

A Risk-Adjusted Analysis of LTC insurance

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2026 September, 9



Contractual profiles and risk sources for LTC coverages has been widely analyzed in the literature

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Traditional valuation approach $\Rightarrow$ More extensive valuation system

Enterprise Risk Management (ERM)

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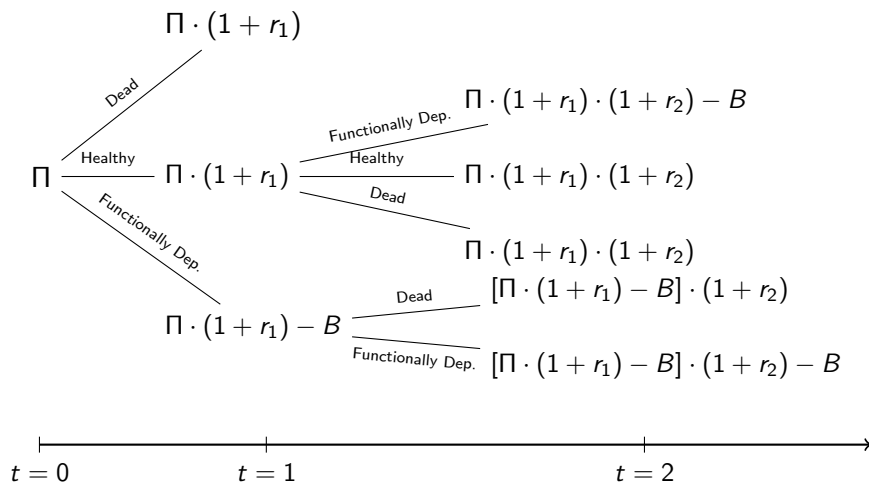
- Forward-looking evaluation of profit flows
- Measure of model risk arising from uncertainty on technical bases
- Risk-adjusted extension via a CRRA utility function

E. Di Lorenzo, G. Piscopo, A. Roviello, and M. Sibillo (2026). "A Risk-Adjusted Analysis of LTC Insurance". In: *Applied Stochastic Models in Business and Industry* 42.4, e70123

Model assumptions

- Homogeneous **portfolio** of LTC coverages (age x , healthy)
- Three-health states (healthy, functional dependent, dead)
- No recovery
- Focus on the benefit period
- Pre-funded plan (single premium Π) with LTC coverage starting at a contractually stipulated age
- Predetermined and constant benefits B

Individual's financial flows



$N(x)$ insureds of age x at $t = 0$

$$\mathcal{F}_t = N(x) \cdot \Pi \cdot \prod_{\tau=1}^{t-1} (1 + r_{\tau}) - \sum_{d=1}^{N(x)} \left[\sum_{\tau=1}^{t-1} B \cdot X_{\tau,d} \cdot \prod_{t'=\tau}^{t-1} (1 + r_{t'}) + B \cdot X_{t,d} \right]$$

$$X_{t,d} = \begin{cases} 1 & \text{if } \Omega_{t,d} \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

$$\Omega_{t,d} := \{s_{t,d} = 2 \mid s_{t-1,d} = 1 \vee s_{t-1,d} = 2\}$$

* $s_{t,d} = 1$ means healthy and $s_{t,d} = 2$ means functionally dependent

When evaluating $\mathcal{F}_t$, the insurer relies on a specific table, assuming it accurately represents future outcomes.

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$\mathcal{Z} = \{Z_1, Z_2, \dots, Z_j, \dots, Z_K\}$ finite set of random scenarios

$p^{(j)}(\Omega_{t,d}) = p(\Omega_{t,d}|Z_j)$ probability given the scenario Z_j for $j = 1, \dots, K$.

How the insurer may manage multiple options?

The insurer assigns a certain degree of reliability to each scenario:

$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_j, \dots, \alpha_K)$ with $\sum_{j=1}^K \alpha_j = 1$

$$\sigma_{\alpha, \mathcal{Z}}^2(t) = \text{Var}_{\alpha}[E(\mathcal{F}_t | \mathcal{Z})]$$



measure of the variability of $\mathcal{F}_t$, due to the randomness of the choice of the tables, the effects of stochastic outflows having been averaged out

M. Coppola, E. Di Lorenzo, and M. Sibillo (2003). "Stochastic analysis in life office management: applications to large annuity portfolios". In: *Applied Stochastic Models in Business and Industry* 19.1, pp. 31–42

Risk-adjusted expected utility

- Yearly gain/loss function:

$$Y_{t,j} := E[\mathcal{F}_{t-1}|Z_j] \cdot (1 + r_{t-1}) - \sum_{d=1}^{N(x)} B \cdot X_{t,d}$$

- CRRA utility function:

$$U(Y_{t,j}) = \frac{Y_{t,j}^{1-\gamma}}{1-\gamma}$$

- Expected utility of gain/loss: $\mathcal{U}_{t,j} = E_{p_j}[U(Y_{t,j})]$

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Risk-adjusted expected utility

Risk-adjusted expected utility:

$$\mathcal{U}_{h_t,j} = h_t \cdot \mathcal{U}_{t,j}$$

where $h_t \in [0, 1]$ is the risk index

$$h_t = h(\xi_\alpha(t), \nu_t) = e^{-\xi_\alpha(t)(1-\nu_t)}$$

where

$$\xi_\alpha(t) = \frac{\sqrt{\sigma_{\alpha, \mathcal{Z}}^2(t)}}{\mu_\alpha(t)} \quad \text{with} \quad \mu_\alpha(t) = \sum_{j=1}^K E(\mathcal{F}_t | Z_j) \cdot \alpha_j$$

and ν_t is a time-dependent risk score associated by the insurer.

