Pricing Commodity Swing Options

Financial Engineering Workshop

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Based on “Pricing commodity swing options” [2] with
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Outline

1. Natural gas market
2. Pricing model
3. Swing contracts
4. Pricing with LSMC
5. Pricing with PPO
6. Conclusions

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Linear contracts

- Each instruments is identified by:
  - ⇒ **Delivery point**: e.g. TTF, PSV, ...
  - ⇒ **Delivery start/end**: when the gas flux starts/ends.
  - ⇒ **Quantity**: The quantity of Gas per hour (MWh/h).

- **Spot** contracts:
  - ⇒ Delivery period is less than one month.
  - ⇒ Payment is at inception.
- **Futures** contract:
  - ⇒ Delivery period is one or more months.
  - ⇒ Payment is on each delivery date.

Delivery periods

- Spot contracts:

Within Day Starts after 3h and ends at the end of the day[1].
Day Ahead Next business day.
Weekend Next weekend.
BOM Remaining days of the months.

- Futures contracts:

Month Entire month.
Quarter The four quarters of the year.
Season Summer and Winter.
Cal. Year Entire year.

Definition: $F_t^\delta(T)$

The futures with expiry T and delivery period δ observed at time t.

[1] each gas day starts at 6AM and ends at 6AM of the day after, see e.g. https://www.eex.com/en/markets/trading-ressources/rules-and-regulations

Options

- The most liquid are the **options on futures**:
  - $\Rightarrow$ The underlying is a monthly futures.
  - $\Rightarrow$ For each underlying futures there is only one expiry.
  - $\Rightarrow$ The expiry is typically close to the delivery start (end of the month).

- Other fairly common contracts:
  - $\Rightarrow$ **Mid curve options**: options on futures where the expiry of the options is far before the expiry of the underlying futures.
  - $\Rightarrow$ **Calendar spread options**: options on the spread between two consecutive futures.

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Motivations

We would like the model to have the following features:

- Calibrate to the options on futures **smile** with a fast and accurate procedure.
- Imply the volatilities of a futures at **early expiries** including the **Samuelson effect**.
- Imply the volatilities for different **tenors** (e.g. the volatilities of the quarters from the volatilities of the monthly futures).

Modelling the instantaneous futures

We start defining the *fictitious spot* process as

$$dx_t := a(t)(1 - x_t)dt + \eta(t, x_t)x_t d\omega_t, \qquad x_0 := 1, \tag{1}$$

then the abstract futures for instantaneous delivery period as:

$$f_t(T) := f_0(T)\mathbb{E}\left[x_t\right] = f_0(T)\left(1 - (1 - x_t)e^{-\int_t^T a(u)du}\right), \tag{2}$$

where:

- $a(t)$ is the mean reversion speed.
- $\eta(t, x_t)$ is the local volatility.
- ω_t is a standard BM under the RN measure.

Observation

For $a = 0$ we obtain the usual Dupire model.

Modelling futures

Futures are defined by averaging on the delivery period δ:

$$F_t^\delta(T) := \frac{1}{\delta}\int_T^{T+\delta} f_t(u)du = F_0^\delta(T)\left(1-(1-x_t^\delta)e^{-\int_t^T a^\delta(u)du}\right) \tag{3}$$

$$dx_t^\delta = a^\delta(t)(1-x_t^\delta)dt + \eta^\delta(t,x_t^\delta)x_t^\delta d\omega_t, \qquad x_0^\delta = 1, \tag{4}$$

where

$$x_t^\delta = \left(1-(1-x_t)G^\delta(t)\right), \qquad a^\delta(t) = a(t) - \partial_t \log G^\delta(t),$$

$$G^\delta(t) = \frac{1}{F_0(T;\delta)}\int_T^{T+\delta} f_0(u)e^{-\int_t^u a^\delta(v)dv}du, \tag{5}$$

$$\eta^\delta(t,x) = \left(1-\frac{1-G^\delta(t)}{x}\right)\eta\left(t, 1-\frac{1-x}{G^\delta(t)}\right)$$

Observation

The futures dynamics is the same up to a change parameters.

