Heterogeneous Expectations and Speculative Behavior in Insurance-linked Securities

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Motivation

- Background:
  - Principle of Insurance: Law of Large Numbers
    - The greater the number of similar exposures, the more predictable the outcome
  - Life Insurance Pricing Risks
    - Mortality-when death occurs
    - Persistency-how long the customer keeps the policy
    - Expenses-how efficient the company operates
    - Investment-return of and on invested cashflows.
  - Risks insured (life insurance)
    - Premature death
    - Outliving accumulated savings

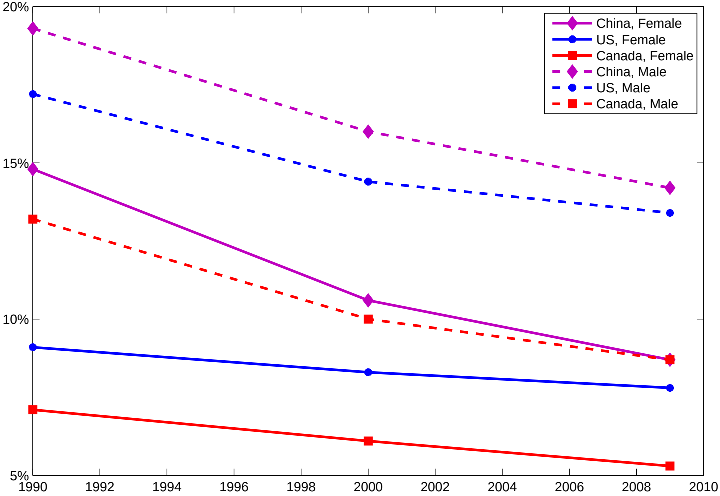


Figure: Mortality Improvement (Source: WHO (2012a). ["World Health Statistics 2011"]. Geneva. Accessed March 2012.)

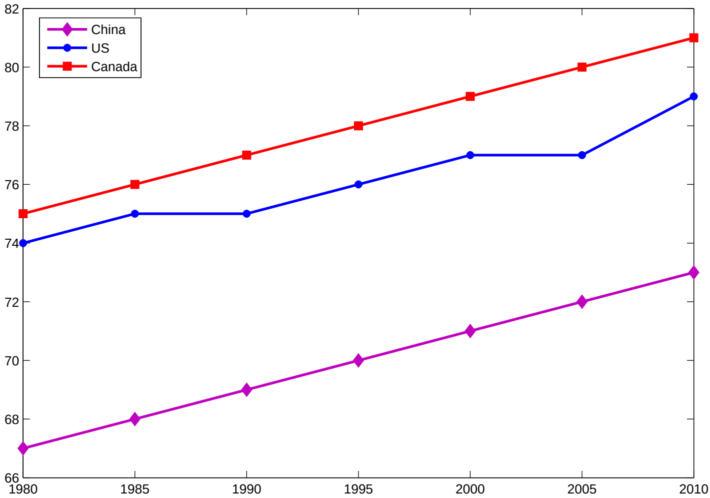
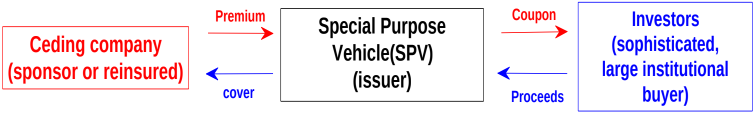


Figure: Life Expectation At Birth (Source: World Bank)

- Solutions:
  - Natural Hedge
    - Natural hedge between life insurance and annuity
    - Ineffective and cost prohibitive
  - Reinsurance
    - capacity constraint
  - Securitization
    - Transfer risk to investors in capital market
    - Derivative securities with payoffs link to certain insurance risk
    - E.g. Swiss Re Mortality Bond (2003), EIB/BNP Longevity Bond (2004)

Securitization: ILS



- Key elements
  - Price
  - Demand v.s supply
  - Ability to hedge risks
  - ...

