

Designing Sustainable, Stable and Equitable Payouts under Longevity Inequality and Uniform Pricing

Presenter: Andrés Villegas

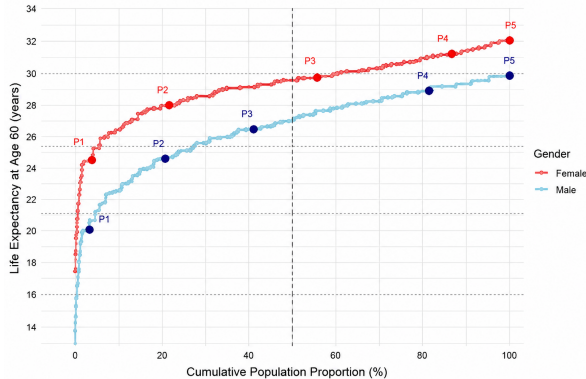
Joint work with: Katja Hanewald, Gayani Thalagoda, Jonathan Ziveyi

UNSW Sydney

L21: The Twenty-First International Longevity Risk and Capital Markets Solutions
Conference
Roma, Italy
September 8-9, 2026

Longevity inequalities and uniform pricing

Australian period life expectancy during 2016-2017 at age 60 (Source: Huang et al., 2024)



Uniform pricing $\Rightarrow$ Unpriced longevity inequality $\Rightarrow$ Inequitable benefit allocations $\Rightarrow$ Low participation

[Download This Paper](#)[Open PDF in Browser](#)[Add Paper to My Library](#)

An Assessment Framework for Equitable Longevity Risk-Sharing Arrangements

31 Pages • Posted: 23 Mar 2026

[Gayani Thalagoda](#)

University of New South Wales (UNSW)

[Katja Hanewald](#)

University of New South Wales (UNSW)

[Andrés Villegas](#)

University of New South Wales (UNSW) - ARC Centre of Excellence in Population Ageing Research (CEPAR)

[Jonathan Ziveyi](#)

University of New South Wales; ARC Centre of Excellence in Population Ageing Research and School of Risk & Actuarial Studies



[There are 2 versions of this paper](#)

Date Written: February 27, 2026

Longevity risk-sharing arrangements

- A longevity risk-sharing arrangement in which participants:
 - Agree to pool their money
 - Receive (variable) payouts while alive
 - Forfeit (at least a part of) their accounts upon death to surviving members
- Two sources of (variable) returns:
 - Investment income
 - “Mortality credits” from the balances of deceased members
- Not generally underwritten, **age-based uniform pricing**
- Those who live longer receive greater cumulative lifetime benefits, **inequitable benefit allocations**

Related work

Piggott et al. (2005); Sabin (2010); Donnelly et al. (2014); Hanewald (2013); Milevsky & Salisbury (2015); Olivieri et al. (2022); Bernhardt and Qu (2024); Bégin and Sanders (2024); Chen and Rach (2023); Dhaene and Milevsky (2024),

What we know

- **Homogeneous participants:** payout design is relatively well understood
Denuit et al. (2022); Jiao et al. (2022); Feng & Liu (2024)
- **Heterogeneous participants:** trade-offs emerge
 - Actuarial fairness cannot be achieved simultaneously for all groups
 - Cross-subsidisation can create inequitable outcomes and adverse selection
 - Premium and/or benefit differentiation are possible responses
 - Joint wealth-longevity heterogeneity:** alternative sharing rules **depend on social cohesion**

Gap: No framework when neither benefit nor price differentiation is feasible or allowed

Our contribution

A generalised two-step framework for designing equitable, stable, and sustainable payouts under uniform pricing

Step 1: Menu of feasible payout paths

$$\begin{aligned} \min_{B(t) \in \mathbb{C}} \{ & S(B(t)) + \lambda E(B(t)) \} \\ \text{s.t. } & \int_0^{\omega-x} e^{-\delta t} S_x(t) B(t) dt = W_0 \end{aligned}$$

- Generates the Pareto frontier
(by varying $\lambda_E/\lambda_S \in [0, \infty)$)
- Makes trade-offs explicit
- Decides what is actuarially feasible

