# An assessment framework for equitable longevity pooling arrangements

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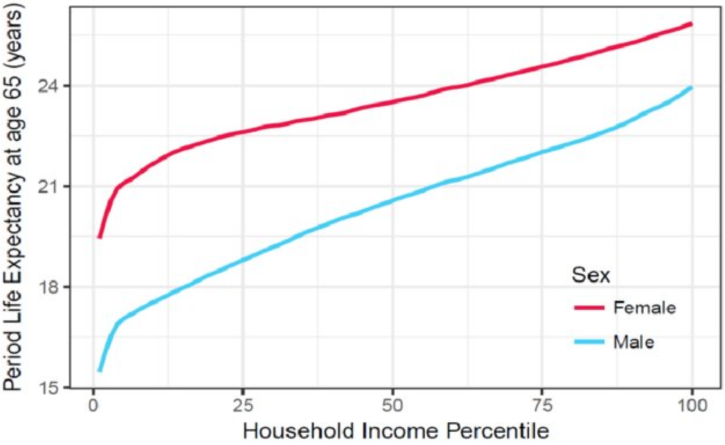
# Longevity inequalities are well known

US period life expectancy in 2014 at age 65 by household income percentile




Source: Holzmann et al (2019) based on data from Chetty et al. (2016)

# Addressing longevity inequalities:

## Link to pension policy reforms


Ensure member outcomes for differing needs and circumstances


**Principle 6**

**Intentionally address longevity inequalities, including across gender, race and class**

Benefits of longevity are not distributed equitably. Advocacy for pay and pension equity, as well as support for informal caregivers, are some of the crucial elements to ensure that financial security and the benefits of longevity can be more accessible to all.

Intentionally address longevity inequalities, including across gender, class and race


Design needs to be in line with objectives related to benefit stability and equity

# Why is equitability important in longevity pooling?

Innovative solutions: Longevity pooling

Mechanism: Pool and share

Key characteristic: Mortality-linked benefits (GSA, Tontines, Pooled Annuity Overlays)

Concerns: Socioeconomic mortality differentials $\rightarrow$ Equitable retirement product design

Differences in lifespan behavior by demographic or socioeconomic status

$\rightarrow$ 'Unfair' wealth transfers that disproportionately benefit some groups

$\rightarrow$ Inequitable distribution of benefits

$\rightarrow$ Unfair allocations

Intentionally addressing longevity inequalities in longevity pooling is important

# Our contribution

**Research question**: How to assess equitability of a pooling arrangement?

**Focus**: In the presence of demographic and socioeconomic longevity inequalities

**Modelling contribution:**

A Gini-based assessment framework to assess equitable longevity pooling arrangements