Securitization: ILS

- Pricing
  - No-arbitrage approach: Cairns, Blake and Dowd (2006), Bauer, Boerger and Russ (2010)
  - Maximum Entropy: Kogure and Kurachi (2010)
  - Indifference pricing approach: Cui (2008), Cox, Lin and Pederson (2010)
  - Tâtonnement approach: Zhou, Li and Tan (2011) and Chen, Sherris, Sun and Zhu (2012)
    - A gradual calibration of supply and demand

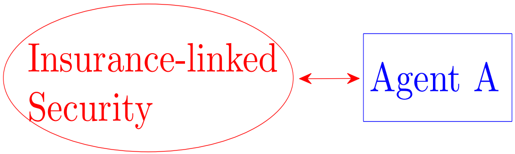
Securitization: Tâtonnement approach

- Agent I (hedger) v.s Agent A (investor)
- Agents are expected utility maximizers.
- Two investment choices in Zhou, Li and Tan (2011)
  - the insurance-linked security
  - agents can lend/borrow at the same risk-free interest rate r

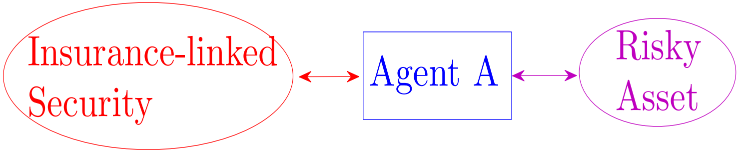
Securitization

$\boxed{\text{Agent A}}$

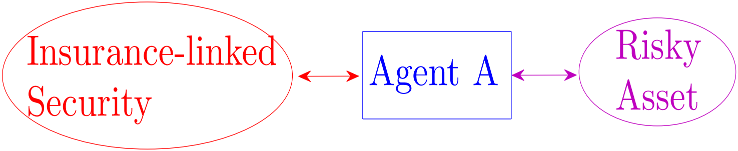
Securitization



Securitization



Securitization



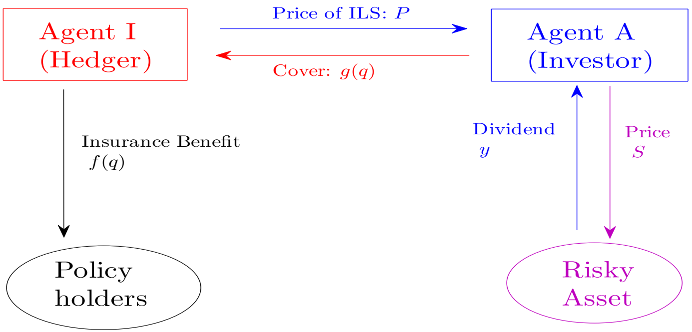
- ILS offers a new class of investment opportunities
- Agent A wants to optimize his whole portfolio.

In this paper

- We follow the Tâtonnement approach and use the BH framework given by Brock and Hommes (1997,1998).
- Main questions:
  - Whether ILS can help the insurance company to hedge its risk?
  - Whether ILS as a new investment tool can increase the investment profit?
  - What relationship between the risky asset and ILS?
- Three cases:
  - Static model with two agents
  - Dynamic model with two agents
  - Dynamic model with three agents

Set-up: One-period

- q: the risk agent I wants to hedge
- Cash flow for agents



- Θ_I, Θ_A: quantities of ILS traded by agents I and A
- Φ_A: quantity of the risky asset traded by agent A

Set-up: One-period

- Agent A
  - wealth process

$$W_A = (\omega_A - \Theta_A P - \Phi_A S)R + \Theta_A g(q) + \Phi_A y, \quad R = 1 + r$$

  - Agent A is a myopic mean-variance maximizer based on his own belief:

$$\sup_{\Theta_A, \Phi_A} E_0^A(W_A) - \frac{k_A}{2} V_0^A(W_A),$$

where E_0^h and V_0^h denote the 'beliefs' of agent h about the conditional expectation and conditional variance at time 0 and k_h denotes the risk aversion of agent h.

