Pricing longevity-linked options

Longevity 11 – Lyon, France

9 September 2015

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Agenda

- Motivation
- Recap: The forward mortality framework
- The problem of options
- Market-consistent mortality dynamics
- Pricing longevity-linked options
- Discussion

Motivation

- Many issued and proposed mortality- and longevity-linked securities are in the form of options:
  - Mortality catastrophe bonds
  - Swiss Re Kortis Bond
  - Survivor caps and floors, LEOs
- Options can be better at hedging tail risks than forwards, such as swaps, so more capital efficient
- However, require more sophisticated models to price

Recap: The forward mortality framework

- Forward mortality rates and time, τ, for age, x, and future year, t

$$\nu^{\mathbb{Q}}_{x,t}(\tau) = \mathbb{E}^{\mathbb{Q}}{}_{\tau}\mu_{x,t}$$

- But do not have the distribution of force of mortality, $\mu_{x,t}$, in market-consistent measure, $\mathbb{Q}$
  - If use age/period/cohort mortality model

$$\ln(\mu_{x,t}) = \alpha_x + \boldsymbol{\beta}_x^{\top}\boldsymbol{\kappa}_t + \gamma_{t-x}$$

  - Have expectation in real-world measure, $\mathbb{P}$

$$\nu^{\mathbb{P}}_{x,t}(\tau) = \exp\left(\alpha_x + \boldsymbol{\beta}_x^{\top}\mathbb{E}^{\mathbb{P}}{}_{\tau}\boldsymbol{\kappa}_t + \frac{1}{2}\boldsymbol{\beta}_x^{\top}\mathbb{V}ar^{\mathbb{P}}_{\tau}(\boldsymbol{\kappa}_t)\boldsymbol{\beta}_x + \mathbb{E}^{\mathbb{P}}{}_{\tau}\gamma_{t-x} + \frac{1}{2}\mathbb{V}ar^{\mathbb{P}}_{\tau}(\gamma_{t-x})\right)$$

Recap: The forward mortality framework

- In real-world measure
  - Random walk for period functions

$$\mathbb{E}^{\mathbb{P}}{}_{\tau}\, \boldsymbol{\kappa}_t = \boldsymbol{\kappa}_\tau + \mu \sum_{s=\tau+1}^{t} X_s$$

$$\mathbb{V}ar_{\tau}^{\mathbb{P}}\,(\boldsymbol{\kappa}_t) = (t-\tau)\Sigma$$

  - Bayesian approach for cohort function

$$\mathbb{E}^{\mathbb{P}}{}_{\tau}\, \gamma_y = D_{\tau-y}\overline{\gamma}_y(t) + (1-D_{t-y})(\beta\tilde{X}_y + \rho(\mathbb{E}^{\mathbb{P}}{}_{\tau}\, \gamma_{y-1} - \beta\tilde{X}_{y-1})$$

$$\mathbb{V}ar_{\tau}^{\mathbb{P}}(\gamma_y) = (1-D_{t-y})\sigma^2 + (1-D_{t-y})^2\rho^2\mathbb{V}ar_{\tau}^{\mathbb{P}}(\gamma_{y-1})$$

Recap: The forward mortality framework

- Use Esscher transform

$$\mathbb{E}^{\mathbb{Q}} X_{x,t} = \frac{\mathbb{E}^{\mathbb{P}}\left[X_{x,t}\exp(-Z_{x,t})\right]}{\mathbb{E}^{\mathbb{P}}\exp(-Z_{x,t})}$$

$$Z_{x,t} = \boldsymbol{\lambda}^{\top}\boldsymbol{\kappa}_t + \lambda^{\gamma}\gamma_{t-x}$$

- To give expectation in market-consistent measure

$$\begin{aligned}\nu^{\mathbb{Q}}_{x,t}(\tau) = \exp\Bigg(&\alpha_x + \boldsymbol{\beta}_x^{\top}\mathbb{E}^{\mathbb{P}}{}_{\tau}\,\boldsymbol{\kappa}_t + \frac{1}{2}\boldsymbol{\beta}_x^{\top}\mathbb{V}ar^{\mathbb{P}}_{\tau}\,(\boldsymbol{\kappa}_t)\boldsymbol{\beta}_x + \mathbb{E}^{\mathbb{P}}{}_{\tau}\,\gamma_{t-x} \\ &+\frac{1}{2}\mathbb{V}ar^{\mathbb{P}}_{\tau}\,(\gamma_{t-x}) - \frac{1}{2}\boldsymbol{\beta}_x^{\top}\mathbb{V}ar^{\mathbb{P}}_{\tau}\,(\boldsymbol{\kappa}_t)\boldsymbol{\lambda} - \frac{1}{2}\boldsymbol{\lambda}^{\top}\mathbb{V}ar^{\mathbb{P}}_{\tau}\,(\boldsymbol{\kappa}_t)\boldsymbol{\beta}_x - \lambda^{\gamma}\mathbb{V}ar^{\mathbb{P}}_{\tau}(\gamma_{t-x})\Bigg) \\ = \exp&\left(-\boldsymbol{\beta}_x^{\top}\mathbb{V}ar^{\mathbb{P}}_{\tau}(\boldsymbol{\kappa}_t)\boldsymbol{\lambda} - \lambda^{\gamma}\mathbb{V}ar^{\mathbb{P}}_{\tau}(\gamma_{t-x})\right)\nu^{\mathbb{P}}_{x,t}(\tau)\end{aligned}$$

