


A Study of Mortality Compression and Longevity Risk

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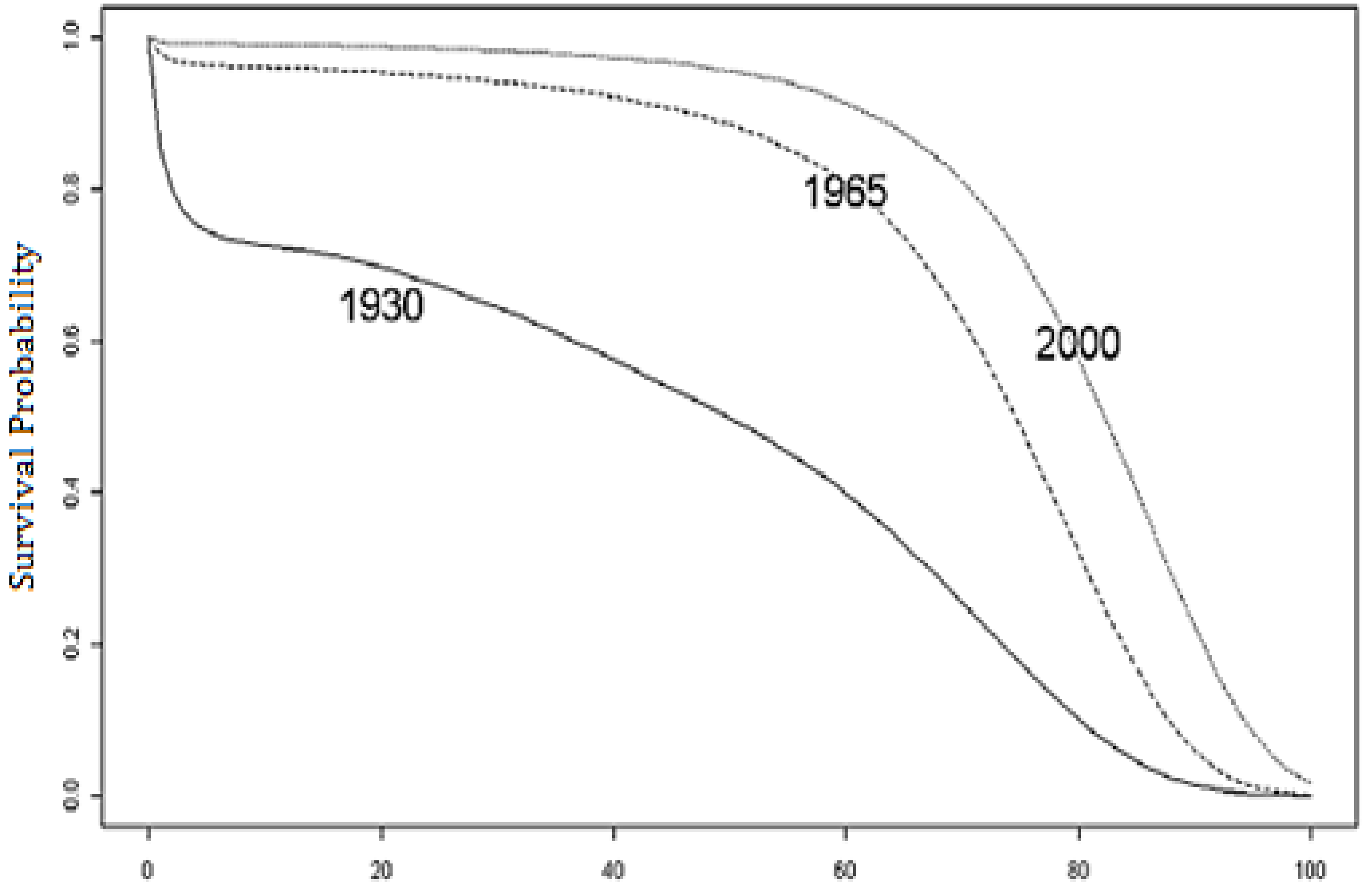
Summary