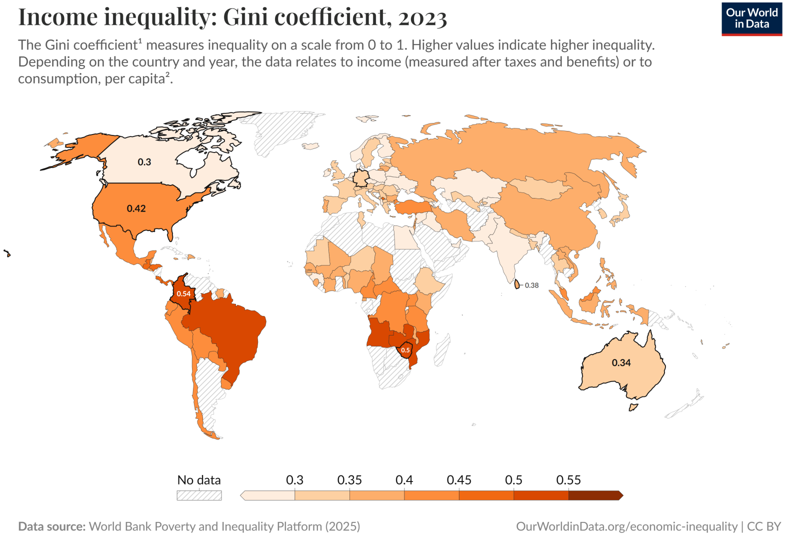
(i) Quantify the wealth transfers

(ii) Inform who is subsidizing whom

(iii) Help determine the best management strategy

**Our findings**:

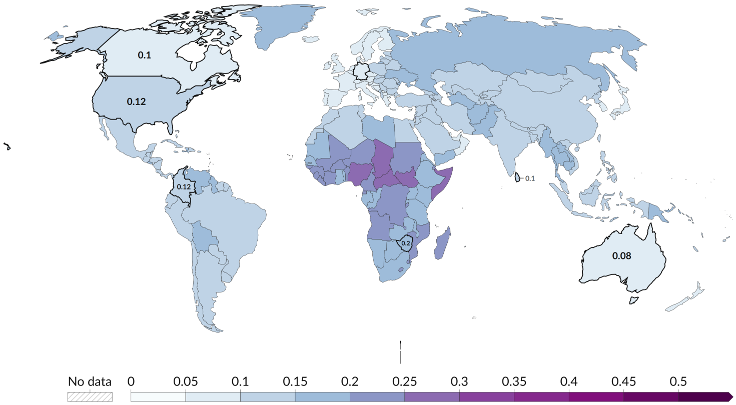
Given similar pre-existing inequalities in longevity risk and initial investment amounts, different payout rules can lead to varying degrees of inequitability




Gini coefficient is a measure of absolute differences between all pairs of outcomes (income) in a distribution

## Lifespan inequality: Gini coefficient in men, 2023

The level of inequality in lifespans within a country, measured between 0 and 1. A higher Gini coefficient[1] indicates greater inequality in ages of death.




**Data source:** Human Mortality Database (2024); Aburto et al. (2023)

OurWorldinData.org/life-expectancy | CC BY

Gini coefficient is a measure of absolute differences between all pairs of outcomes (lifetimes) in a distribution

Can we use Gini-based metrics to assess inequalities in mortality pooling arrangements?

# Defining inequalities in a pooling arrangement

| **Gini Metric** | **Notation** | **Description** |
| --- | --- | --- |
| Wealth Gini | $\Delta W$ | Measures absolute differences in wealth invested |
| Lifetime Gini | $\Delta T_x$ | Measures absolute differences in lifespans |
| Benefit Gini | $\Delta B$ | Measures absolute differences in PV of benefits |

# The Lifetime Gini

Average absolute difference in the lifetime of two random participants in the pool

$$\Delta T_x = \sum_{k=1}^{K} \sum_{h=1}^{K} \frac{n_k n_h}{n^2} \int_0^\infty \int_0^\infty f_{x_k}(a) f_{x_h}(b) |a-b| \, da \, db$$

$K$: Number of groups in the pool

$n_k$: Number of people in group $k$

$n$ : Total number of participant in the pool

$f_{x_k}$: Age-at-death distribution for people in group $k$

# The Benefit Gini

Average absolute difference in the present value of lifetime benefits received per dollar invested

$$\Delta B = \sum_{k=1}^{K} \sum_{h=1}^{K} \frac{n_k W_k n_h W_h}{\left(\sum_{j=1}^{K} n_j W_j\right)^2} \int_0^{\infty} \int_0^{\infty} f_{x_k}(a) f_{x_h}(b) \left| \int_0^{a} e^{-\delta u} B_k(u)\, du - \int_0^{b} e^{-\delta v} B_h(v)\, dv \right| da\, db$$

$K$: Number of groups in the pool

$n_k$: Number of people in group $k$

$n$ : Total number of participant in the pool

$f_{x_k}$: Age-at-death distribution of participants in group $k$

$W_k$: Initial wealth invested by participants in group $k$

$B_k(t)$: Benefit per dollar invested received at time $t$ by alived participants in group $k$