Step 2: Welfare maximisation

$$B^*(t) \in \arg \max_{B(t) \in \mathbb{C}} \sum_{k=1}^K w_k E[u_k],$$

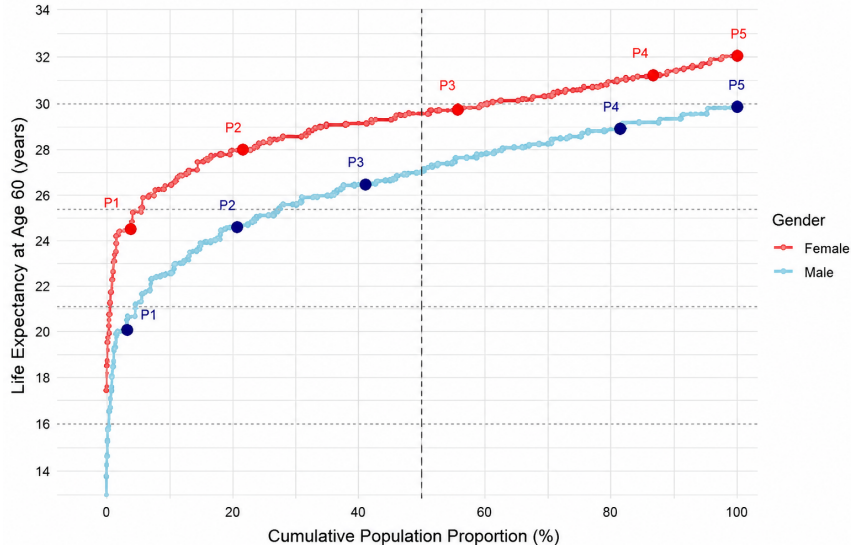
- Selects collectively preferred payout path
- Generalized welfare criteria
(pure self-interest, inequity-averse)

$B(t)$: fund payout, $E(B(t))$: equity loss, $S(B(t))$: stability loss.

Key idea: disentangling actuarial feasibility from welfare aggregation

Numerical Application: Australian Socio-economic Profiles

Australian period life expectancy during 2016-2017 at age 60 (Source: Huang et al., 2024)



Pool setup

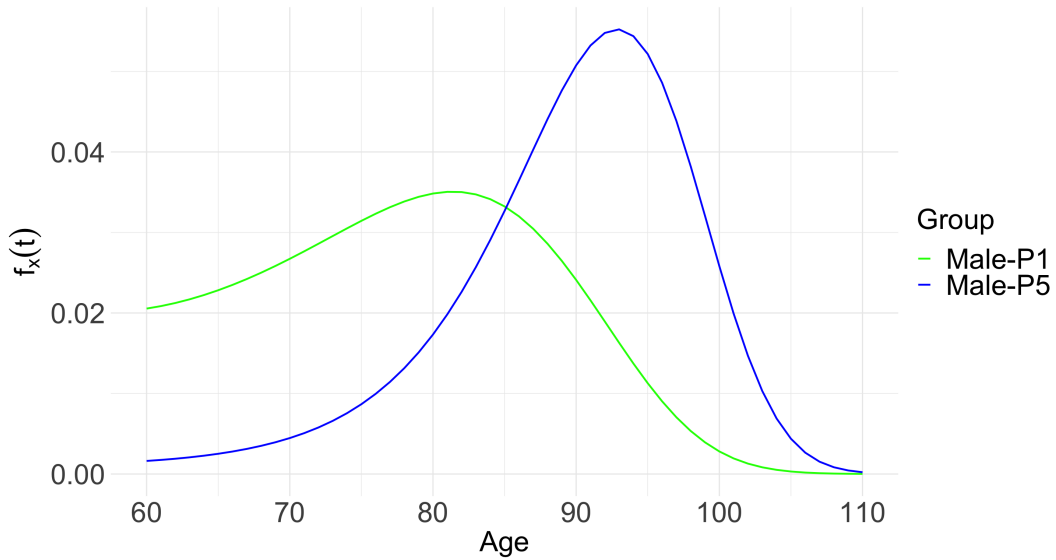
- Age: 60-old males

- Groups:

