theta$ tail dynamics

$$
\begin{aligned}
\mathsf{MtM}(t) &= V(t) + g(S(t), \sigma_S(t)) \\
dS(t) &= \mu_S S(t)dt + \beta_{\sigma_S}\sigma_S S(t) + S(t)dJ_S(t) \\
d\Theta(t) &= \mu_\Theta \Theta(t)dt + \sigma_\Theta \Theta(t)dW_\Theta(t) + \rho_\theta \Theta(t)dJ_S(t) \\
d\sigma_S(t) &= -\gamma(\sigma_S(t) - \bar{\sigma}_S)dt + \frac{\rho_{\sigma_S}}{\mu_{J_S}}\bar{\sigma}_S dJ_S(t), \\
\Delta_{1d}(C_{\mathsf{VM}}(t)) &= -\Delta_{1d}(\mathsf{MtM}(t)),\ C_{\mathsf{VM}}(0) = -\mathsf{MtM}(0) \\
\Delta_{1d}(\mathsf{NAV}(t)) &= \Delta_{1d}(\Theta(t)) + \Delta_{1d}(\mathsf{MtM}(t)), \\
& \quad \mathsf{NAV}(0) = \Theta(0) - L + \mathsf{MtM}(0).
\end{aligned}
$$

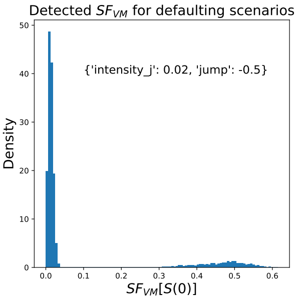
- $S(t)$ is the WWR underlying of the generic pay-off function g.
- $V(t)$ is the non-WWR part of the portfolio and is uncorrelated to $S(t)$.
- Since we may have OPT products, we included the implied vol $\sigma_S(t)$.
- $dJ_S(t)$ provides the correlated gap dynamics for S, σ_S (ρ_{σ_S}) and Θ (ρ_θ).

When the magic is gone: t_d margins are not "contractual"

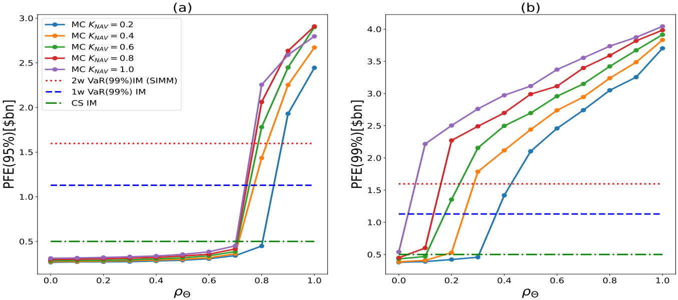
Since we are interested in margin-driven defaults, we focus on the the t_d^+ coll-SF (SF_{VM}) conditional upon default; that is always 0 according to B3/IMM/XVA!



- Our joint model detect defaults path-wise by using the *going concern* condition $\text{NAV}(t) > 0$. Upon default, SF_{VM} is obtained from t_d^- and t_d^+ VM requirements.
- The SF_{VM} distribution has most of the weight around 0 and a long tail centred around the jump size (the "discontinuous" defaults).



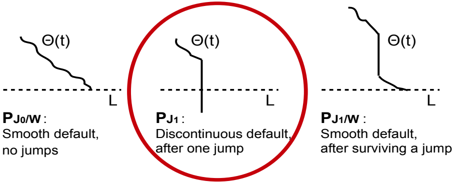
Archegos: *ex post* forecasts...



- The plot shows the predicted PFE$\approx$ SF_{VM} for the model calibrated on the CS Archegos portfolio, with WWR ρ_Θ as free parameter.
- For large levels of ρ_Θ ($\rho_\Theta \approx 1$ for Archegos), the model PFE is in the region $3bn to $4.2bn, to be compared with $5.5bn closeout losses.

Additional readings

- The non-factorised model can be used for capital as well. In that context, the EAD $\approx \langle \mathrm{SF_{VM}} \rangle$ can be material for levered and WWR counterparties also in the presence of IM ($\approx 0^+$ for B3/IMM/XVA frameworks!).
- While the non-factorised approach differs from existing IMM/XVA implementations, we have shown that the analytic structure of the model can be used to provide more parsimonious solutions to compute $\mathrm{SF_{VM}}$, such as:
  - ◇ Re-using existing IMM/XVA scenarios with small additional trimmings.
  - ◇ Using a closed-form solution, directly applicable e.g. for non MC frameworks such as SA-CCR or stress testing.



