Guaranteeing the unsustainable: A framework for mixed pension schemes

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Introduction & motivation

- Population is ageing: by 2050 the world's population of people aged 60 and over is expected to double **(United Nations, 2020)**
- Ageing = fertility ↘ + life expectancy ↗: e_{65} expected to increase by 3.9 years among women and 4.5 among men by 2065 while current fertility rates of 1.67 (<2.1 replacement rate) (OECD Publishing 2021)
- In Europe the common trend of the pension crisis is a wave of parametric adjustments such as increases in the retirement ages or a decrease in pension indexation (Whitehouse 2009a,b; OCDE 2011; OECD 2013, 2012, 2017)
- Other countries combine PAYG and funding instead. E.g. Sweden allocates 86.5% of the pension contributions to PAYG.
- Indeed, academic research shows that diversification benefits arise when combining both PAYG and funding (Dutta et al. 2000; Devolder and Melis 2015; Alonso-García and Devolder 2016; Boado-Penas et al. 2021)

Pure PAYG

In a pure PAYG, we have:

$$\pi_t \cdot \overbrace{C_t}^{=\bar{s}_t \cdot w_t} = \overbrace{P_t}^{=\bar{p}_t \cdot r_t} \tag{1}$$

where π_t, $\bar{p}_t$ and $\bar{s}_t$ represent the contribution rate, average pensions and salary in the pension scheme and r_t and w_t are the *r*etired population and the *w*orking population. A minor rewrite gives:

$$\pi_t = \frac{P_t}{C_t} = \frac{\bar{p}_t}{\bar{s}_t}\frac{r_t}{w_t} = BR_t \cdot DR_t \tag{2}$$

where BR_t and DR_t represent the benefit ratio and dependency ratio respectively.

⚠️ if $BR_t = \bar{br}$ to ensure equity between workers & retirees, then $\pi_t \nearrow$ when $DR_t \nearrow$.

However, most social planners fix $\pi_t = \pi$.

$\Rightarrow$ a systematic deficit arises when $DR_t \nearrow$ in pure PAYG.

$\Rightarrow$ we consider a system where funding and PAYG coexist.

Stochastic number of contributors & financial returns

- We consider that the target total pension expenditure P, total contribution rate π and average salary $\bar{s}$ are deterministic.
- The number of contributors is modeled through $w = \{w_t\}$ by an Ornstein-Uhlenbeck process. It means the number of contributors at time t is given by

$$w_t = w_0 \cdot e^{-at} + b(1 - e^{-at}) + \delta \int_0^t e^{-a(t-s)} \, \mathrm{d}B_s \ ,$$

where $B = \{B_t\}$ is a SBM, $a > 0$ is reversion speed, $\delta > 0$ is the volatility and b denotes the so-called long-term mean.[1]

- The fund value at time t is modeled as a GBM:

$$F_t = F_0 e^{\mu t + \sigma W_t} \ ,$$

where $\mu, \sigma > 0$ and $W = \{W_t\}$ a SBM independent of B.

[1] Note that for every t the random variable w_t is normally distributed with the mean $\mathbb{E}[w_t] = (w_0 - b)e^{-at} + b$ and variance $\mathrm{Var}[w_t] = \frac{\delta^2}{2a}(1 - e^{-2at})$.

1-year balance w/ and w/out buffer fund

The total contribution rate π is further split in θ, which is invested into the fund F and $1-\theta$ that goes to the PAYG.

Let $F_0 = \theta \cdot \bar{s} \cdot w_0$, at the end of the first year we have

$$R_1 := \overbrace{(1-\theta)\cdot \bar{s}\cdot w_1}^{\text{PAYG year 1 contributions}} + \overbrace{\theta \cdot \bar{s}\cdot w_0}^{\text{contributions in 0 to F}=F_0} \cdot \underbrace{\max\{F_1/F_0, 1\}}_{\text{return with 0\% guarantee}} - P\,. \quad (3)$$

Despite the 0% guarantee, we might want to lock away surplus in a buffer fund. To be general, let us assume the buffer fund could eventually also be partially invested in risky assets with no guarantee. The balance at time 1 is then, using Equation (3)

$$\begin{aligned} R_1^p &:= R_1 + \overbrace{(1-p\%)\cdot B_0 + p\% \cdot B_0 \cdot e^{\mu+\sigma W_1}}^{B_1} \\ &= (1-\theta)\cdot \bar{s}\cdot w_1 + F_0 \max\left(e^{\mu+\sigma W_1}, 1\right) - P + B_1\,. \end{aligned} \quad (4)$$

Various questions arise

What is the **probability of ruin** under this framework? and does adding a buffer earning 0% risk-free rate substantially decrease the probability of ruin?