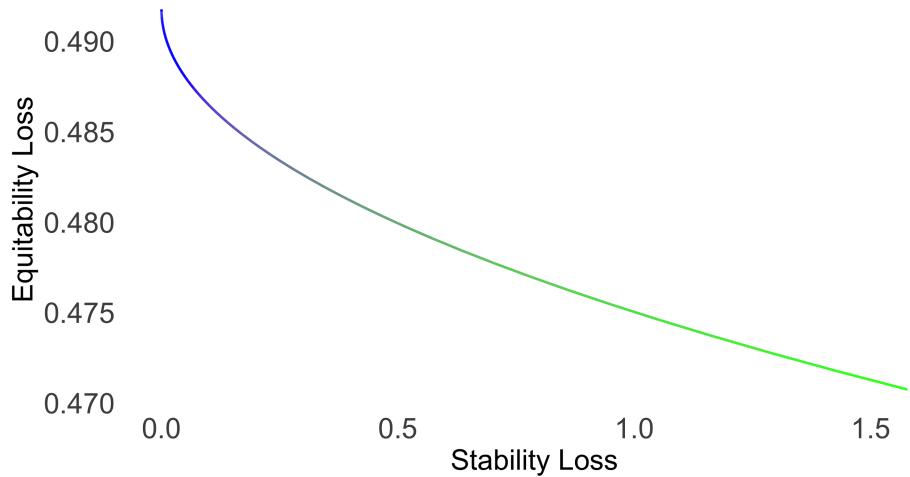
Group 1: Single, non-homeowner, <\$499, D1-IRSAD: Tend to leave early and subsidize the long-lived group

Group 2: Married, homeowner, > \$1,000, D10-IRSAD: Tend to live longer and collect more payouts

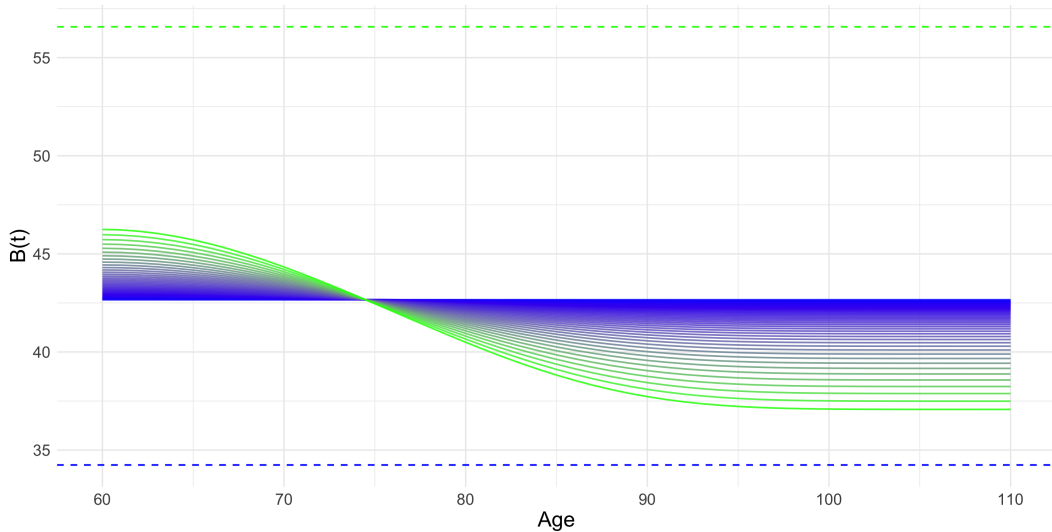
- Pool composition: (50%, 50%)
- Initial investment: (\$100k, \$100k)
- Closed pool



Step 1: Pareto frontier



Step 1: Menu of actuarially feasible tontine payouts



Collective Choice

- Case 1: Pure self-interest
- Case 2: Inequity-averse preferences (Fehr and Schmidt, 1999)

Step 2: Welfare selection — Case 1 (no inequity aversion)