Options on futures

- Options on futures: $C^\delta(t,T,K) := \mathbb{E}_0\left[\left(F_t^\delta(T)-K\right)^+\right]$.
- Options on the fictitious spot: $c^\delta(t,k) := \mathbb{E}_0\left[(x_t^\delta - k)^+\right]$.
- The two are related by the following relation:

$$\begin{aligned} C^\delta(t,T,K) &= F_0^\delta(T)e^{-\int_t^T a^\delta(u)du}c^\delta(t,k_f) \\ k_f &:= 1-\left(1-ke^{\int_t^T a^\delta(u)du}\right), \qquad k=\frac{K}{F_0^\delta(T)} \end{aligned} \tag{6}$$

Extended Dupire

$$\partial_t c^\delta(t,k) = \left(-a(t)-a(t)(1-k)\partial_k - \frac{k^2\eta^\delta(t,k)^2}{2}\partial_{kk}\right)c(t,k) \tag{7}$$

Local volatility calibration

1. Select the mean reversion speed (e.g. $a(t) = 1$).
2. Map the market volatilities into spot volatilities using (6):
   $C^{1m}(t, T, K) \to c^{1m}(t, k_f)$.
   Vanilla options are usually on the monthly futures.
3. Solve the extended Dupire for $\eta^{1m}(t, k)$.
   $\Rightarrow$ In [4] they derive asymptotic relations between the local and the market volatility and they exploit them to solve the extended Dupire equation with a fixed point algorithm.

Remarks:

- When we change $a(t)$, $\eta^{1m}(t, k)$ must be re-calibrated.
- The dynamics for other tenors can be implied using (5).

Implying volatilites at early expiries

- Increasing $a(t)$ imply that futures are more volatile near expiration: **Samuelson effect**.
- *Mid curve options* are strongly impacted: $MCO(t,T,K,\delta) = \mathbb{E}_0\left[\left(F_t^{\delta}(T) - K\right)^+\right]$.

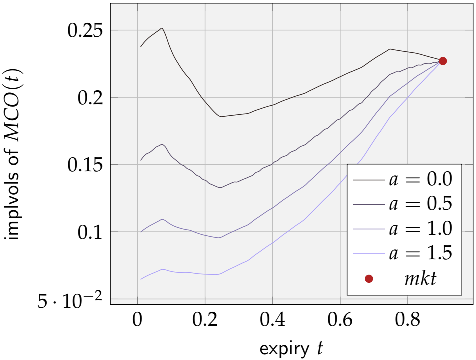

Figure 1: MCOs implied volatilities.

- *Eval date*: $29/3/2018$
- *Underlying futures*: MAR19 $(T \sim 11m,\, \delta = 1m)$ on NG TTF
- *Strike*: ATM

Implying volatilites for different tenors

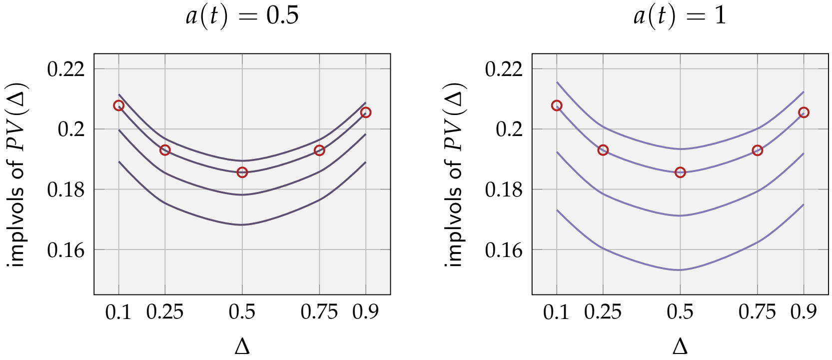

Figure 2: Implied volatilities of options on futures with different tenors $PV(t, T, \delta)$. Red dots are the market quotes ($\delta = 1m$).

- *Eval date*: 29/3/2018
- *Underlying futures*: JUL19 on NG TTF
- $\delta = 1d, 1m, 3m, 6m$ (from top to bottom).
- *Strikes*: Expressed as Black deltas

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Contract definition

- Guarantee a flexible daily supply of gas during the delivery period: on each day the buyer can choose the quantity within daily and global constraints. The gas price (strike) can be either known at inception, or fixed at a forward date.