Set-up: One-period

- Agent I
  - Hedging term: Basis risk (BR)
    $$BR = f(q) - \Theta_I(g(q) - RP).$$
  - If $BR \leq 0$, then it means this type of ILS can completely hedge all the risks and even more, the profit can be obtained by ILS.
  - Otherwise, it means there exists the basis risk between this type of ILS and the risk hedge demand.
  - Objective:
    $$\inf_{\Theta_I} E_0^I(BR) + \frac{k_I}{2} V_0^I(BR).$$

- Notations:

$$\mathcal{Y}_h = E_0^h(y),\ \mathcal{G}_h = E_0^h(g(q)), \quad (h = A, I)$$

$$\sigma^2_{h,y} = V_0^h(y),\ \sigma^2_{h,g} = V_0^A(g(q)),\ \sigma^2_{h,f} = V_0^A(f(q)),$$

$$cov_{A,gy} = cov_0^A(g(q), y),\ cov_{I,gf} = cov_0^I(g(q), f).$$

- Equilibrium: Walrasian scenario

$$\begin{cases} \Theta_A + \Theta_I = 0 \\ \Phi_A = 0 \end{cases} \tag{1}$$

Theorem

The equilibrium prices of ILS and the risky asset respectively are

$$P^* = R^{-1} \frac{(\mathcal{G}_I + k_I cov_{I,gf}) k_A \sigma^2_{A,g} + \mathcal{G}_A k_I \sigma^2_{I,g}}{k_A \sigma^2_{A,g} + k_I \sigma^2_{I,g}} \tag{2}$$

$$S^* = R^{-1} \left(\mathcal{Y}_A + k_A cov_{A,gy} \frac{\mathcal{G}_I + k_I cov_{I,gf} - \mathcal{G}_A}{k_A \sigma^2_{A,g} + k_I \sigma^2_{I,g}} \right) \tag{3}$$

ILS v.s. Risky Asset

- The relationship between the risky asset and ILS

$$\frac{\mathcal{Y}_A - RS^*}{k_A} = \frac{\mathcal{G}_A - RP^*}{k_A} + \frac{\sigma^2_{A,g} - cov_{A,gy}}{\sigma^2_{I,g}} \frac{\mathcal{G}_I + k_I cov_{I,gf} - RP^*}{k_I}. \tag{4}$$

ILS v.s. Risky Asset

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- "CAPM" format

$$\mathcal{Y}_A - RS^* = \frac{cov_{A,gy}}{\sigma^2_{A,g}} (\mathcal{G}_A - RP^*). \tag{5}$$

ILS v.s. Risky Asset

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- "CAPM" format

$$\mathcal{Y}_A - RS^* = \frac{cov_{A,gy}}{\sigma^2_{A,g}} (\mathcal{G}_A - RP^*). \tag{5}$$

- If $cov_{A,gy} = 0$, then

$$S^* = R^{-1} \mathcal{Y}_A.$$

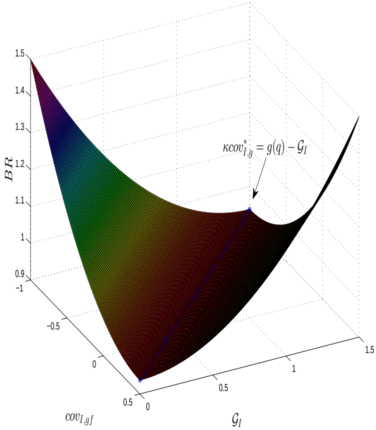
- If $cov_{A,gy} \neq 0$, then

$$\mathcal{Y}_A - RS^* = \frac{cov_{A,gy}}{\sigma^2_{A,g}} (\mathcal{G}_A - RP^*) = k_A cov_{A,gy} \Theta^*_A.$$