Proposition

Let $F_0 = \Upsilon_0 = \theta \cdot \bar{s} \cdot w_0$. The 1-year ruin probability without a buffer fund:

$$\mathbb{P}[R_1 \leq y | w_0, \Upsilon_0] = \mathbb{E}\left[\Phi\left(\frac{P + y - \Upsilon_0 e^{\max\{\mu + \sigma W_1, 0\}} - \mathbb{E}_{w_0}[w_1]}{(1-\theta) \cdot \bar{s} \cdot \sqrt{\mathrm{Var}_{w_0}[w_1]}}\right)\right], \quad (5)$$

whereas the 1-year ruin probability in presence of a buffer fund is given by

$$\mathbb{P}[R_1 + B_0 \leq y | w_0, \Upsilon_0] = \mathbb{E}\left[\Phi\left(\frac{P + y - \Upsilon_0 e^{\max\{\mu + \sigma W_1, 0\}} - B_0 - \mathbb{E}_{w_0}[w_1]}{(1-\theta) \cdot \bar{s} \cdot \sqrt{\mathrm{Var}_{w_0}[w_1]}}\right)\right], \quad (6)$$

where Φ cdf of N(0,1) and $W = \{W_t\}$ is a standard Brownian motion.

Of course from Proposition 1 it is clear that $\mathbb{P}[R_1 + B_0 \leq y | w_0, F_0]$ is strictly decreasing in B_0. The buffer earns a 0% return and can hence be viewed as a way of locking funds away to finance future deficits.

Various questions arise (C'td)

However, the buffer fund might become too big, then *should I invest back into F where I obtain a (risky) higher average return?*

Proposition

The 1-year ruin probability linked to the balance level (4) *is given by*

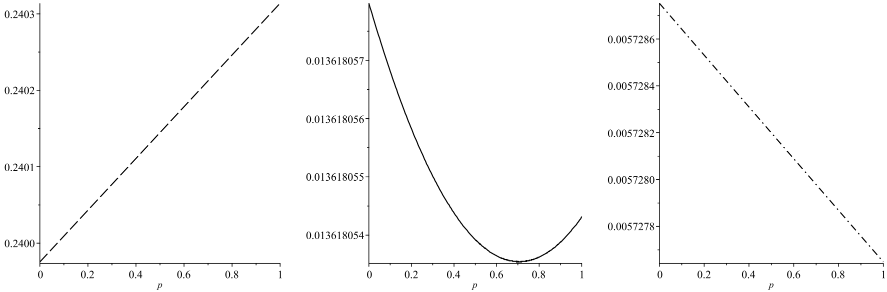
$$\psi(B_0, w_0, p) := \mathbb{P}_{w_0,B_0}[R_1^p < 0]$$
$$= \mathbb{E}\left[\Phi\left(\frac{P - F_0 \max\{e^{\mu+\sigma W_1}, 1\} - B_0 - B_0 p\%(e^{\mu+\sigma W_1} - 1)}{\bar{s} \cdot (1-\theta) \cdot \sqrt{\mathrm{Var}_{w_0}[w_1]}} - \frac{\mathbb{E}_{w_0}[w_1]}{\sqrt{\mathrm{Var}_{w_0}[w_1]}}\right)\right],$$

where Φ *denotes the standard normal distribution function.*

Since the part of the buffer fund invested in risky assets is not protected, we do not know whether we increase or decrease the ruin probability with a bigger p%.

Example[2]

Figure: Possible behaviour of the ruin probability depending on p for $B_0 = 10^6$.



(a) $\mu = 0.02$. (b) $\mu = 0.214$. (c) $\mu = 0.25$.

For small values of μ, the smallest ruin probability is attained at $p = 0$, i.e. the buffer fund should not be invested, see Figure 2a. If the return is quite big, for instance, $\mu = 25\%$, then an investment becomes quite lucrative. The ruin probability is minimised if one invests 100% of the buffer, see Figure 2c.

[2] $a = 0.017,\ b = 4.5 \cdot 10^6,\ \delta = 8000,\ w_0 = 10^7,\ \sigma = 0.1$

Calibration

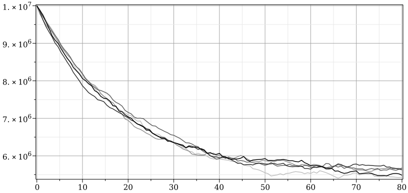
- We use Eurostat EU 27 countries data from 2020 to calibrate our OU process for the number of contributors: $a = 0.055$, $b = 5.56 \cdot 10^6$, $\delta = 35000$, $w_0 = 10^7$.
- The average salary/wages $\bar{s}$ per year totals EUR $36 \cdot 10^3$.
- Contribution rate 20.88%.
- The average pension per year $\bar{p}$ that needs to be covered by contributions amounts to EUR $21 \cdot 10^3$.
- The total pension expenditure amounts to

$$P := 21000 \cdot 3.48 \cdot 10^6 = 73.08 \cdot 10^9 \ .$$

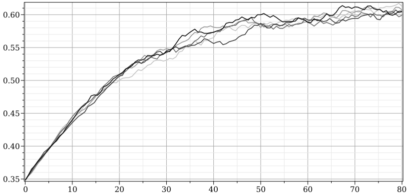
$$C_0 := \pi \cdot \bar{s} \cdot w_0 = 0.2088 \cdot 36 \cdot 10^3 \cdot 10^7 = 75.168 \cdot 10^9 \ .$$