Agenda

1. CCR stress testing revival
2. WWR and highly leveraged counterparties
3. Modelling WWR and leverage bottom-up
4. **Modelling WWR and leverage top-down**

M. Pykhtin proposal: a Gaussian copula to model WWR

- In his recent paper, MP uses a Gaussian copula to correlate the portfolio MPOR returns distribution (PDF $\omega(x)$,CDF $\Omega(x)$) with the probability distribution that governs the default event. For N simulated scenarios ω_i:

$$\begin{aligned} Z_i &= \Phi^{-1}(\Omega(\omega_i)) \\ X_i &= -\rho Z_i + \sqrt{1-\rho^2}\epsilon_i \end{aligned}$$

  Where conditioning upon default $X_i < \Phi^{-1}(\mathrm{PD}_{\mathrm{MPOR}})$ selects the scenarios Z_i that drive the exposure ($\mathrm{PD}_{\mathrm{MPOR}}$ being the PD over the MPOR horizon)

- In practice, this means:
  - ◇ For $\rho = 0$, the conditioning selects a random sample of the scenarios ω_i
  - ◇ For $\rho = 1$, the conditioning selects the worst $N \cdot \mathrm{PD}_{\mathrm{MPOR}}$ scenarios ω_i

MP showed that $\rho > 0$ increases the EE for Normal returns; we now use the same formalism to explore alternative choices of $\omega(x)$, focussing on the PFE...

Modelling $\omega(x)$ with a Gaussian mixture

- Mixture models combine the effect of distinct risk dynamics that can occur with different likelihoods. For scenario ω_i and using two generic distributions $\alpha_1(x)$ and $\alpha_2(x)$:

$$\omega_i = \begin{cases} \phi_{1,i} \in \alpha_1(x) & p_1 = 1 - p \\ \phi_{2,i} \in \alpha_2(x) & p_2 = p \end{cases}$$

  where the CDF of the mixture distribution can be expressed as:

$$\Omega(x) = (1 - p) \cdot \mathrm{CDF}_{\alpha_1}(x) + p \cdot \mathrm{CDF}_{\alpha_2}(x)$$

- Anticipating the next steps, we introduce the following calibration scheme:
  - ◇ $\alpha_1(x)$ is the BAU portfolio MPOR returns distribution
  - ◇ $\alpha_2(x)$ is the stressed portfolio MPOR returns distribution
  - ◇ p is the likelihood of the stress event
- And to use MP approach, we need to calibrate ρ as well...

Two flavours (among many!) of stressed calibration

$\omega(x)$ is a bimodal distribution that accounts (according to likelihood) for the larger volatility that accompanies periods of stress.

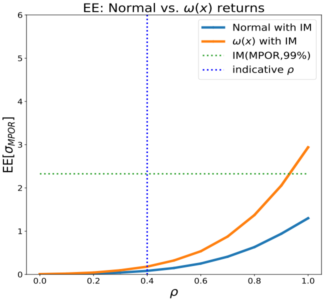
- We construct $\omega(x)$ for two different flavours of market distress:
  - A global financial crisis scenario (GFC benchmark) $\rightarrow$ to be applied to e.g. large broker-dealers with diversified portfolios
  - An idiosyncratic volatility shock scenario (Archegos benchmark) $\rightarrow$ to be applied to e.g. hedge funds with concentrated / illiquid portfolios
- In the following, we use the following parameterisation for $\alpha_1(x)$ and $\alpha_2(x)$:
  - $\alpha_1(x) = N(0, \sigma)$, with σ being the BAU volatility
  - $\alpha_2(x) = N(0, K \cdot \sigma)$, with K being a shock factor

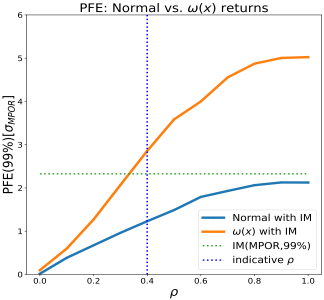
For MC implementations, $\omega(x)$ can be generated by sampling simulated paths of the two calibrations according to p; and more granular specifications of the shocks can be easily specified (e.g. as for the current stressed calibration methodology).

Global financial crisis calibration for large banks cptys

Back-of-the-envelope calibration

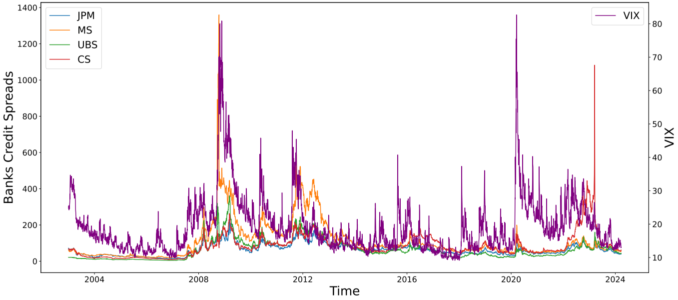
- PD≈ 50bps, MPOR=4w
- $p \approx$ 350bps, based on VIX> 2VIX$_{\mathrm{LT}}$ (≈ 3m/8y stress in a economic cycle)
- $K \approx 2$, based on VIX threshold





The (post IM) stressed PFE is very material and commensurate with the IM!