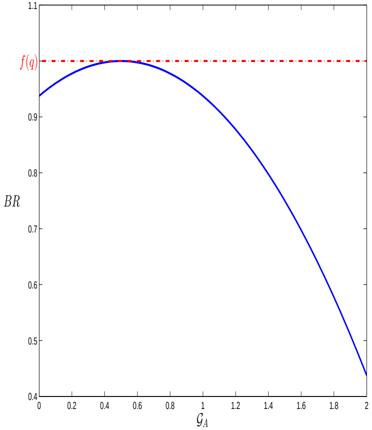
Basis Risk

- $BR = f(q) - \Theta_I^*(g(q) - RP^*)$ measures the level of the risk hedge through ILS.
- If $\sigma_{A,g} = \sigma_{I,g} = \varpi$ and $k_A = k_I = \kappa$, then

$$
\begin{aligned}
BR &= f(q) - \Theta_I^*(g(q) - RP^*) \\
&= \frac{\kappa}{4\varpi^2}\left(cov_{I,gf} - \frac{g(q) - \mathcal{G}_I}{\kappa}\right)^2 + \left(f(q) - \frac{(g(q) - \mathcal{G}_A)^2}{4\kappa\varpi^2}\right) \\
&= \frac{1}{4\kappa\varpi^2}\Big(\kappa cov_{I,gf} - (g(q) - \mathcal{G}_I)\Big)^2 + \left(f(q) - \frac{(g(q) - \mathcal{G}_A)^2}{4\kappa\varpi^2}\right)
\end{aligned}
$$



(a) $\mathcal{G}_A = 1$



(b) $cov_{I,gf} = 1$ and $\kappa cov_{I,gf} = g(q) - \mathcal{G}_I$

Figure: Basis risk at $g(q) = 0.5$, $f(q) = 1$, $\sigma_{I,g} = \sigma_{A,g} = 1$, $k_I = k_A = 1$.

Set-up: Multi-period

- $\Theta_{h,t}$ and $\Phi_{h,t}$ respectively are the net position of the risky asset and ILS held by agent h.
- The wealth process of Agent A is

$$W_{A,t+1} = W_{A,t}R + \Theta_{A,t}R_{P,t+1} + \Phi_{A,t}R_{S,t+1}$$

- Similarly, the basis risk of agent I would be

$$BR_{t+1} = f(Q_{t+1}) - \Theta_{I,t}R_{P,t+1}.$$

  where $R_{P,t+1} = g(Q_{t+1}) + P_{t+1} - RP_{t+1}$ and $R_{S,t+1} = S_{t+1} + y_{t+1} - RS_t$.

- At every time, the two agents will choose the position of the risky asset and ILS to optimize their utilities based on their own beliefs

$$\begin{cases} \text{Agent } A: & \sup_{\Theta_{A,t},\Phi_{A,t}} E_t^A(W_{A,t+1}) - \frac{k_A}{2} V_t^A(W_{A,t+1}), \\ \text{Agent } I: & \inf_{\Theta_{I,t}} E_t^I(BR_{t+1}) + \frac{k_I}{2} V_t^I(BR_{t+1}), \end{cases}$$

where E_t^h and V_t^h respectively are the conditional expectation and variance of agent h ($h = A, I$) at time t based on the publically available information $\mathcal{F}_t = \{P_0, \cdots, P_{t-1}, S_0, \cdots, S_{t-1}\}$.