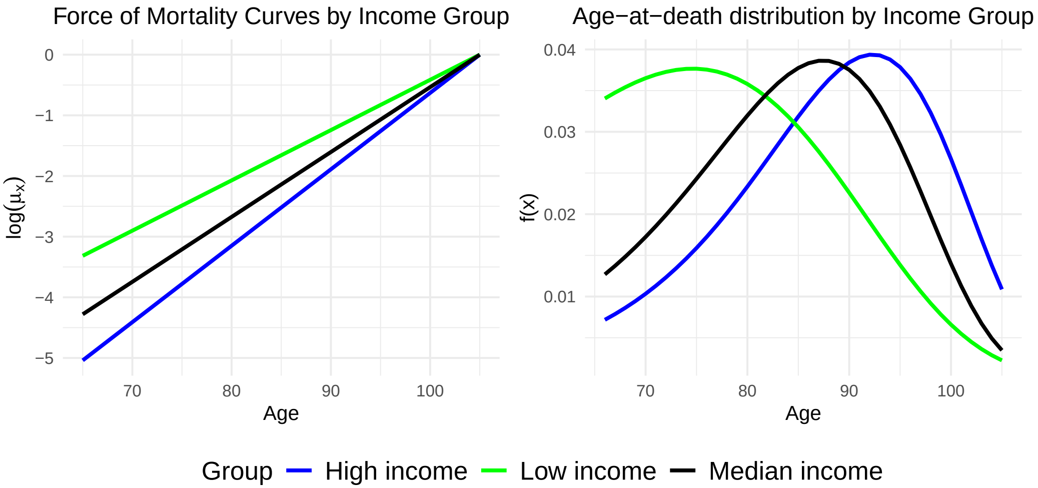
# Decomposing inequalities

$$\underbrace{\Delta B}_{\text{total inequality}} = \underbrace{\sum_{k=1}^{K} \left( \frac{n_k W_k}{\left(\sum_{i=1}^{K} n_i W_i\right)^2} \right)^2 \Delta B_{kk}}_{\text{within}} + \underbrace{\sum_{k=2}^{K} \sum_{h=1}^{k-1} 2 \frac{n_k W_k \, n_h W_h}{\left(\sum_{i=1}^{K} n_i W_i\right)^2} \Delta B_{kh}}_{\text{between}}$$

$$= \sum_{k=1}^{K} \left( \frac{n_k W_k}{\left(\sum_{i=1}^{K} n_i W_i\right)^2} \right)^2 \Delta B_{kk} + \sum_{k=2}^{K} \sum_{h=1}^{k-1} \frac{n_k W_k \, n_h W_h}{\left(\sum_{i=1}^{K} n_i W_i\right)^2} \underbrace{(B_{hk} + B_{kh})}_{\text{inequitability}}$$

# Application to lifetime income-based groups

- Use data from Chetty et al. (2016) on income linked mortality heterogeneity



# Pool setup

- Age: 65-old males
- Groups: Low-income/high-mortality and High-income/low mortality
- Pool composition: (50%, 50%)
- Initial investment: ($1,000, $1,000)
- Closed pool

# Payout design challenge

The benefit at time $t$ of the pooling product is given by:

$$B(t) = B(0)\frac{S^{\text{Pricing}}(t)}{S^{\text{Realised}}(t)}$$

subject to

$$\int_0^\infty e^{-\delta t} S^{\text{Pricing}}(t) B(0) dt = \text{Initial Wealth Invested}$$

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$$\boxed{\text{How to choose } S^{\text{Pricing}}(t)?}$$

# Payout design challenge: How to choose $S^{\text{Pricing}}(t)$?

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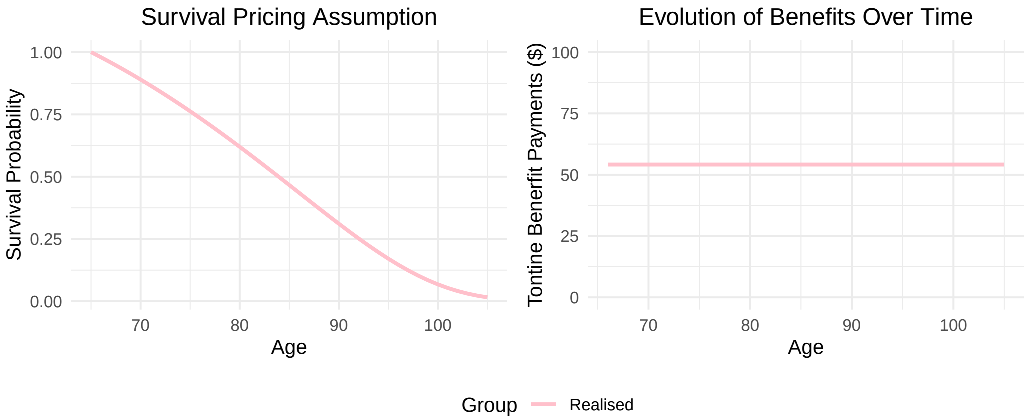