Global financial crisis calibration for large banks cptys: ρ



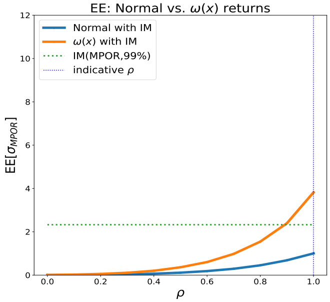
- The WWR is driven by the correlation between market volatility (VIX) and banks PDs (CDSs)
- Assets or log-returns correlations? Similar signal, $\rho \approx 0.3 - 0.5$.

	CDS-VIX ρ (assets)	CDS-VIX ρ (log-returns)
JPM_cds5Y	0.59	0.32
DB_cds5Y	0.28	0.35
MS_cds5Y	0.58	0.26
UBS_cds5Y	0.53	0.23
CS_cds5Y	0.37	0.22

Volatility shock calibration for hedge funds cptys

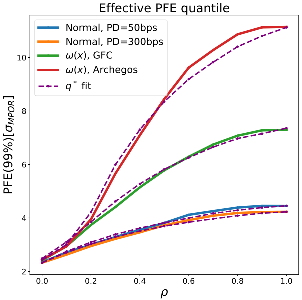
Back-of-the-envelope calibration

- PD≈ 300bps, MPOR=2w
- $p \approx$ 100bps commensurate with the PD
- $K \approx 3.6$, based on Archegos benchmark (Viacom down-move over MPOR)



For $\rho \approx 1$, the level of the stressed PFE is in line with Archegos closeout losses!

The effective PFE quantile: when heavy tails matter...



$$q^* \approx 1 - (\mathsf{PD}_{\mathsf{MPOR}})^{\rho}(1 - q_{\mathsf{PFE}})^{1-\rho^2+\rho}$$
$$\rho = 0 \rightarrow q^* = q_{\mathsf{PFE}}$$
$$\rho = 1 \rightarrow q^* = 1 - (1 - q_{\mathsf{PFE}}) * \mathsf{PD}_{\mathsf{MPOR}}$$

- ◇ For PD=50bps and $\rho = 0.4$, $(1 - q^*) \approx$ 1.1bps
- ◇ For PD=300bps and $\rho = 1$, $(1 - q^*) \approx$ 0.1bps

- WWR correlations can shift materially the quantile of the MPOR returns distribution at which the PFE is computed! As such, and differently from the case of ordinary CCR metrics, tails matter!
- The fitting formula proposed works well across different levels of PDs and quantiles and independently on distributional assumptions.

Interpretation of K and ρ

- the $\omega(x)$ stressed PFE is significantly larger than the PFE generated with Normal returns and the copula, where the latter is itself larger than the ordinary PFE already:

$$\mathsf{PFE}_{\rho,\omega(x)} > \mathsf{PFE}_{\rho,N(x)} > \mathsf{PFE}_{N(x)}$$

- The effects of ρ and $\omega(x)$ are aligned but distinct:
  - ◇ ρ shifts the effective quantile of the MPOR returns distribution that drives the PFE (see previous slide).
  - ◇ The usage of the K shocks and of the mixture model provide weight to the tails of the MPOR returns distribution, so as to produce closeout exposures compatible with historical defaults.

MC framework for CCR stress testing

- We used the copula approach introduced by MP to model WWR; and we constructed a MC framework that can be used for CCR stress testing of collateralised and overcollateralised counterparties.
- In the presence of WWR, PFE is driven by extreme quantiles. As such, the heaviness of the tails becomes a critical driver of the exposure.
- To construct suitable heavy-tailed MPOR returns distributions, we used mixture distributions; that have the advantages of being: i) easily interpretable in terms of market risk factors shocks, and ii) directly implementable in existing MC frameworks.
- Last word of caution: watch out for the number of scenarios, that becomes the critical bottleneck...

Wrapping up

Summary

- CCR stress testing is good for you.
- Defaults of levered counterparties rarely occur in sunny days.
- We served modelling of WWR ad leverage two-ways:
  - ◇ Bottom-up, accounting for the balance sheet / portfolio tail dynamics.
  - ◇ Top-down, using a combination of copula and mixture models.

Acknowledgments

- FAn thanks the organisers for the kind invitation and the audience for the attentive participation.
- FAn thanks the colleagues at the PRA and in the industry for the many inspiring discussions on the topics presented.