

Option-Implied Volatility and Variable Annuity Reserves under VM-21

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- **The implied-volatility overlay lowers the VM-21 reserve by 1.1 to 2.3 percent against each of four backward-looking rules.** About **\$3,000 to \$6,600** per \$1 million policy.
- **The reduction is the choice of dates.** The signal's choice of dates adds **1.39** percentage points of reserve reduction over the same cuts placed on random dates.
- **The reserve magnifies the timing gap.** On a fixed-strike guarantee, each point of lead in portfolio volatility is worth **1.6 to 2.3** points of reserve.



- ① Motivation and research question
- ② Valuation framework
- ③ Fund design and volatility signals
- ④ Reserve effects and their channels
- ⑤ Conclusion and future work



- **Guaranteed living benefits convert a fund balance into promised income.** The insurer carries the longevity and market risk the policyholder gives up, and holds statutory reserves against it.
- **The promise is large.** \$1.5 trillion of guaranteed living benefits in 2015, 35% of U.S. life-insurer liabilities (Kojen and Yogo, 2022).
- **The exposure peaks when capital is scarce.** In 2008 guarantee reserves at the ten largest issuers reached 52% of capital (Drexler et al., 2017).



- **The reserve is set on a distribution of outcomes.** The guarantee is projected over thousands of stochastic market scenarios, and the reserve is the average shortfall in the **worst 30%** of them.
- **The aggregate reserve consists of two components.** A **stochastic reserve** from the company's own scenarios and assumptions, and an **additional standard projection amount** computed under prescribed assumptions.
- **The standard projection sets a floor under company assumptions.** The reserve cannot fall below the amount the prescribed assumptions produce; the company's own modelling sets the reserve above that floor.

- **The account value is affected from two perspectives.**
 - Liability side: policyholder behaviour and contract charges
 - Asset side: how much of the separate account sits in equity, and the market returns that allocation earns
- **Our focus is the asset perspective.** The equity allocation runs through four steps to the reserve:
 - **Equity allocation** sets the account's share of each day's market move
 - **Account-value path** compounds those moves, one path per scenario
 - **Guarantee shortfall** is how far the account value ends below the guarantee
 - **Distribution of shortfalls** across scenarios, whose worst 30% sets the reserve



- **A target-volatility fund scales equity exposure inversely to estimated volatility.** Exposure falls as estimated volatility rises and recovers as it subsides, holding portfolio volatility near a target.
- **The design is the industry's mainstream response to guarantee risk.** About **\$400 billion** sat in these funds by 2015 (Chandumont, 2016), and linking one to a guaranteed variable annuity lowers the VM-21 reserve (Chiu et al., 2025).
- **Prevailing mandates estimate volatility from past returns.** Exposure adjusts after the move has been observed (Ramachandran, 2018).

- **TV allocation raises risk-adjusted returns and is used in retirement products.**
 - Scaling equity exposure inversely to realised volatility raises risk-adjusted returns in equity portfolios (Moreira and Muir, 2017) and is documented in insurance retirement products (Olivieri et al., 2022)
- **Option-implied volatility improves TV strategies, measured on portfolio performance outside insurance.**
 - Substituting the VIX for historical volatility in a TV strategy raises risk-adjusted performance (Cirelli et al., 2017; Božović, 2024)
 - Within variable annuities, implied volatility has been used to set the guarantee fee (Cui et al., 2017)

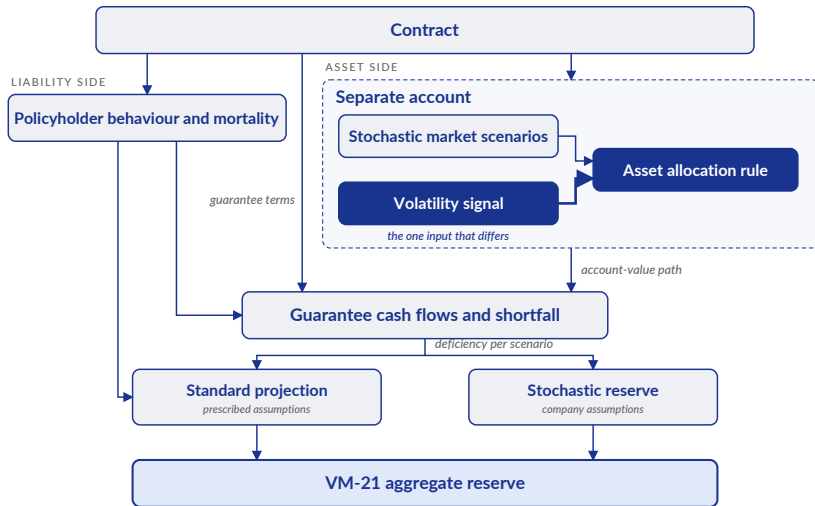
- **Research gap**

- Target-volatility funds in variable annuities de-risk on realised, backward-looking volatility
- A forward-looking signal for that de-risking has not been tested on the statutory reserve