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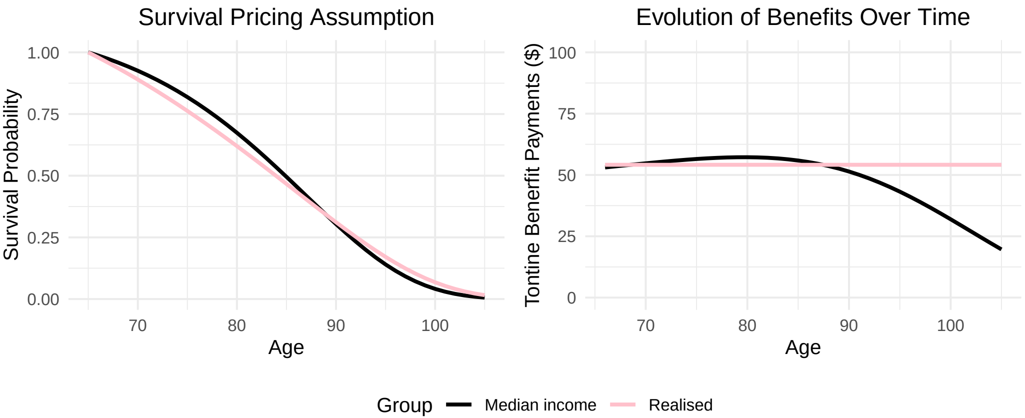

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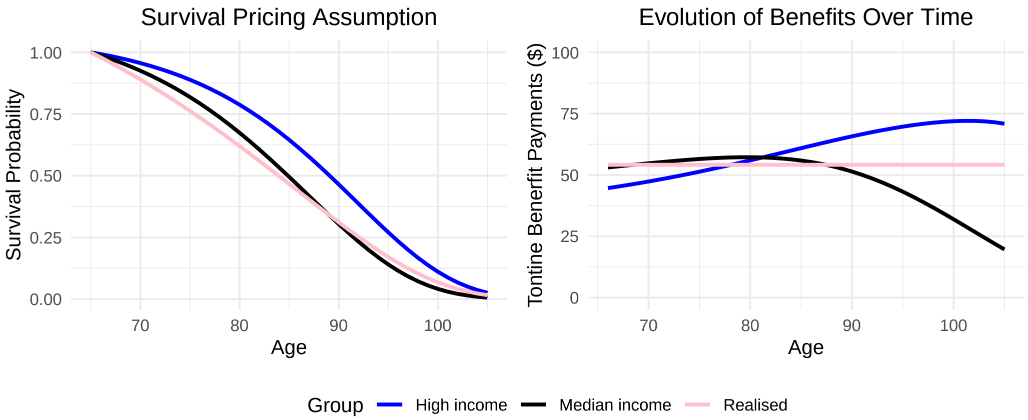