- A slight initial surplus arises on the pure PAYG.
- The fund is $\mu = 0.02$ and $\sigma = 0.10$.

Figure: Simulated paths of the evolution of the working population and the dependency ratio over period of 80 years



(a) Realisations of the OU process, describing the evolution of the working population over a period of 80 years.



(b) Realisations of the dependency ratio over a period of 80 years.

1-year ruin probability

Potential problems

$$0.95 \cdot w_1 \times 36 \cdot 10^3 \times 0.2088 + \Upsilon_0 \cdot F_1 < P \ , \qquad \text{in a mixed scheme,}$$
$$w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 < P \ . \qquad \text{pure PAYG.}$$

If we choose the worst case scenario in both financial return & population evolution then $w_1 = 9.7 \cdot 10^6$ and $F_1 = 3.1 \cdot 10^9$, i.e.

$$\begin{aligned} 0.95 \cdot w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 + F_1 &= 0.95 \cdot 9.7 \cdot 10^6 \cdot 36 \cdot 10^3 \cdot 0.2088 + 31 \cdot 10^8 \\ &= 72.38 \cdot 10^9 < 73.08 \cdot 10^9 = P \ , \\ w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 &= 72.91 \cdot 10^9 < 73.08 \cdot 10^9 = P \ . \end{aligned}$$

It means a deficit in the PAYG system in both scenarios. In particular, the mixed PAYG seems to be even worse.

1-year ruin probability (C'td)

However, if the return on investment is at least 0 (guaranteed by the state), then the balance with the investment increases but still stays under P for the worst case scenario:

$$\begin{aligned} 0.95 \cdot w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 + \max\{F_1, F_0\} &= 0.95 \cdot 9.7 \cdot 10^6 \cdot 36 \cdot 10^3 \cdot 0.2088 + 36.54 \cdot 10^8 \\ &= 72.92 \cdot 10^9 < P \,. \end{aligned}$$

But *what is the probability of a deficit in both scenarios*? We can easily calculate that abandoning the 0-return guarantee of the state

$$\begin{aligned} \mathbb{P}[0.95 \cdot w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 + F_1 \le P] &= 0.2669 \,, \\ \mathbb{P}[w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 \le P] &= 0.1191 \,. \end{aligned}$$

Also, in terms of probability, the mixed scheme seems to perform worse, as the ruin probability is higher than in the pure PAYG system. Adding the guarantee, one gets

$$\mathbb{P}[0.95 \cdot w_1 \cdot 36 \cdot 10^3 \cdot 0.2088 + F_1 \vee F_0 \le P] = 0.0277 \,.$$

The probability of a deficit is substantially reduced and is now **approximately 1/5** of the ruin probability in the PAYG system. However, a 1-year period is very short, the investment cannot unfold its potential and the ruin probability is extremely high.

>1 time horizon[3]

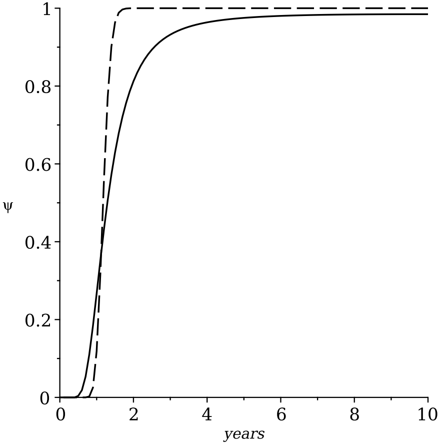


Figure: The ruin probability without investments (dashed line) and with investments (no guarantees) (solid line) in dependence on the time t.

[3]The mathematical expressions used for $t > 1$ are left from the presentation to be concise but is available in the paper.

Conclusion

- The mixed scheme performs better than the pure PAYG in terms of ruin probability, even if the funded part has no nominal guarantee.
- Including a buffer fund that earns 0% risk-free rate decreases substantially the ruin probability.
- However, if we allocate a part of the buffer into the risky markets the result is less clear as it depends on whether the one-year return is positive or not.

Next steps:

- Finetune the long-term horizon development.
- Using the VaR of the deficit in order to assess how to invest the buffer fund.

Thank you for your attention

Questions?

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