On Discrete-Time Greek Hedging of Longevity Risk

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Abstract

In this paper, we study Greek hedging of longevity risk with standardized q-forwards. We consider 'delta' and 'gamma', which represent the sensitivity of a mortality-dependent portfolio to changes in the underlying period effects. We further propose to incorporate 'vega', which measures the sensitivity to changes in the period effects' volatility, a property that researchers are increasingly concerned with. Our results reveal several informative facts about standardized q-forward hedges. One example is that while the success of a delta-gamma hedge depends heavily on the reference ages of the q-forwards used, the effectiveness of a delta-vega hedge is robust relative to the choice of reference ages. Another example is that owing to the unique convexities of annuity liabilities and q-forwards, a delta-only hedge does not perform satisfactorily in mitigating left-tail risk but the problem can be overcome by including an additional Greek letter. We present the proposed hedging methodologies in both static and dynamic set-ups, and apply them to situations when population basis risk is absent and present.

Key words: Conditional heteroscedasticity models; Index-based longevity hedges; Population basis risk; q-forwards; The Lee-Carter model

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