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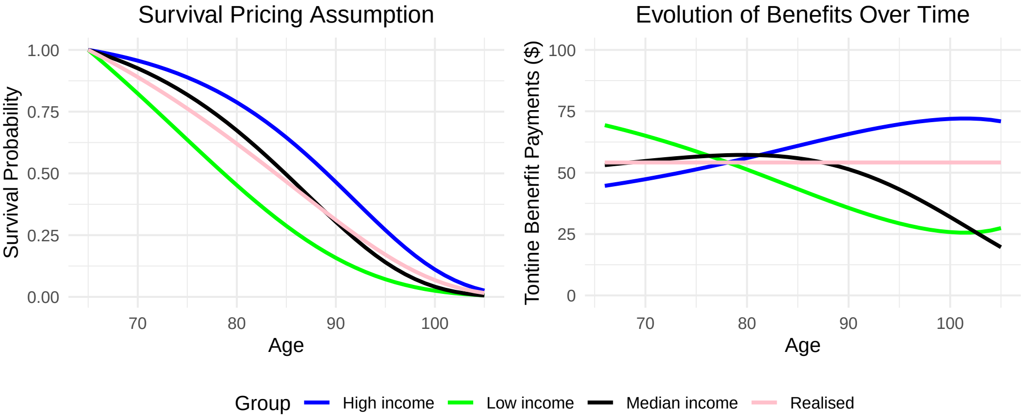
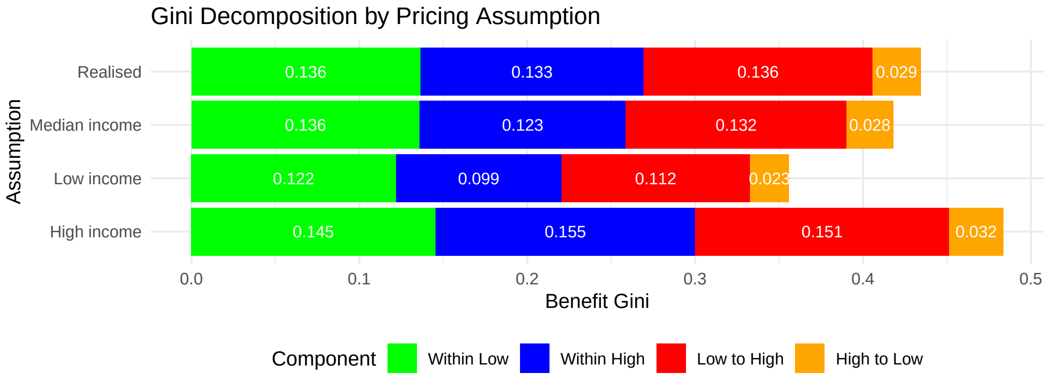


# Benefit equitability: Implications of mortality assumption

# Quantifying wealth transfers and equitability with the Benefit Gini

$$\Delta B = \frac{1}{4}\Delta B_{low,low} + \frac{1}{4}\Delta B_{high,high} + \frac{1}{4}B_{low,high} + \frac{1}{4}B_{high,low}$$




Given the same pre-existing inequalities, different payout rules can lead to varying degrees of inequitability

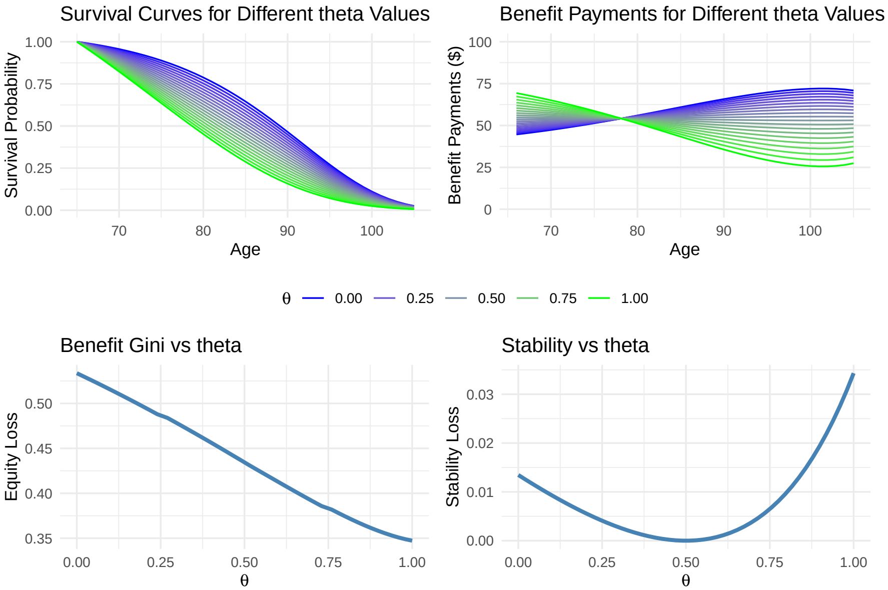
# Designing a stable and equitable payout