- Motivation
- Proposed Approach
- Simulation Study
- Applications
- Discussions

Rectangularization of Survival Curve



Survival Curves of Taiwan Female

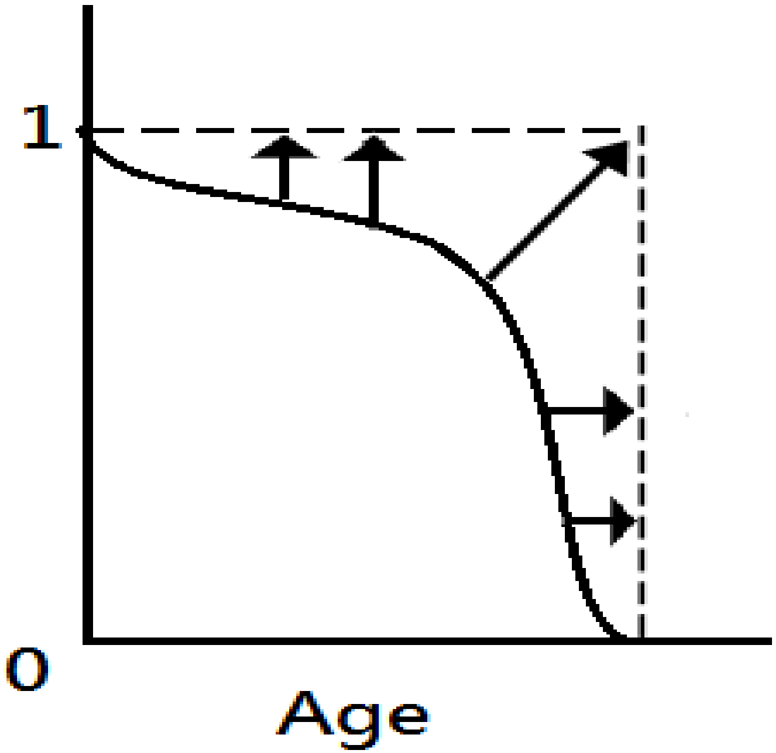
Rectangularization and Lifespan

- Regarding the theory of lifespan, there are two opinions: life with or without a limit.
- In either case, the rectangularization seems to be a consensus.

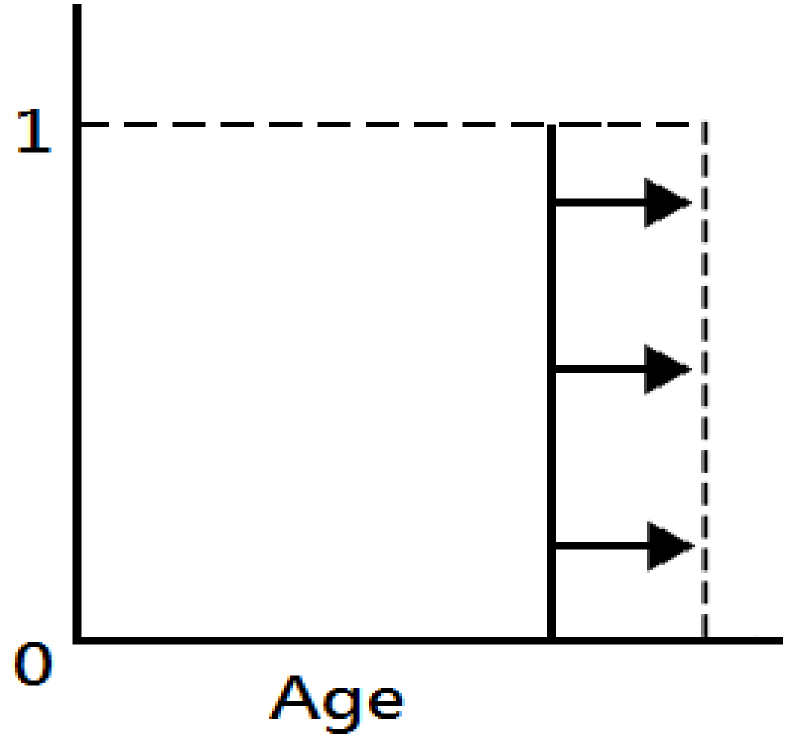
→Premature deaths (including infants) will gradually decrease and some postulates that the distribution of death number will behave like a normal curve (at least for the part with age higher than the mode).