- Calibration:
  - Either from genuine market information (e.g., longevity swap rates, q-forward prices); or
  - From internal deterministic mortality assumption (e.g., reserving assumption, reinsurance quotation)

The problem of options

- To price options, we need the distribution of the force of mortality in the market-consistent measure
  - E.g., equity forward contrasts required drift of share price, r, in risk-neutral measure
  - C.f., Black-Scholes formula requires both drift (r) and volatility (σ) of share price in risk-neutral measure
- Forward mortality framework just gives expectation of distribution of $\mu_{x,t}$ in market-consistent measure

Market-consistent mortality dynamics

- Re-write this expectation as

$$\nu^{\mathbb{Q}}_{x,t}(\tau) = \exp\left(\alpha_x + \boldsymbol{\beta}_x^\top \left[\mathbb{E}^{\mathbb{P}}{}_\tau\, \boldsymbol{\kappa}_t - \mathbb{V}ar^{\mathbb{P}}_\tau\,(\boldsymbol{\kappa}_t)\boldsymbol{\lambda}\right] + \frac{1}{2}\boldsymbol{\beta}_x^\top \mathbb{V}ar^{\mathbb{P}}_\tau\,(\boldsymbol{\kappa}_t)\boldsymbol{\beta}_x \right.$$
$$\left. + \left[\mathbb{E}^{\mathbb{P}}{}_\tau\, \gamma_{t-x} - \lambda^\gamma \mathbb{V}ar^{\mathbb{P}}_\tau(\gamma_{t-x})\right] + \frac{1}{2}\mathbb{V}ar^{\mathbb{P}}_\tau\,(\gamma_{t-x})\right)$$

- Compare with expectation if market-consistent distribution known

$$\mathbb{E}^{\mathbb{Q}}{}_\tau \boldsymbol{\kappa}_t = \mathbb{E}^{\mathbb{P}}{}_\tau\, \boldsymbol{\kappa}_t - \mathbb{V}ar^{\mathbb{P}}_\tau\,(\boldsymbol{\kappa}_t)\boldsymbol{\lambda}$$
$$\mathbb{V}ar^{\mathbb{Q}}_\tau(\boldsymbol{\kappa}_t) = \mathbb{V}ar^{\mathbb{P}}_\tau\,(\boldsymbol{\kappa}_t)$$
$$\mathbb{E}^{\mathbb{Q}}{}_\tau \gamma_y = \mathbb{E}^{\mathbb{P}}{}_\tau\, \gamma_{t-x} - \lambda^\gamma \mathbb{V}ar^{\mathbb{P}}_\tau(\gamma_{t-x})$$
$$\mathbb{V}ar^{\mathbb{Q}}_\tau\,(\gamma_{t-x}) = \mathbb{V}ar^{\mathbb{P}}_\tau\,(\gamma_{t-x})$$

Market-consistent mortality dynamics

- We define new processes for the period and cohort functions in the market-consistent measure to match these moments
  - Period functions: random walk with modified drift