- Consider $S^{Pricing}(t) = \theta S^{low}(t) + (1-\theta)S^{high}(t)$
- Choose $\theta$ to find a trade-off between benefit stability, measured by

$$\text{Stability} = \sum_{t=1}^{\omega-x} \left[ (B_t - B_{t-1})^2 \right]$$

and equitability measured by $\Delta B$.

# Designing a stable and equitable payout



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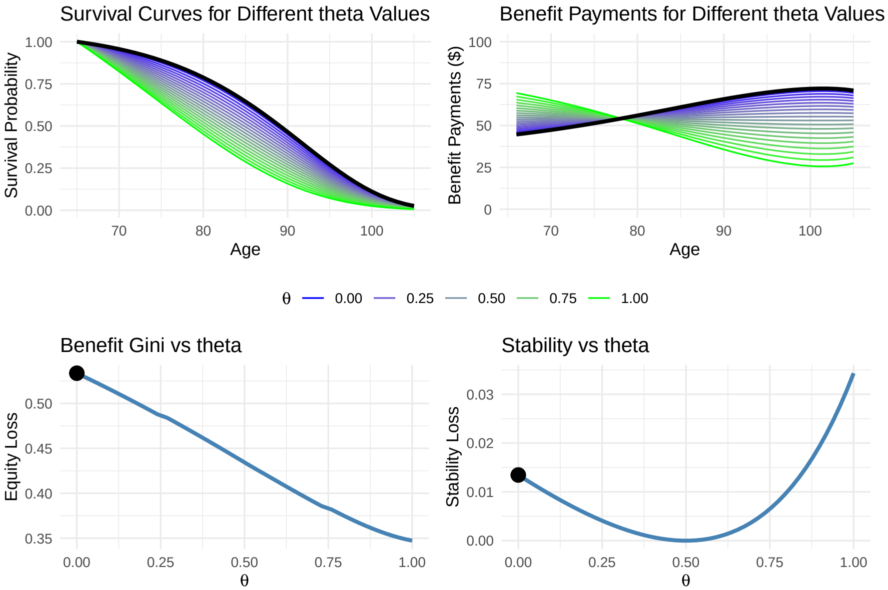

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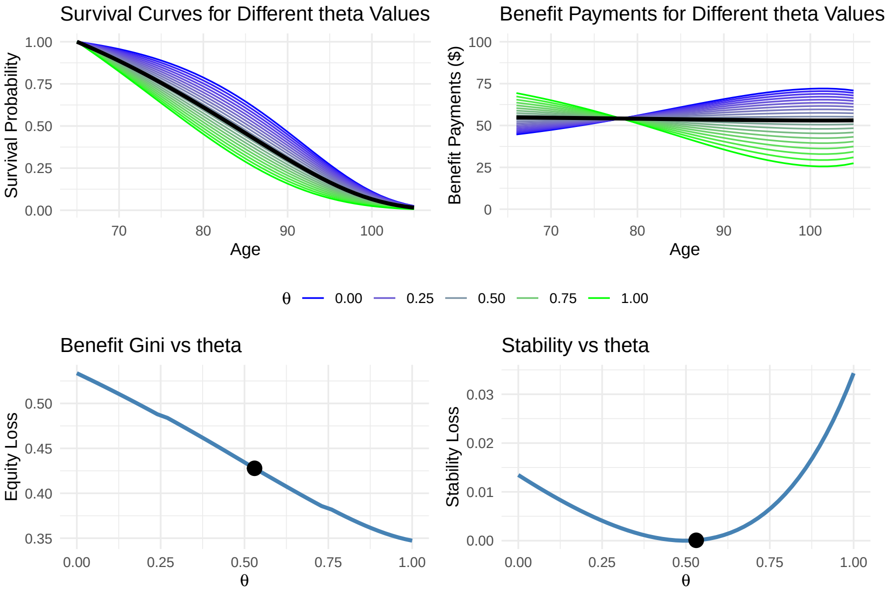