Lifetime benefit per dollar invested

$$\mathcal{B}(T_{x,i,k}) = \int_0^{T_{x,i,k}} e^{-\delta t} B(t) dt$$

Utility from lifetime benefits

$$u(\mathcal{B}(T_{x,i,k})) = \mathcal{B}(T_{x,i,k})$$

Expected utility

$$E[u_k] = E[u(\mathcal{B}(T_{x,i,k}))] = E[\mathcal{B}(T_{x,i,k})]$$

$$E[u_k] = \int_0^{\infty} e^{-\delta t} S_k(t) B(t) dt$$

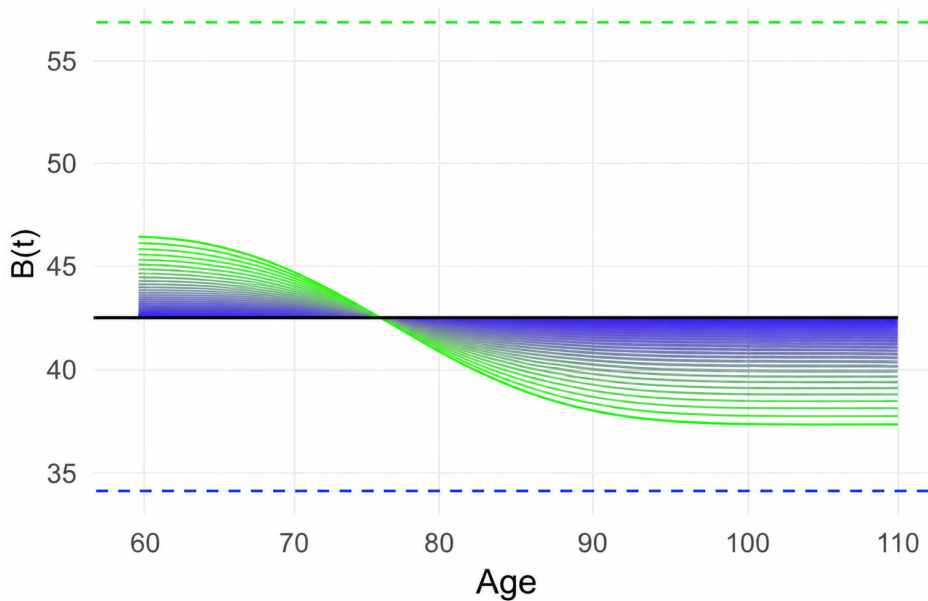
Aggregate social welfare

$$\mathcal{W} = \sum_{k \in K} c_k E[u_k]$$

δ : benefit discount rate, $S_k(t)$: survival probability, c_k : contribution weight

Interpretation: Participants care only about their own lifetime benefits.

Case 1: Optimal $B^*(t)$



Step 2: Welfare selection — Case 2 (with inequity aversion)

Utility from lifetime benefits with inequity aversion

$$u(\mathcal{B}(T_{x,i,k})) = \mathcal{B}(T_{x,i,k}) - \underbrace{\alpha_k \sum_{h=1}^K c_h \max(\mathcal{B}(T_{x,j,h}) - \mathcal{B}(T_{x,i,k}), 0)}_{\text{Disadvantageous inequity (envy)}} - \underbrace{\beta_k \sum_{h=1}^K c_h \max(\mathcal{B}(T_{x,i,k}) - \mathcal{B}(T_{x,j,h}), 0)}_{\text{Advantageous inequity (guilt)}}$$