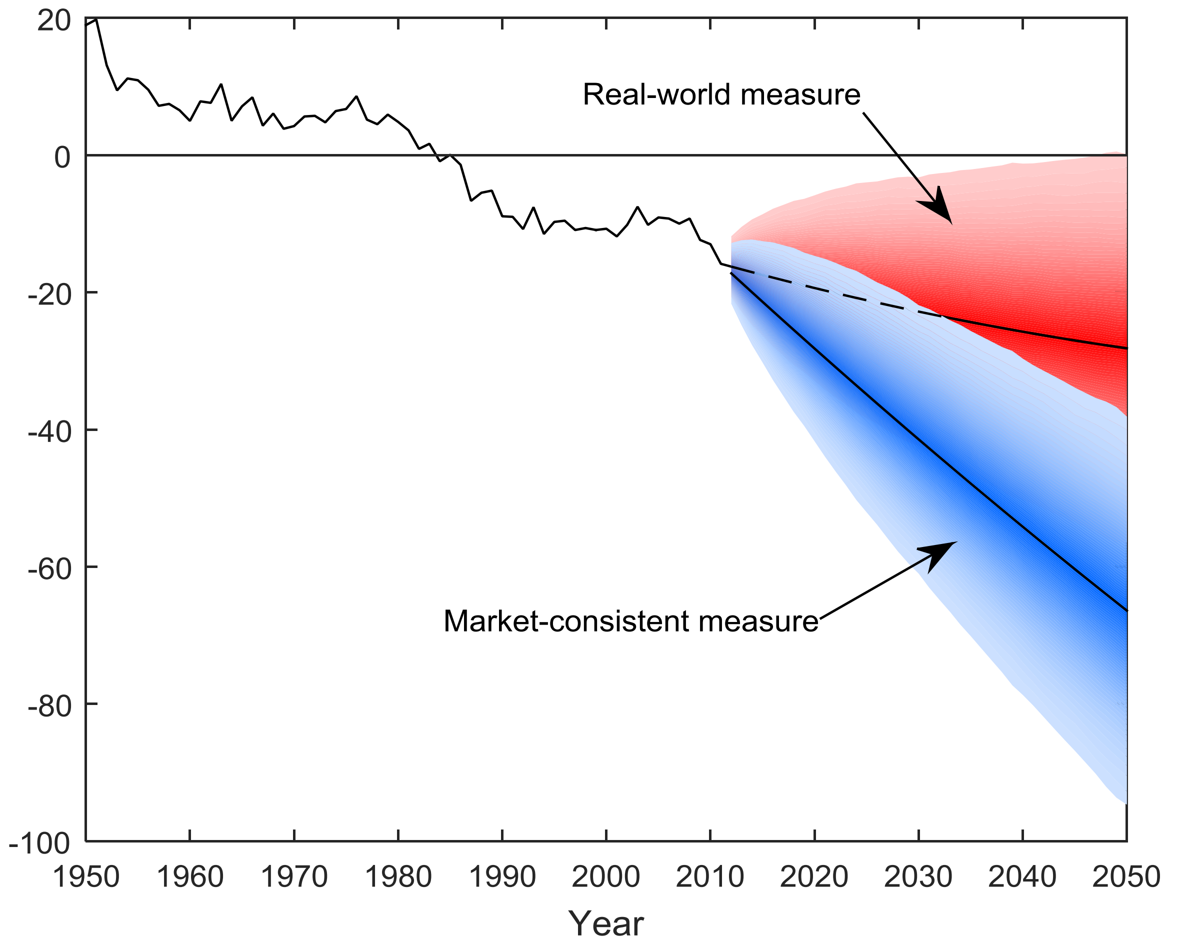
$$\mathbb{E}^{\mathbb{Q}}{}_{\tau}\, \boldsymbol{\kappa}_t = \boldsymbol{\kappa}_\tau + \mu \sum_{s=\tau+1}^{t} X_s - \Sigma \boldsymbol{\lambda}(t-\tau)$$
$$= \boldsymbol{\kappa}_\tau + \tilde{\mu} \sum_{s=\tau+1}^{t} X_s$$

    - C.f., Cairns (2006) and Plat (2009)
  - Cohort function: Bayesian approach with modified drift

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}{}_{\tau}\, \gamma_y = {} & D_{\tau-x}\left(\overline{\gamma}_y(\tau) - \lambda^{\gamma}\mathbb{V}ar^{\mathbb{Q}}_{\tau}\left(\gamma_y\right)\right) \\ & + (1-D_{\tau-y})\left[\beta\tilde{X}_y + \rho\left(\mathbb{E}^{\mathbb{P}}{}_{\tau}\, \gamma_{y-1} - \beta\tilde{X}_{y-1} + \lambda^{\gamma}\mathbb{V}ar^{\mathbb{Q}}_{\tau}\left(\gamma_{y-1}\right)\right) - \lambda^{\gamma}\mathbb{V}ar^{\mathbb{Q}}_{\tau}\left(\gamma_y\right)\right] \\ = {} & D_{\tau-y}\overline{\gamma}^{\mathbb{Q}}_y(\tau) + (1-D_{\tau-y})\left[\beta^{\mathbb{Q}}\tilde{X}^{\mathbb{Q}}_y + \rho\left(\mathbb{E}^{\mathbb{Q}}{}_{\tau}\, \gamma_{y-1} - \beta^{\mathbb{Q}}\tilde{X}^{\mathbb{Q}}_{y-1}\right)\right]\end{aligned}$$