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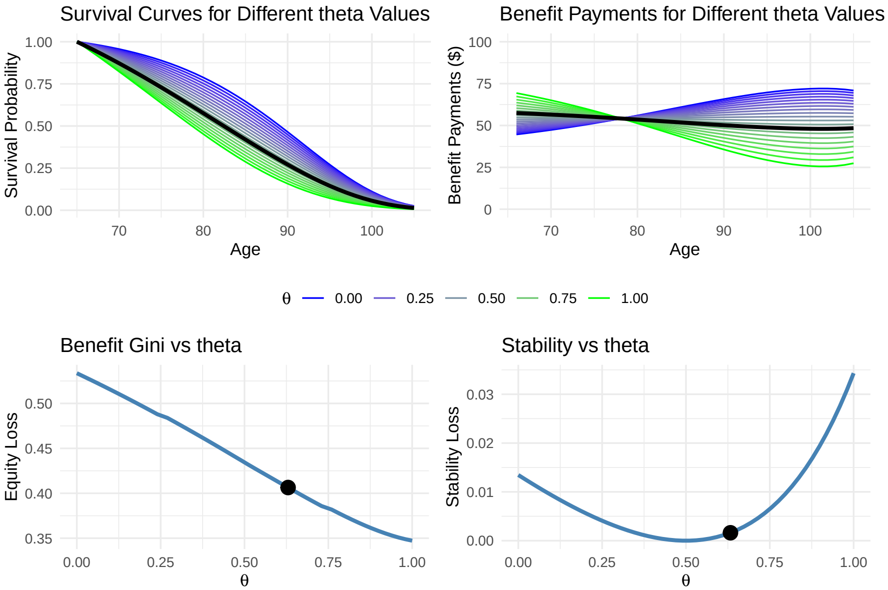

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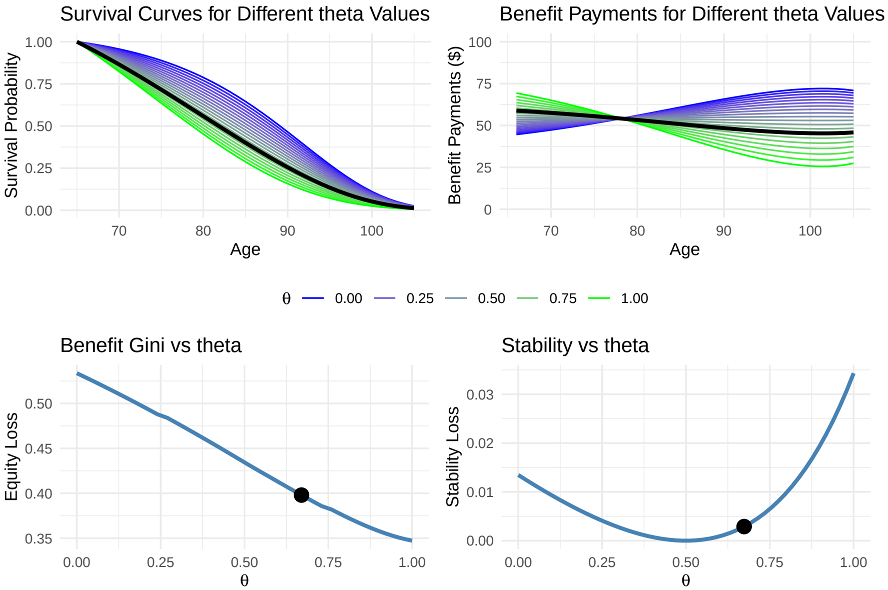