Expected utility

$$E[u_k] = E[u(\mathcal{B}(T_{x,i,k}))]$$

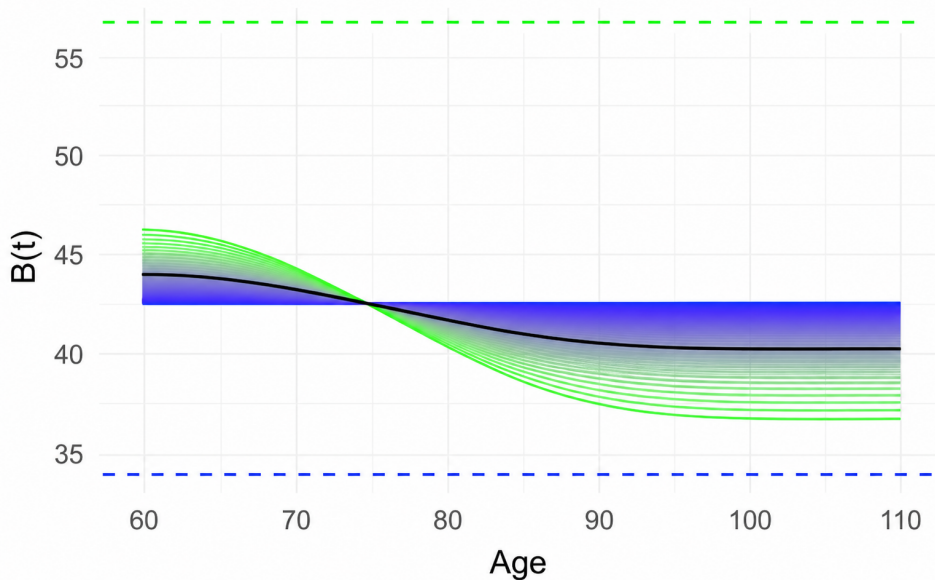
Aggregate social welfare

$$\mathcal{W} = \sum_{k \in K} c_k E[u_k]$$

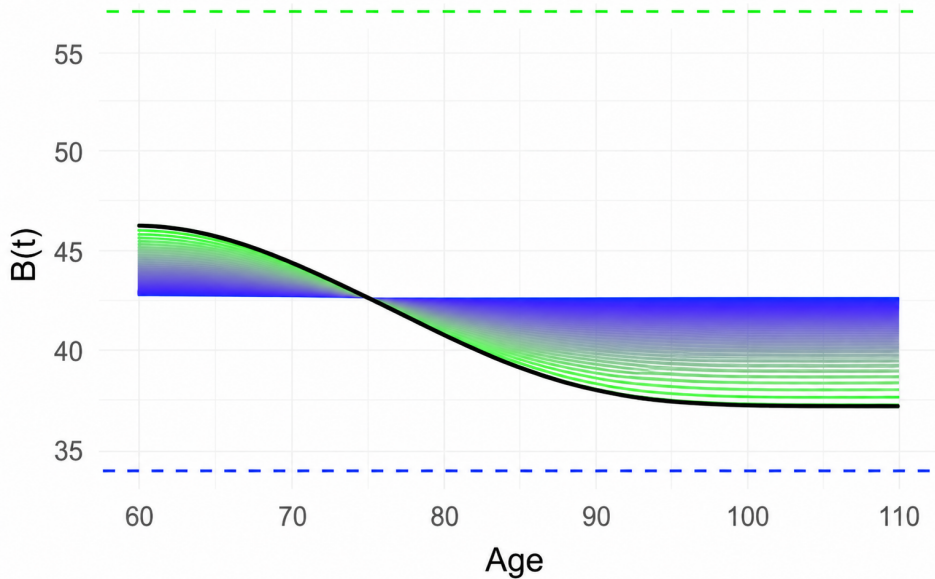
α_k : envy parameter, β_k : guilt parameter, c_k : contribution weight

Interpretation: Participants value their realized lifetime benefits and experience utility losses from advantageous and disadvantageous benefit differences.

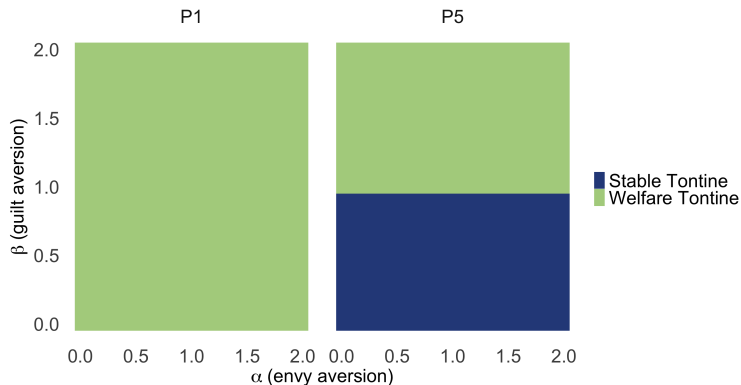
Case 2: Optimal $B^*(t)$



Case 3: Optimal $B^*(t)$



Participation: Equitable payout Vs. stable payout



Equitable designs can increase participation

Conclusion

Uniform pricing can hide sociodemographic longevity inequality, leading to unequal lifetime benefit outcomes