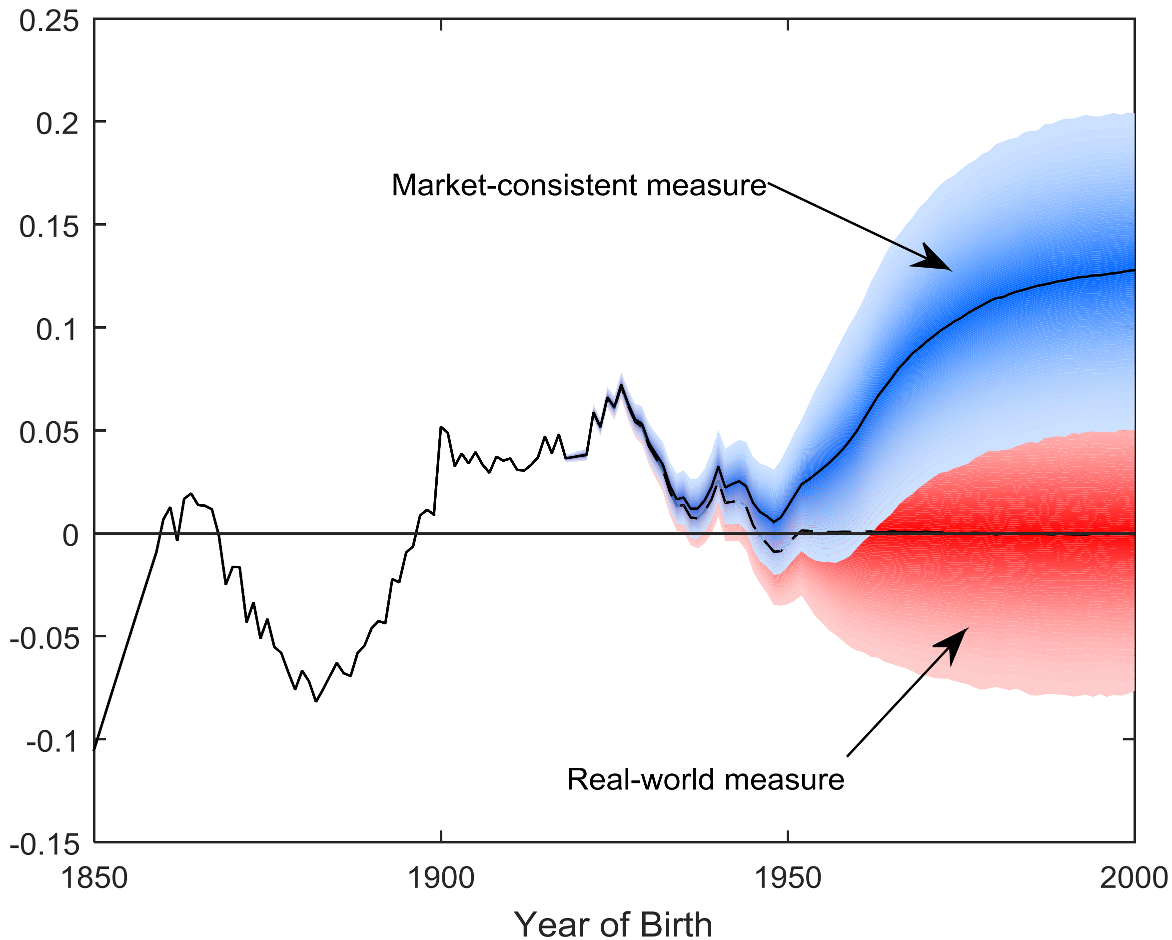
Market-consistent mortality dynamics

- Period functions



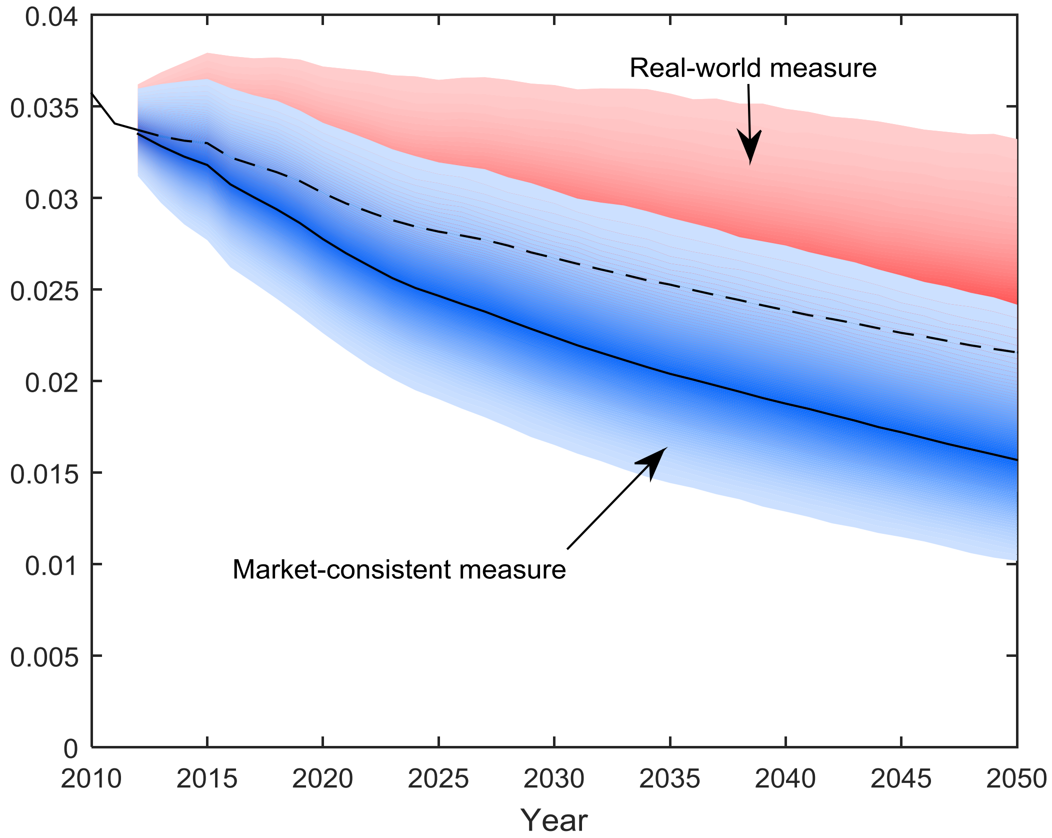
Market-consistent mortality dynamics

- Cohort function



Market-consistent mortality dynamics

- Force of mortality

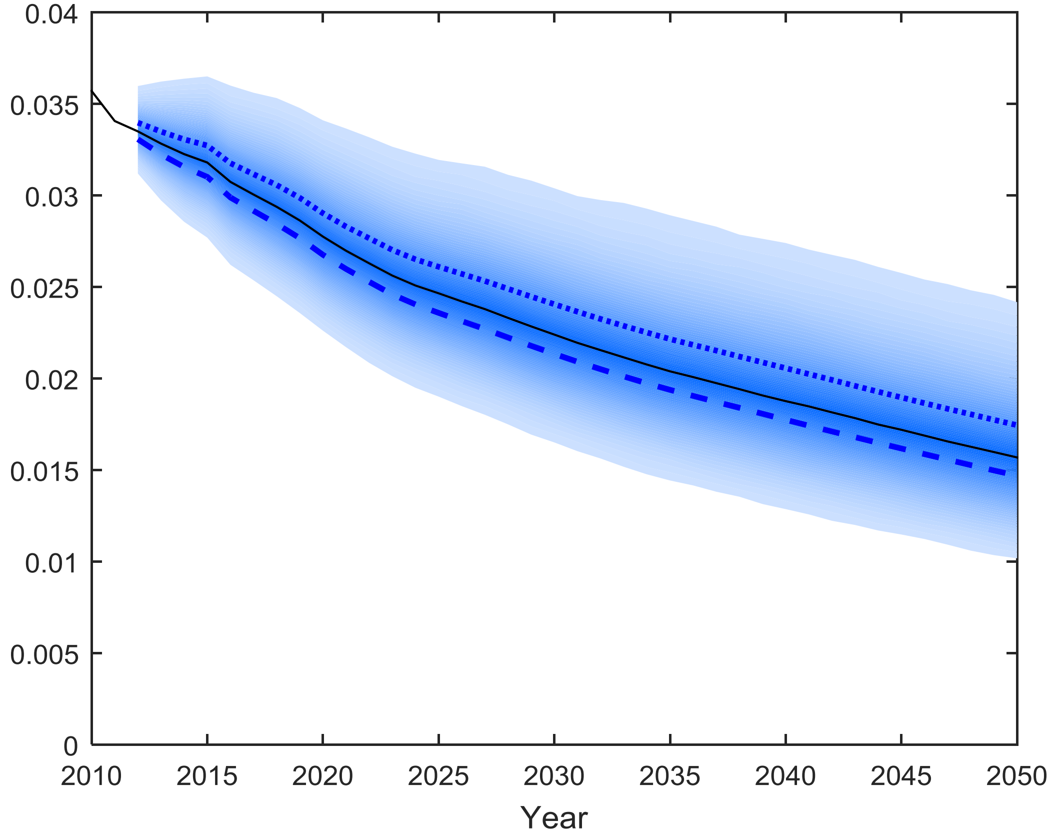


Pricing longevity-linked options

- Two types of option:
  - Those with payoff based on observable mortality rates, $\mu_{x,T}$, at expiry, T
  - Those with payoff based on forward mortality rates, $\nu_{x,t}^{\mathbb{Q}}(T)$, at expiry, T
  - C.f., interest rate options based on either short rate at expiry or yield curve at expiry
- However, using forward mortality framework, we can generate forward mortality rates at expiry, so this does not present a problem