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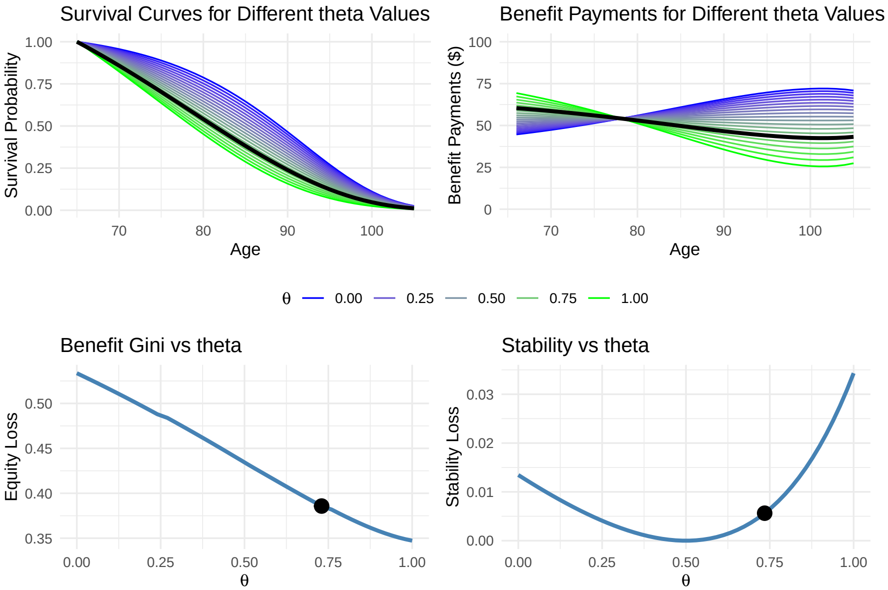

# Conclusion

- Differences in lifespan behavior by socioeconomic status $\rightarrow$

  'Unfair' wealth transfers that disproportionately benefit some groups $\rightarrow$

  Inequitable distribution of benefits $\rightarrow$ Unfair allocations

- Contribution: We develop a coherent framework to assess the equitability

  1. Quantify the wealth transfers due to pooling participants with unequal wealth and lifespans
  2. Inform who is subsidizing whom
  3. Help detect whether a benefit structure leads to an equitable longevity pooling arrangement

- A practical assessment tool for retirement income providers to compare payout rules

# Next steps

1. Decide the best risk management strategy
   - To pool or not to pool
   - Optimum benefit payout rule: Given pre-existing longevity & wealth inequality, which design minimizes unfair transfers
2. Who to pool with whom

Thank you!

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# Inequalities in pooling

**1. Total fund value at $t$**

$$F(t) = \sum_{i \in \mathcal{L}_t} F_i(t)$$

Payout rule

Management action : *pool*/*unpool*

**2. After investment return**

$$F_i(t+1)^- = F_i(t)\big(1 + r_i(t)\big)$$

**3. Mortality credits**

$$MC(t+1) = \sum_{j \in \mathcal{D}_{t+1}} F_j(t+1)^-$$

Initial wealth : $\{W_i(0)\}$

Longevity behaviour: $\{f_{x_i}(t)\}$

Pool composition: $\{p\}$

**4. Risk-sharing allocation**

$$MC_i(t+1) = MC(t+1)\big(w_i(t+1)/\sum_{j \in \mathcal{L}_{t+1}} w_j(t+1)\big)$$

Benefit profiles : $\{B_i(t)\}$

**5. Account update**

$$V_i(t+1) = F_i(t+1)^- + MC_i(t+1)$$

**6. Benefit payment**

$$B_i(t+1) = d(t+1)\, M(t+1)\, \big(w_i(t+1)/\sum_{j \in \mathcal{L}_{t+1}} w_j(t+1)\big)$$

# Assessment framework

i. Define a Gini coefficient-based decomposition to detect disproportionate wealth transfers between groups

ii. Show that under equal initial wealth contributions, the benefit Gini coefficient ($\Delta B$) is always less than or equal to the Longevity Gini coefficient ($\Delta T_x$), scaled by a factor $B^*$, that is,

$$\Delta B \leq B^* \cdot \Delta T_x.$$

iii. Propose a post-hoc diagnostic test to assess whether a given benefit lead to an equitable longevity pooling arrangement in the presence of inequalities in both longevity and initial wealth invested