Delivery parameters:

- *Period*: $\{t_i\}_{i=1}^{n_f}$.
- *Underl.*: day ahead $F_{t_i}^{1d}(t_i)$.
- *Daily*: N_{min}, N_{max}.
- *Total*: C_{min}, C_{max}.

Strike parameters:

Fixed strike

- *K*: daily price.

Floating strike

- *Strike period*: $\{s_i\}_{i=1}^{n_s}$.
- *Underlying*: $F_t^{\delta}(T)$

Term-sheet example

- From now on we will study only monthly swing which will be identified by the delivery month (e.g. `JUL18` in this example).

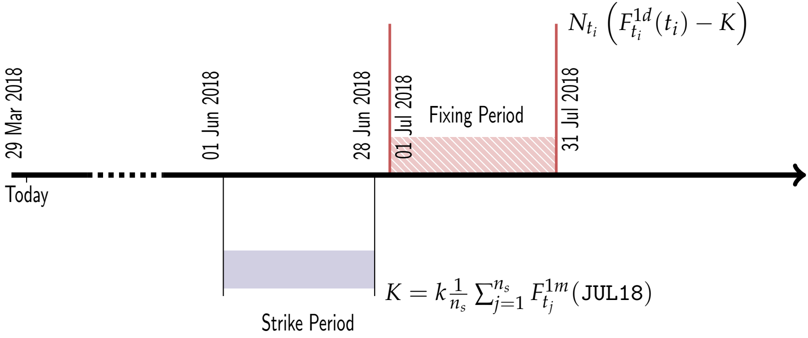

Figure 3: Term-sheet data for a swing option contract with delivery in July 2018. The strike is fixed by averaging the JUL18 futures contract observed in the month of June.

Pricing problem

The price of a swing is:

$$P = \max_{N \in \mathcal{N}} \sum_{i=1}^{n_f} \mathbb{E}_0 \left[N_{t_i} \left(F_{t_i}^{1d}(t_i) - K \right) P_0(t_i^p) \right], \tag{8}$$

where

- $P_0(t_i^p)$ is the discount up to the payment date t_i^p
- N_t is the quantity nominated at time t:
- $K = k \sum_{i=1}^{n_f} F_{t_i}^{\delta}(T)$ for floating strike.
- N is the delivery plan $\{N_{t_i}\}_{i=1}^{n_f}$.
- $\mathcal{N}$ is the set of all admissible strategies, i.e. subject to:

$$N_{min} \leq N_t \leq N_{max}, \qquad C_{min} \leq \sum_{i=1}^{n_f} N_{t_i} \leq C_{max}, \tag{9}$$

Bang bang strategies

A *bang bang strategy* is a delivery plan in which the choices are always the minimum or the maximum allowed:

- $N_t \in \{N_{min}, N_{max}\}$

Bardou et al. [1] (theorem 2.4)

If C_{min} and $C_{max} - C_{min}$ are multiple of $N_{max} - N_{min}$
$\Rightarrow$ the optimal strategy is bang bang.

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LSMC algorithm

On each fixing day we have to solve the following problem:

$$W_{t_i} = \max_{N_{t_i}} \left\{ N_{t_i}(F^{1d}_{t_i}(t_i) - K) + \mathbb{E}\left[W_{t_{i+1}} \frac{P_0(t^p_{i+1})}{P_0(t^p_i)} \,\middle|\, F^{1d}_{t_i}(t_i), C_{t_i}, K \right] \right\}, \quad (10)$$

where we have defined the total quantity $C_{t_i} := \sum_{t<t_i} N_t$.

We solve the problem with the following steps:

1. Build a grid for C_{t_i} at each fixing date.
2. Estimate of the value functions on the grids by means of regressions with a backward procedure.
3. Compute the swing option price with a standard forward Monte Carlo simulation.