- Assume that agent I adopts the fundamental analysis to analyze the price trend while agent A prefers using the technical methods to estimate the price value, that is

$$E_t^A(S_{t+1}) = S_{t-1} + \gamma_A(S_{t-1} - \mathcal{S}_{t-1}^{\sharp}), \quad \gamma_A > 0$$
$$E_t^A(P_{t+1}) = P_{t-1} + \alpha_A(P_{t-1} - \mathcal{P}_{t-1}^{\sharp}), \quad \alpha_A > 0$$
$$E_t^I(P_{t+1}) = P_{t-1} + \alpha_I(P_{t-1}^{*} - P_{t-1}), \quad \alpha_I \in [0,1].$$

- For agent A, the reference prices are calculated by the moving average method

$$\mathcal{S}_t^{\sharp} = \frac{1}{2}(S_{t-1} + S_t),$$
$$\mathcal{P}_t^{\sharp} = \frac{1}{2}(P_{t-1} + P_t).$$

- For agent I, the fundamental price comes from

$$E_t^I\left(g(Q_{t+1}) + \mathcal{P}_{t+1}^* - R\mathcal{P}_t^*\right) + k_I cov_{I,Pf} = 0.$$

- In the case where the coupon process $\{g(Q_t)\}$ is IID, $E_t^I(g(Q_{t+1})) = \mathcal{G}_I$ which is a constant and a standard notion of 'fundamental' is

$$\mathcal{P}_t^* = \frac{\mathcal{G}_I + k_I cov_{I,Pf}}{R-1}.$$

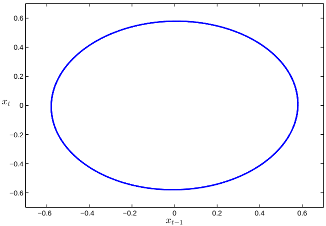
Proposition

The equilibrium prices of ILS and the risky asset are

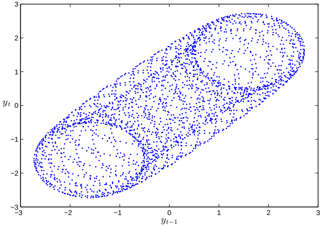
$$\overline{P} = r^{-1}\frac{\frac{r}{k_A\sigma^2_{A,P}}\mathcal{G}_A + \frac{r+\alpha_I}{k_I\sigma^2_{I,P}}(\mathcal{G}_I + k_I cov_{I,Pf})}{\frac{r}{k_A\sigma^2_{A,P}} + \frac{r+\alpha_I}{k_I\sigma^2_{I,P}}},$$

$$\overline{S} = r^{-1}\left(\mathcal{Y}_A + k_A cov_{A,PS}\frac{r+a_I}{rk_I\sigma^2_{I,P} + (r+\alpha_I)k_A\sigma^2_{A,P}}(\mathcal{G}_I + k_I cov_{I,Pf} - \mathcal{G}_{\mathcal{A}})\right),$$

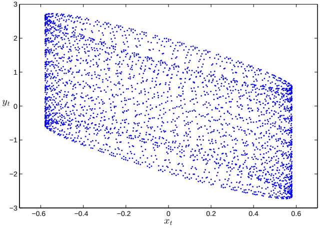
which are stable when $\alpha_A k_I\sigma^2_{I,P} < 2R(k_A\sigma^2_{A,P} + k_I\sigma^2_{I,P})$ *and* $\gamma_A < 2R$.



(a) Price of ILS



(b) Price of the risky asset



(c) ILS v.s. Risky Asset

Figure: $\alpha_A k_I \sigma_{I,P}^2 = 2R(k_A \sigma_{A,P}^2 + k_I \sigma_{I,P}^2)$ and $\gamma_A = 2R$.