M. T. Kronborg and S. F. Jarner (2015). "Why You Should Care About Investment Costs: A Risk-Adjusted Utility Approach". In: *Journal of Behavioral and Experimental Finance* 6, pp. 56–66

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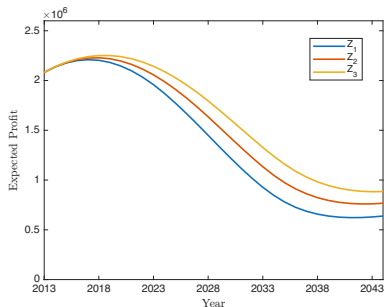
Assumptions

- Three possible scenarios: $\mathcal{Z} = \{Z_1, Z_2, Z_3\}$;
- Starting age: $x = 70$;
- Time interval: $[2013, 2044]$;
- Annual benefit payout: $B = 24000\text{€}$;
- Single premium: $\Pi = 20815\text{€}$ according to Z_2 ;
- Technical discount rate: $r' = 2,5\%$;
- $N(x) = 100$;

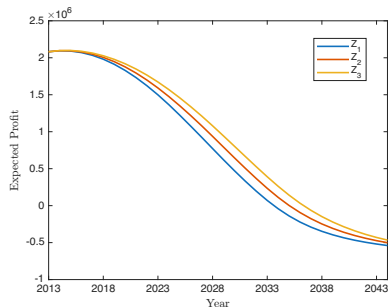
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Example



Conditional expected periodic profit with $r = 3, 5\%$



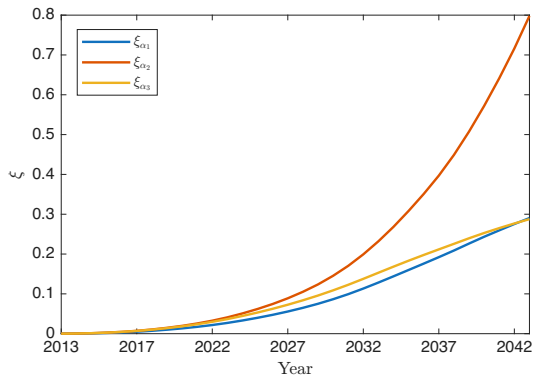
Conditional expected periodic profit with $r = 1, 5\%$

Example

Opportunity Cost Rate	Scenario	Final Expected Profit	Mean NPV	Minimum	Years of Loss
3,5%	Z_1	6.3848×10^5	1.4017×10^6	6.2244×10^5	0
	Z_2	7.6717×10^5	1.5249×10^6	7.599×10^5	0
	Z_3	8.8432×10^5	1.6374×10^6	8.8263×10^5	0
2, 5%	Z_1	-7.8080×10^4	1.0363×10^6	-7.8080×10^4	4
	Z_2	9.3132×10^{-10}	1.1423×10^6	9.3132×10^{-10}	0
	Z_3	7.1195×10^4	1.2395×10^6	7.1195×10^4	0
1, 5%	Z_1	-5.3945×10^5	7.5311×10^5	-5.3945×10^5	11
	Z_2	-5.0097×10^5	8.4456×10^5	-5.0097×10^5	9
	Z_3	-4.659×10^5	9.286×10^5	-4.659×10^5	8

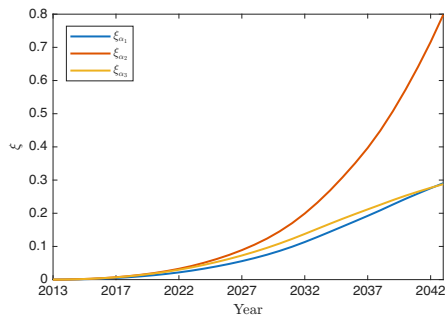
Table: Key summary indicators for different opportunity cost rates

Example



Variation index for $\alpha_1 = (0.1, 0.8, 0.1)$, $\alpha_2 = (0.8, 0.1, 0.1)$, $\alpha_3 = (0.1, 0.1, 0.8)$

Example

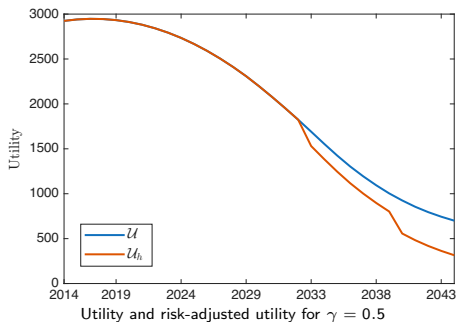
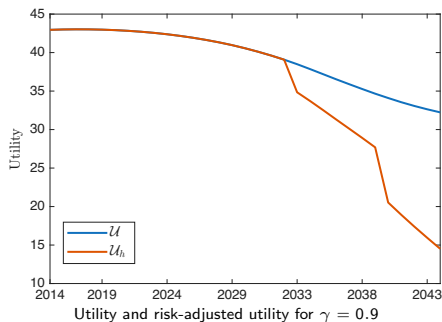


Variation index for $\alpha_1 = (0.1, 0.8, 0.1)$,
 $\alpha_2 = (0.8, 0.1, 0.1)$, $\alpha_3 = (0.1, 0.1, 0.8)$

- $\nu_t = 1$ for $t \in [2013 - 2033]$;
- $\nu_t = 0.7$ for $t \in [2034 - 2040]$;
- $\nu_t = 0.5$ for $t \in [2041 - 2044]$.

Example

$$\mathcal{U}_{h_t,k} = h_t \cdot \mathcal{U}_{t,k} = h_t \cdot E_{p_k}[U(Y_{t,k})] \text{ risk-adjusted expected utility}$$



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Thank you!