Prices vs $a(t)$

- Constraints:
  - $N_{min} = 0\, MWh$
  - $C_{min} = 12.5\, MWh$
  - $N_{max} = 1\, MWh$
  - $C_{max} = 20\, MWh$

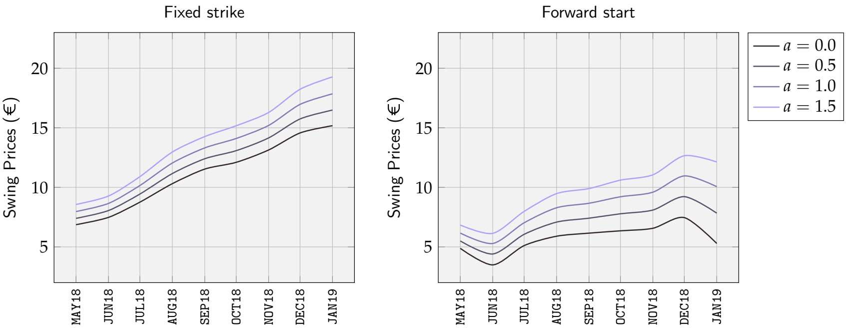

Figure 4: Swing option prices by varying the delivery period.

- Day ahead volatility increases as $a(t)$ increases.
- Monthly futures early volatility decreases as $a(t)$ increases.

Theorem (14) verification

- Two constraints types:
  - $\Rightarrow$ Outside theorem (14) hypothesis: $C_{min} = 12.5\, MWh$.
  - $\Rightarrow$ Within theorem (14) hypothesis: $C_{min} = 12\, MWh$.
- Two sets of strategies:
  - $\Rightarrow$ All strategies: $N_t \in [N_{min}, N_{max}]$.
  - $\Rightarrow$ Only bang bang: $N_t \in \{N_{min},\ N_{max}\}$.

	All Strategies	Only Bang-Bang
Out of Theorem Hyp.	7.97 ± 0.04	7.76 ± 0.04
Within Theorem Hyp.	8.46 ± 0.04	8.46 ± 0.04

Table 1: MAY18 fixed strike swing option prices in four different cases. The constraints are the same as before and $a(t) = 1$. One-sigma statistical errors are displayed.

Optimal strategies

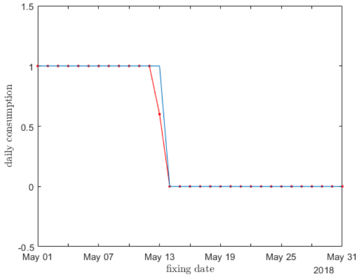

Figure 5: Daily consumption selected by the optimal strategy for the `MAY18` fixed-strike swing in a sample path. Left panel displays the out-of-theorem scenario, while the right panel the within-theorem. The blue lines refer to the case where only bang-bang strategies are allowed, while red lines to the case without restrictions. In the bang-bang case (right panel) the two lines coincide.

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Problem setup

- The problem is represented as an *agent* that can observe and interact with the *environment* by taking *actions*.
- a *state* is an observation of the environment.
- After taking an action the agent gets a *reward* and the state is updated.
- This *step* is repeated for a finite number of time.

- For the pricing of swing options:
  - The environment is represented by the time t, the commodity price $F_t^{1d}(t)$, the total consumption C_t and the strike K for forward start options.
  - The action is the quantity N_t.
  - The steps are the fixing dates.

Agent policy

- The function, potentially stochastic, defining the action in terms of the state is called *policy*.
  - $\Rightarrow$ In our setting the actions are random variables $\tilde{N}^{\theta}_{t_i}$ distributed according to independent Gaussian distributions ϕ centered on the value $N^{\theta}_{t_i}$, which is determined by a neural network

$$\pi^{\theta}_{t_i}(n) := \mathbb{Q}\left\{ \tilde{N}^{\theta}_{t_i} = n \mid s_{t_i}, C^{\theta}_{t_i} \right\} = \phi(n; N^{\theta}_{t_i}, \xi_i) \tag{11}$$

- The algorithm we use is of *actor critic* type, which means that there is another explicit function, the critic, that estimates the value function a each step.
  - $\Rightarrow$ In our setting we use the same net for the policy N^{θ}_{t} and the estimate of the value function V^{θ}_{t}.
- The *training* goal is to find the optimal policy, i.e. the policy that maximizes the expected value of the future rewards.