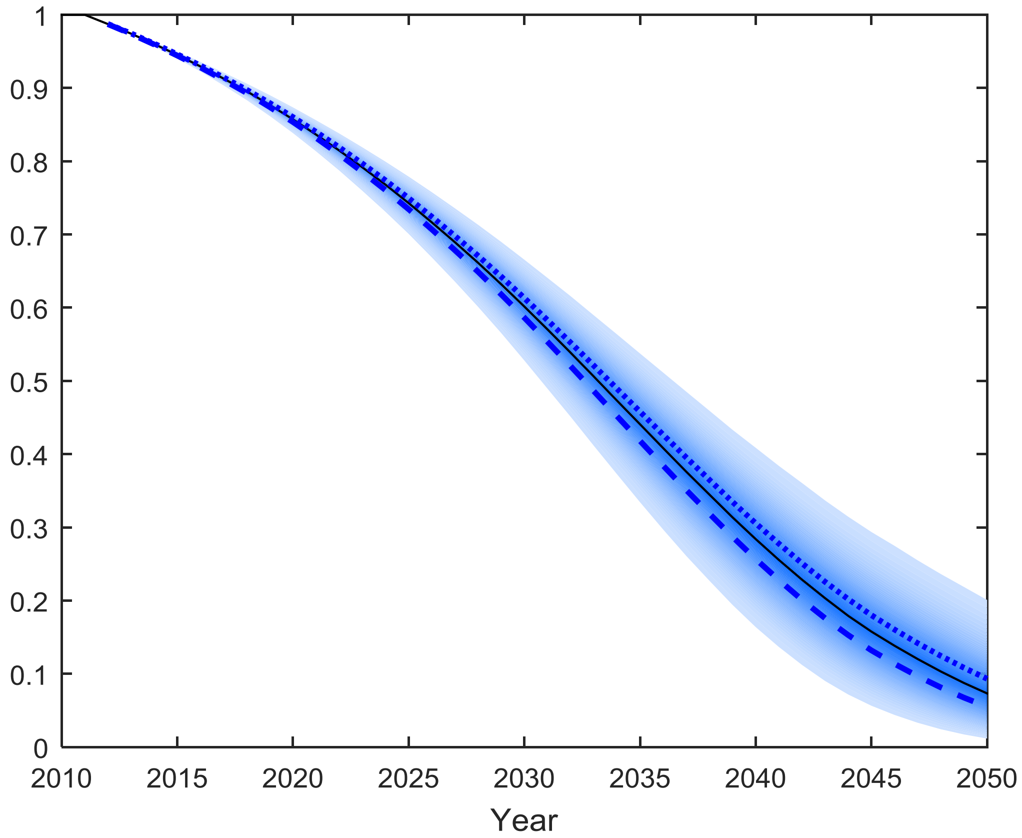
Pricing longevity-linked options

- Force of mortality caps and floors



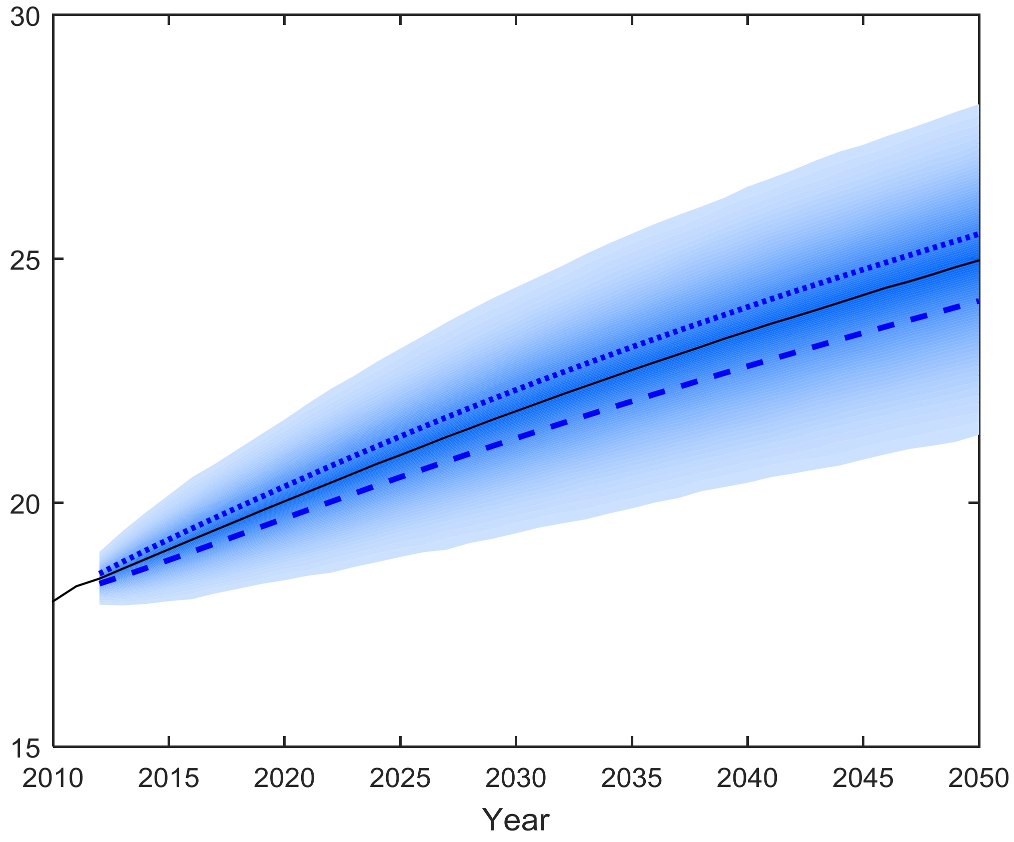
Pricing longevity-linked options

- Survivor caps and floors



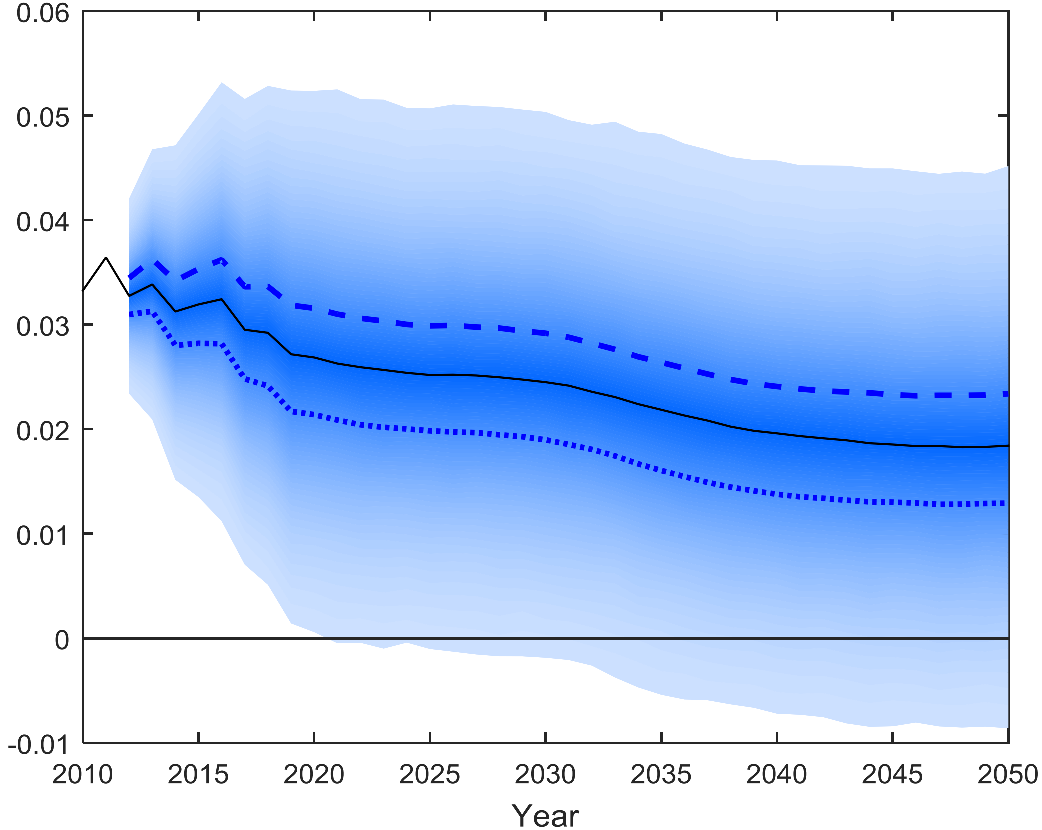
Pricing longevity-linked options

- Period life expectancy caps and floors



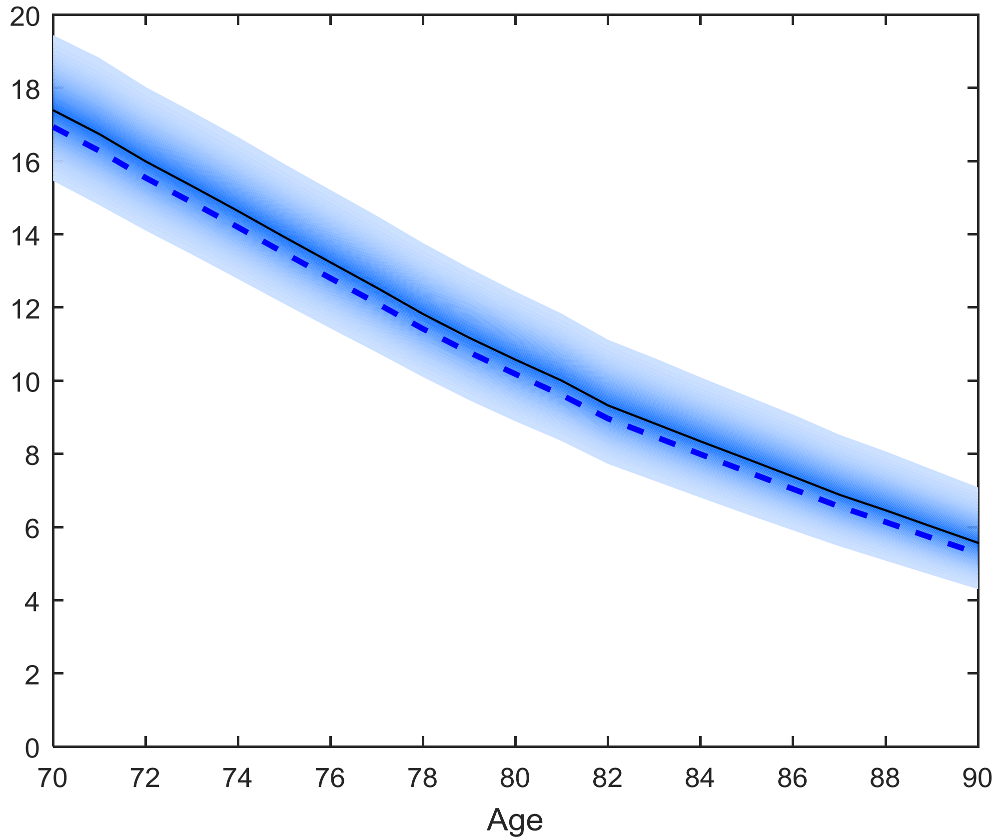
Pricing longevity-linked options

- Mortality improvement rate caps and floors