Set-up: Three agents

- Except agents I and A, there is another agent who is just interested in the risky asset.
- The wealth process of agent B is

$$W_{B,t+1} = W_{B,t}R + \Phi_{A,t}R_{S,t+1}.$$

- Equilibrium conditions

$$\begin{aligned} n_{I,t-1}\Theta_{I,t} + n_{A,t-1}\Theta_{A,t} &= 0 \\ n_{A,t-1}\Phi_{A,t} + n_{B,t-1}\Phi_{B,t} &= 0 \end{aligned}$$

where $n_{h,t}$ is the population fraction of agent h ($h = I, A, B$) and $n_{I,t} + n_{A,t} + n_{B,t} = 1$

- The equilibrium price satisfies

$$
\begin{aligned}
& \frac{n_{A,t-1}}{k_A} E_t^A(R_{S,t+1}) + \frac{\sigma^2_{A,S} - cov_{A,PS}}{\sigma^2_{B,S}} \frac{n_{B,t-1}}{k_B} E_t^B(R_{S,t+1}) \\
= \quad & \frac{n_{A,t-1}}{k_A} E_t^A(R_{P,t+1}) + \frac{\sigma^2_{A,P} - cov_{A,PS}}{\sigma^2_{I,P}} \frac{\mathbf{n_I}}{k_I} (E_t^I(R_{P,t+1}) + k_I cov_{I,Pf})
\end{aligned}
$$

- The equilibrium price satisfies

$$
\begin{aligned}
& \frac{n_{A,t-1}}{k_A} E_t^A(R_{S,t+1}) + \frac{\sigma_{A,S}^2 - cov_{A,PS}}{\sigma_{B,S}^2} \frac{n_{B,t-1}}{k_B} E_t^B(R_{S,t+1}) \\
= \quad & \frac{n_{A,t-1}}{k_A} E_t^A(R_{P,t+1}) + \frac{\sigma_{A,P}^2 - cov_{A,PS}}{\sigma_{I,P}^2} \frac{\mathbf{n_I}}{k_I} (E_t^I(R_{P,t+1}) + k_I cov_{I,Pf})
\end{aligned}
$$

- Special case: $\mathbf{n_I} = 0$, then

$$
E_t^A(R_{p,t+1}) = \frac{cov_{A,PS}}{\sigma_{A,S}^2} E_t^A(R_{S,t+1}). \tag{6}
$$

- Agents cannot switch to each other, that is to say, $n_{I,t} \equiv \mathbf{n_I}$, $n_{A,t} \equiv \mathbf{n_A}$, $n_{B,t} \equiv \mathbf{n_B}$ and $\mathbf{n_I} + \mathbf{n_A} + \mathbf{n_B} = \mathbf{1}$.
- Agents A and B can switch to each other.
  - The population fractions are updated by the well known logit model probabilities

$$n_{h,t} = \frac{\exp(\beta_h U_{h,t})}{Z_t}, \tag{7}$$

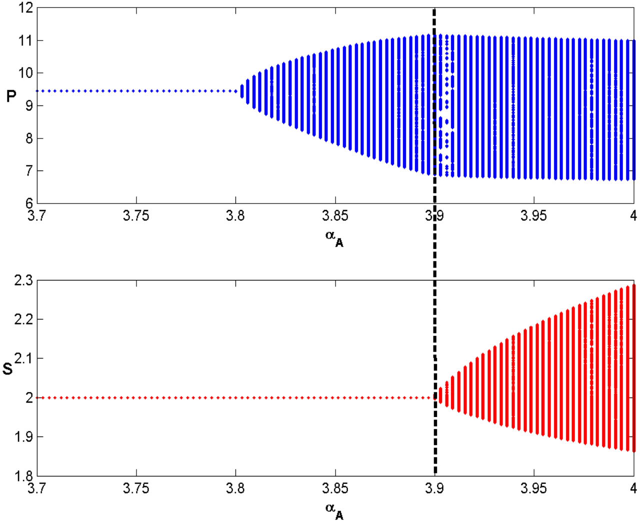
    where $Z_t = \exp(\beta_A U_{A,t}) + \exp(\beta_B U_{B,t})/(1 - \mathbf{n_I})$.
  - $U_{h,t}$ is the performance measure