The advantage function

- We define the *action-value* function:

$$\tilde{Q}^{\theta}_{t_i}(n) := \mathbb{E}\left[r_{r_i}(n) + \tilde{Q}^{\theta}_{t_{i+1}}(\tilde{N}^{\theta}_{t_{i+1}})D(t_i, t_{i+1}) \,\middle|\, F^{1d}_{t_i}(t_i), C_{t_i} \right], \quad (12)$$

where $r_{t_i}(n) := n(F^{1d}_{t_i}(t_i) - K)$ is the reward at time t_i.

- If the action at time t_i is integrated over its law under the policy, we can write the value function as

$$\tilde{V}^{\theta}_{t_i} := \mathbb{E}\left[r_{t_i}(\tilde{N}^{\theta}_{t_i}) + \tilde{V}^{\theta}_{t_{i+1}} D(t_i, t_{i+1}) \,\middle|\, F_{t_i}, C_{t_{i-1}} \right] \quad (13)$$

- The *advantage function* $A^{\theta}_{t_i}(n)$ measures the average improvement in expected reward given by choosing the action n, over the average reward of the current policy $\tilde{V}^{\theta}_{t_i}$.

$$A^{\theta}_{t_i}(n) = \tilde{Q}^{\theta}_{t_i}(n) - \tilde{V}^{\theta}_{t_i}. \quad (14)$$

Training procedure

- A reasonable idea is to train the agent to favour large values of $A^\theta_{t_i}$. Indeed, one can prove that the gradient of the expected total cumulated reward equals:

$$\sum_i \mathbb{E}\Big[A^\theta_{t_i} (\nabla_\theta \log \pi^\theta_{t_i}(n))|_{n=\tilde{N}^\theta_{t_i}} \Big]. \tag{15}$$

- In practice one substitutes the unknown A^θ with a pathwise quantity which gives (approximately) the same gradient:

$$\hat{A}^\theta_i := \sum_{l=0}^{n_f - i - 1} D(t_i, t_{i+l}) \lambda^l \left[r_{t_{i+l}}(\tilde{N}^\theta_{t_{i+l}}) + D(t_{i+l}, t_{i+l+1}) V^\theta_{t+l+1} - V^\theta_{t+l} \right], \tag{16}$$

where λ is introduced to lower the variance.

Proximal policy optimization [5]

- The parameters θ are update according to:

$$\theta_{k+1} = \theta_k + \rho \cdot \mathbb{E}\left[\nabla_\theta \sum_{i=1}^{n_f} \left(L^A_{T_i}(\theta) - \beta L^V_{t_i}(\theta) \right) \Big|_{\theta=\theta_k} \right] \tag{17}$$

$$L^A_{t_i}(\theta) := \min \left\{ \frac{\pi^\theta_{t_i}(\tilde{N}^{\theta_k}_{t_i})}{\pi^{\theta_k}_{t_i}(\tilde{N}^{\theta_k}_{t_i})} \hat{A}^{\theta_k}_i, \operatorname{clip}\left(1-\varepsilon, \frac{\pi^\theta_{t_i}(\tilde{N}^{\theta_k}_{t_i})}{\pi^{\theta_k}_{t_i}(\tilde{N}^{\theta_k}_{t_i})}, 1+\varepsilon \right) \hat{A}^{\theta_k}_i \right\} \tag{18}$$

$$L^V_{t_i}(\theta) := \left(V^\theta_{t_i} - (\hat{A}^{\theta_k}_i + V^{\theta_k}_{t_i}) \right)^2 \tag{19}$$

- where the expected value is estimated over a batch of episodes and, ε and β are hyper-parameters and
  - $\Rightarrow$ L^V measures how well V^θ represents the value function $\tilde{V}^\theta$.
  - $\Rightarrow$ ρ is a learning rate.
  - $\Rightarrow$ $\operatorname{clip}(a, x, b)$ is a cap and floor function.