Pricing longevity-linked options

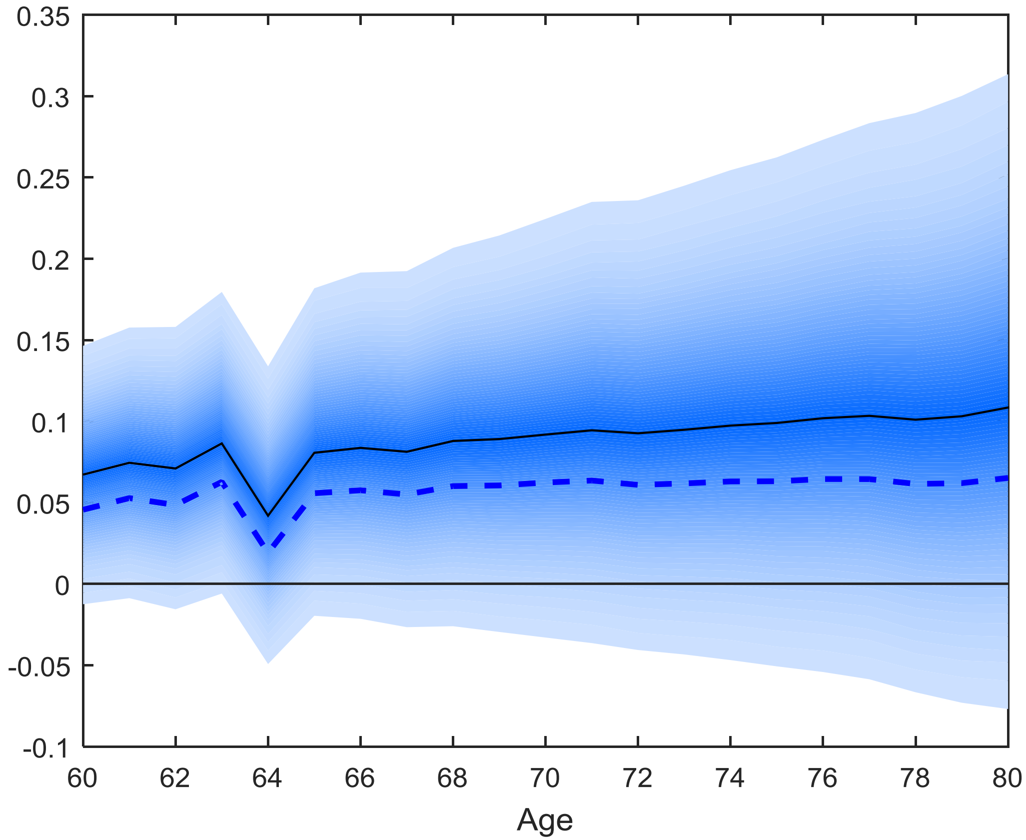
- Guaranteed annuity rates



- 10-year deferral period, to age x

Pricing longevity-linked options

- Longevity swaptions



- 10-year deferral period, swap starting at age x

Discussion

- In Hunt and Blake (2015b,c), introduced a framework that gave market-consistent forward rates
  - Relatively easily calibrated from market information or from internal deterministic mortality assumption
- We have extended this framework to consistently specify the dynamics of force of mortality, $\mu_{x,t}$, in market-consistent measure
- Can be used to price large range of longevity-linked options consistent with each other and with forward mortality rates
- However, beware deep out-of-the-money options (but true of any option-pricing framework)

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Questions?

- Thank you very much for your attention and your feedback