- **Our research**

- Add a signal-triggered de-risking overlay to a target-volatility fund
- Compare an option-implied signal with four backward-looking signals
- Measure whether the aggregate reserve falls, by how much, and through what channel

From Policy to Aggregate Reserve



- **Premium:** \$1 million single premium for both guarantees
- **Issue age and horizon**
 - GMMB: age 45, 20 years to maturity
 - GMIB: age 55, 10 years to annuitisation
- **Benefit base**
 - GMMB: **fixed strike**
 - GMIB: **5% roll-up with annual ratchet**
- **Settlement**
 - GMMB: lump sum at 65
 - GMIB: 20-year term certain from 65
- **Shortfall:** account value below the guarantee

$$AR = SR + ASPA = SR + \max(0, PPA - SR - U)$$

- **Components**

- SR: CTE_{70} of the guarantee deficiency, on the company's assumptions
- PPA: the same calculation under prescribed assumptions
- U : $CTE_{70} - CTE_{65}$ of the deficiency
- ASPA: the excess of $PPA - U$ over SR, floored at zero

- **Two regimes**

- Slack: $PPA - U \leq SR$, so $AR = SR$
- Binding: $PPA - U > SR$, so $AR = PPA - U$ and SR drops out



- **Three calibrated market models**, each carried through the full valuation
 - **SVJ**: stochastic variance with Poisson jumps in returns, on the Eraker (2004) estimates; the primary calibration
 - **SV CJ**: a variance jump arrives with each return jump, so a crash also lifts volatility
 - **Severe jump**: SVJ dynamics with the Bates (2008) jump sizes; excess kurtosis 50 against 6, the stress calibration
- **Simulation**: 10,000 daily paths over 20 years under each model, with returns under the physical measure



- **Two assets in the separate account**
 - **Equity index:** follows the calibrated jump-diffusion; expected return 8.5%, a 3% risk-free rate plus a 5.5% equity premium
 - **Cash account:** earns the 3% risk-free rate
- **Daily portfolio return:** equity weight times the index return, the remainder in cash; no leverage, the weight is capped at one

$$w_{j,t} = \underbrace{\min\left(1, \frac{\sigma^*}{\hat{\sigma}_t}\right)}_{\text{target-volatility base}} \left[1 - \underbrace{\gamma \mathbb{I}\{\bar{s}_{j,t} > q_{j,p,k(t)}\}}_{\text{signal overlay}} \right]$$

- **Base weight**

- 15% volatility target divided by estimated volatility
- at 30% estimated volatility the weight is 0.5

- **Signal overlay**

- triggers when the signal is in the top 10% of its own history to date
- the day's weight is halved: 0.5 becomes 0.25



- **Forward-looking signal:** an option-implied volatility index
 - generated in the same asset model, with the noise of the traded VIX
 - its predictive power for future volatility is checked against the historical VIX
- **Backward-looking signals:** three from the volatility-managed portfolio literature and a GJR-GARCH filter
 - Realised variance, 1 month
 - Realised variance, 6 months
 - EWMA
 - GJR-GARCH, fitted out of sample and frozen

The Exposure and Reserve Effect under Each Signal



Signal	Trigger frequency	Mean equity	Median ΔAR
Implied-volatility overlay	9.57%	77.28%	-1.87%
GJR-GARCH filter	9.60%	77.55%	-0.81%
Realised variance, 1 month	9.61%	77.43%	-0.72%
EWMA	9.60%	77.48%	-0.33%
Realised variance, 6 months	9.61%	77.16%	+0.17%
Random dates	9.57%	—	-0.48%
TV base	—	79.86%	reference

- ΔAR relative to the base, medians over 3,690 settings
at the median setting the guarantee is about twice the premium and the base reserve \$324,120
- mean equity $\bar{w}_j = \frac{1}{NT} \sum_i \sum_t w_{j,i,t}$
- Moving the implied overlay's cuts to random dates removes **1.39** of the 1.87 points; the share the dates account for is **98.8%** SVJ, **97.2%** SVCJ, **48.3%** severe jump.

The Amplification from Portfolio to Reserve



GMMB, fixed strike

Change relative to the TV base (%)	Portfolio realised volatility	Guarantee tail shortfall	Aggregate reserve
Implied-volatility overlay	-3.29	-1.16	-1.87
GJR-GARCH filter	-2.59	-0.24	-0.81
Realised variance, 1 month	-2.71	-0.20	-0.72
EWMA	-2.49	+0.23	-0.33
Realised variance, 6 months	-2.47	+1.06	+0.17
IV advantage over GJR-GARCH	0.70	0.92	1.06