Training setup

- We use the PPO implementation found in OpenAI Baselines `https://github.com/openai/baselines`.
- Default PPO hyper-parameters:
  - $\Rightarrow$ $\lambda = 0.95$, $\varepsilon = 0.2$, $\rho = 0.0003$, $\beta = 0.5$.
  - $\Rightarrow$ The parameters θ are updated once every 2048 episodes.
- Two distinct feed-forward neural networks are used to compute respectively the action and the value function.
- The networks inputs are:
  - $\Rightarrow$ t_i expressed as a year fraction.
  - $\Rightarrow$ C_{t_i} remapped linearly so that its domain is $[-0.5, 0.5]$.
  - $\Rightarrow$ $\log(F_{t_i}^{1d}(t_i)/F_{t_i}^{1d}(t_i))$.
  - $\Rightarrow$ The output layer is linear, i.e. no activation function is applied.
- The networks features are:
  - $\Rightarrow$ hyperbolic tangent activation function.
  - $\Rightarrow$ The output layer is linear, i.e. no activation function is applied.

Neural network fine tuning

- We consider a swing of 1 week with the following constraints:
  - $N_{min} = 0\,MWh$
  - $C_{min} = 3\,MWh$
  - $N_{max} = 1\,MWh$
  - $C_{max} = 5\,MWh$
- We try different architectures:
  - 1 X 64 (wide).
  - 5 X 4 (deep).
  - 5 X 64 (wide and deep).

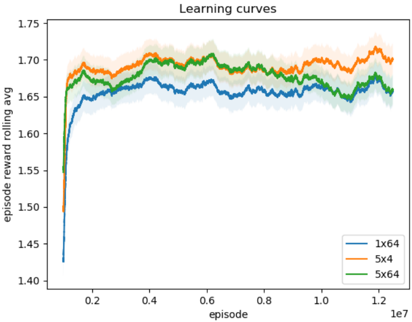

Figure 6: Learning curves. On the horizontal axis the number of training episodes. The solid lines are the moving average of the realized rewards on the last 10^6 episodes. The shadows represent the 98% confidence intervals.

Comparison with LSMC

- We repeat the exercise done before

	All Strategies	Only Bang-Bang
Out of Theorem Hyp.	7.92 ± 0.04	7.74 ± 0.04
Within Theorem Hyp.	8.40 ± 0.04	8.37 ± 0.04

Table 2: `MAY18` fixed strike swing option prices with PPO.

	All Strategies	Only Bang-Bang
Out of Theorem Hyp.	7.97 ± 0.04	7.76 ± 0.04
Within Theorem Hyp.	8.46 ± 0.04	8.46 ± 0.04

Table 3: `MAY18` fixed strike swing option prices with LSMC.

Bang bang strategies in PPO

- the trained agent successfully identifies the optimal bang bang strategies.

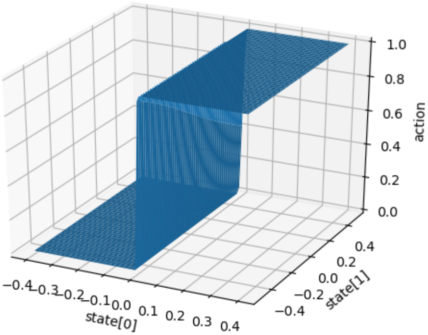

Figure 7: Normalized consumption as a function of normalized log-spot and consumption, on the fourth decision date.

Additional comparisons

- To check the robustness of our results, we repeated the comparison with alternative values of the main model parameters

a	$\Delta\sigma$	RL	LSMC
1.0	0%	7.92 ± 0.04	7.97 ± 0.04
0.5	0%	7.35 ± 0.04	7.40 ± 0.04
1.0	10%	11.36 ± 0.05	11.68 ± 0.05
0.5	10%	10.74 ± 0.05	10.87 ± 0.05

Table 4: Swing option prices with RL and LSMC. We consider the reference case with $C_m = 12.5\,MWh$ not satisfying theorem (14), and vary both the mean reversion speed a and the surface of market implied volatilities; the latter is perturbed by a parallel shift $\Delta\sigma$.

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Conclusions

- We developed a model for energy futures which can efficiently be calibrated to the market quotes.
- The model is able to imply non trivial volatilities for different tenors while remaining parsimonious.
- We analyzed the swing options prices and the optimal strategies using the standard LSMC.
- We compared LSMC with PPO showing that the latter is suitable for this type of problems.

Further developments

- Extend the proposed model to multi-factor , see e.g. [3].
- Test the PPO algorithm in a setting (e.g. multi-factor) where LSMC cannot be used.

Main references I

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