Contribution

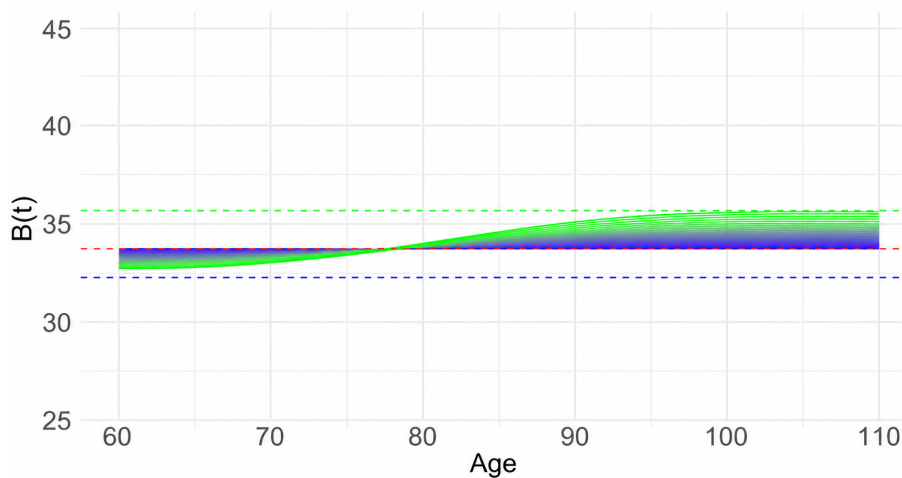
A generalised two-step framework that:

- Identifies the menu of feasible payout designs
- Separates actuarial feasibility from welfare choice

Practical relevance

- Retirees: more equitable benefit outcomes
- Funds: potential for broader participation and larger pools
- Regulators: transparent and auditable design choices

Use case: EU directive



Interventions depend largely on the source of inequality and the normative interpretations of 'fairness'

Thank you!

Andrés Villegas
a.villegas@unsw.edu.au

References

Milevsky & Salisbury. (2015). Optimal retirement income tontines. *Insurance: Mathematics and economics*, 64, 91–105.

Huang, F., Hui, F., & Villegas, A. (2025). Towards fairer retirement outcomes: Socio-economic mortality differentials in australia. *Available at SSRN 5253598*.

Fehr, E., & Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. *The quarterly journal of economics*, 114(3), 817–868.

Dhaene, J., & Milevsky, M. A. (2024). Egalitarian pooling and sharing of longevity risk aka can an administrator help skin the tontine cat? *Insurance: Mathematics and Economics*, 119, 238–250.

Moenig, T., & Zhu, N. (2025). Adverse selection in tontines. *The Geneva Risk and Insurance Review*, 50(1), 6–38.

Benefit Equitability (Thalagoda et al., 2025)

$$\begin{aligned}
 \underbrace{\Delta B}_{\text{total inequality}} &= \underbrace{\sum_{k=1}^K \left(\frac{p_k W_k}{\left(\sum_{i=1}^K p_i W_i \right)^2} \right)^2 \Delta B_{kk}}_{\text{within}} + \underbrace{\sum_{k=2}^K \sum_{h=1}^{k-1} 2 \frac{p_k W_k p_h W_h}{\left(\sum_{i=1}^K p_i W_i \right)^2} \Delta B_{kh}}_{\text{between}} \\
 &= \sum_{k=1}^K \left(\frac{p_k W_k}{\left(\sum_{i=1}^K p_i W_i \right)^2} \right)^2 \Delta B_{kk} + \sum_{k=2}^K \sum_{h=1}^{k-1} \frac{p_k W_k p_h W_h}{\left(\sum_{i=1}^K p_i W_i \right)^2} \underbrace{(B_{hk} + B_{kh})}_{\text{inequitability}} \\
 &= \frac{1}{4} \Delta B_{\text{Low,Low}} + \frac{1}{4} \Delta B_{\text{High,High}} + \underbrace{\frac{1}{4} B_{\text{Low,High}} + \frac{1}{4} B_{\text{High,Low}}}_{\text{inequitability}}
 \end{aligned}$$