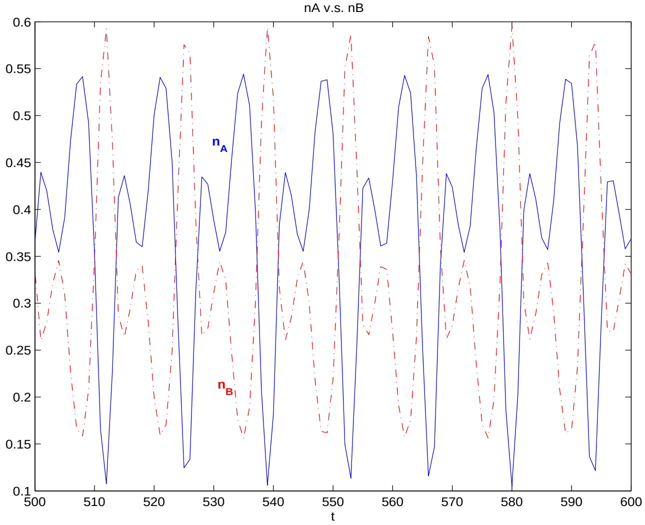
$$U_{h,t} = \pi_{h,t} + \eta_{h,t} U_{h,t-1},$$

    where η_h represents the 'memory strength' of agent h and $\pi_{h,t}$ is given by realized profits for agent h, that is

$$\begin{aligned} \pi_{A,t} &= R_{P,t} \cdot \Theta_{A,t-1} + R_{S,t} \cdot \Phi_{A,t-1} \\ \pi_{B,t} &= R_{S,t} \cdot \Phi_{A,t-1}. \end{aligned}$$

- NS bifurcation



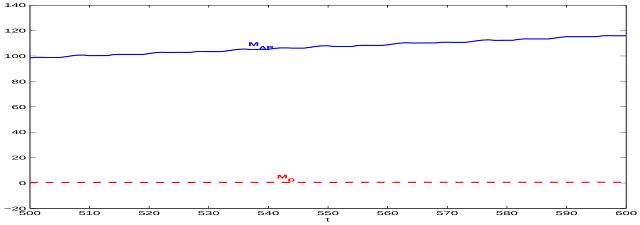

nA v.s. nB
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
500
510
520
530
540
550
560
570
580
590
600
t
n_A
n_B

- The impact of the existence of ILS
  - Only if investing in the risky asset (i.e. choosing Strategy B), is the aggregate wealth

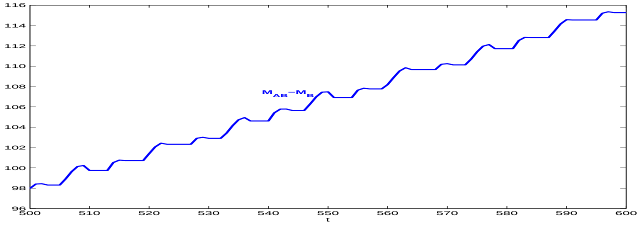
$$M_{B,t+1} = M_{B,t} + \pi_{B,t+1}. \tag{8}$$

  - If choosing the better one between Strategy A and Strategy B, the aggregate wealth is

$$M_{AB,t+1} = M_{AB,t} + \pi_{A,t+1} \cdot \mathcal{I}_{\pi_{A,t+1} > \pi_{B,t+1}} + \pi_{B,t+1} \cdot (1 - \mathcal{I}_{\pi_{A,t+1} > \pi_{B,t+1}}). \tag{9}$$



(a) Aggregate wealth



(b) Difference between different strategies

Conclusion

- We construct a heterogeneous agent model to link the ILS market and the tradition financial market.
- Through the study of basis risk, we analyze the reason of success or failure about the issuance of ILS.
- We study the reaction of the ILS and the tradition risky asset and the role of the ILS in investment portfolio.
- We can discuss the impact of different parameters.
- Future works:
  - Market-maker scenario
  - Inventory effect
  - Optimizing the VaR of insurance companies
  - ...