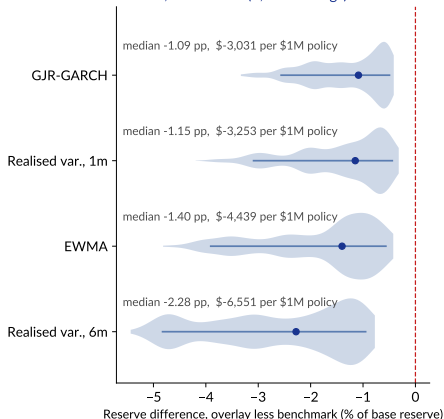
GMIB, benefit basis follows the fund

Change relative to the TV base (%)	Portfolio realised volatility	Guarantee tail shortfall	Aggregate reserve
Implied-volatility overlay	-2.96	-2.27	-1.94
GJR-GARCH filter	-2.36	-1.69	-1.51
Realised variance, 1 month	-2.52	-1.66	-1.36
EWMA	-2.28	-1.35	-1.24
Realised variance, 6 months	-2.25	-1.08	-0.89
IV advantage over GJR-GARCH	0.60	0.58	0.43

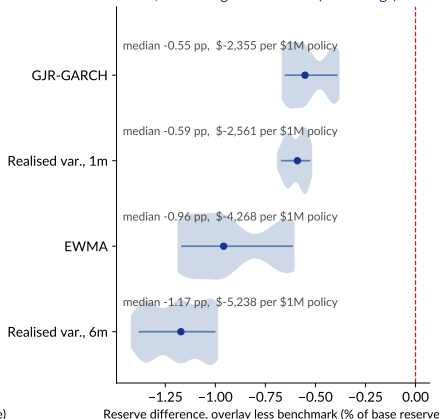
The Reserve Advantage of the Implied-Volatility Overlay



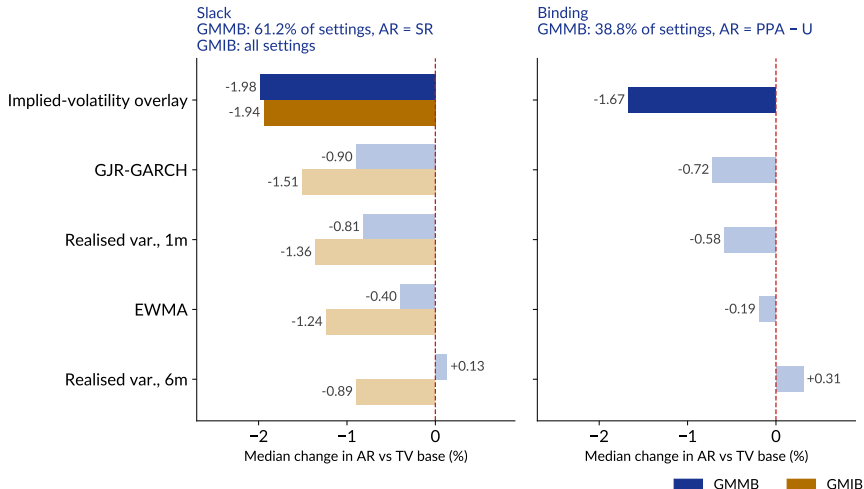
GMMB, fixed strike (3,690 settings)



GMIB, ratcheting benefit basis (90 settings)



Slack and Binding Regimes of the Standard Projection



Assumption varied	Alternative tested	Overlay's own ΔAR
Mortality basis	no improvement, accelerated improvement, no mortality	−1.86 to −1.87%
Interest rate and equity premium	four (r , ERP) pairs, (0%, 11.3%) to (5%, 2.0%)	+2.61 to −3.55%
Trading cost	10 and 20 bp per side on traded notional	−1.12%, −0.60%
Signal observation error	dispersion doubled	−1.59%
Scenario sampling	ten independent seeds	−1.53 to −2.31%
Implied-volatility floor	max(IV, RV) in the stochastic reserve, and through the full path	−1.82%, −1.73%

The least favourable seed set has a lower reserve than the 1-month realised-variance rule in 85.9% of settings.

- **Under every alternative the median ordering remains unchanged.**
The implied-volatility overlay keeps the lowest median reserve against each backward benchmark. Its own reduction **changes sign** across the interest-rate and equity-premium pairs.



- **Forward-looking volatility information is worth reserve capital.** The implied-volatility overlay carries **1.1 to 2.3** percent less VM-21 reserve on the maturity guarantee than any of the four backward-looking rules.
- **The value comes from when the fund de-risks.** Placing the same cuts on random dates gives back **1.39** of the 1.87 points; the signal's dates account for 48 to 99 percent of the reduction across the three calibrations.
- **The statutory reserve rewards timing more than the portfolio does.** A fixed-strike guarantee turns each point of volatility lead into **1.6 to 2.3** points of reserve.



- **A state-dependent variance risk premium as a second forward-looking signal.** In the model the premium is observation error; in data it carries information that a rule can condition on.
- **Stochastic mortality in the income guarantee.** Both guarantees settle on a fixed term here; a stochastic mortality model lets the benefit itself carry longevity risk.
- **Stochastic interest rates.** Rates are constant; the annuitised value of the income guarantee moves with them.



Thank you.

Comments and questions are welcome.