About the Human Longevity

- Life with a limit!



- Life without a limit!



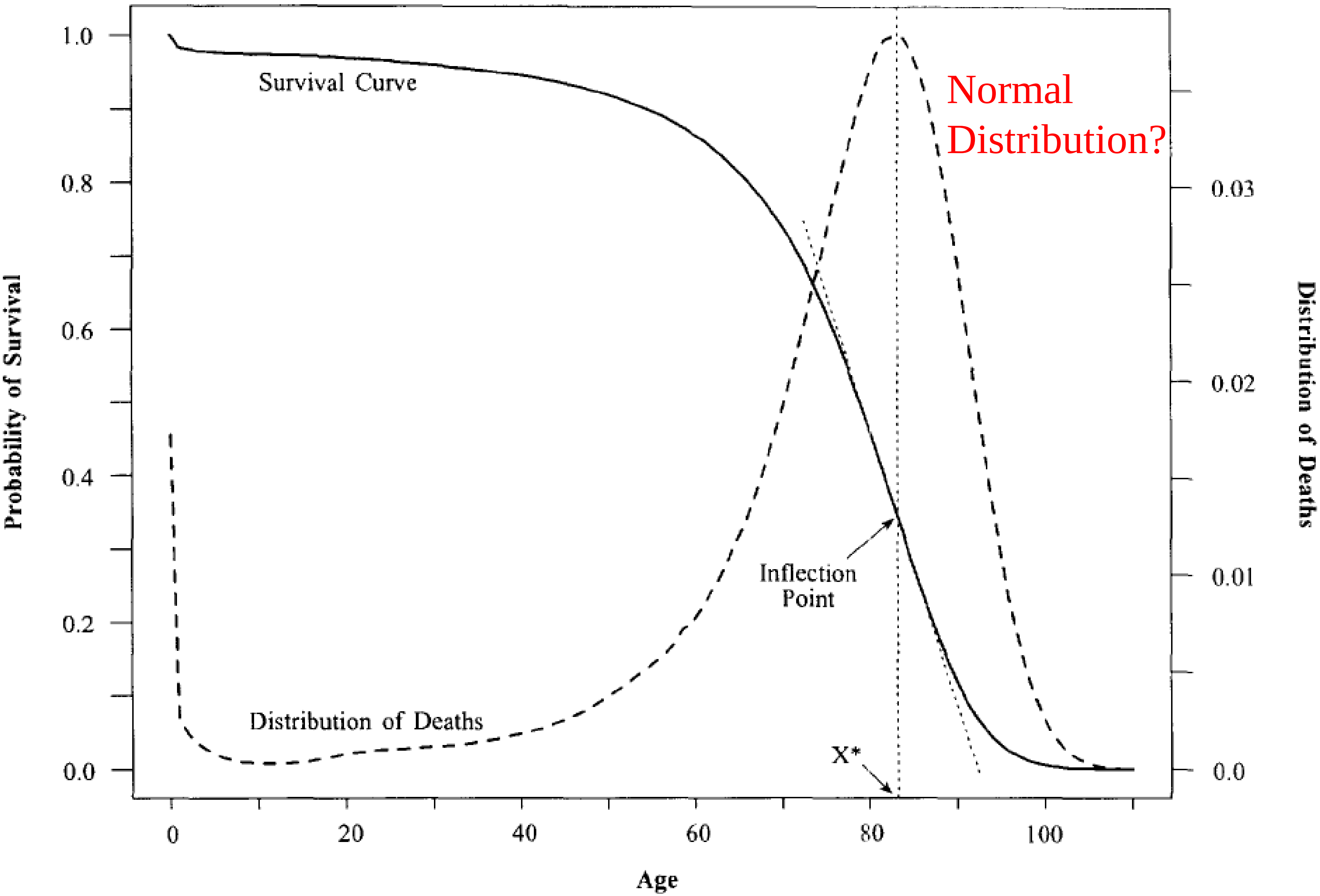
What is Mortality Compression?

- According to Fries (1980), Mortality Compression is

→ Rectangularization of the survival curve

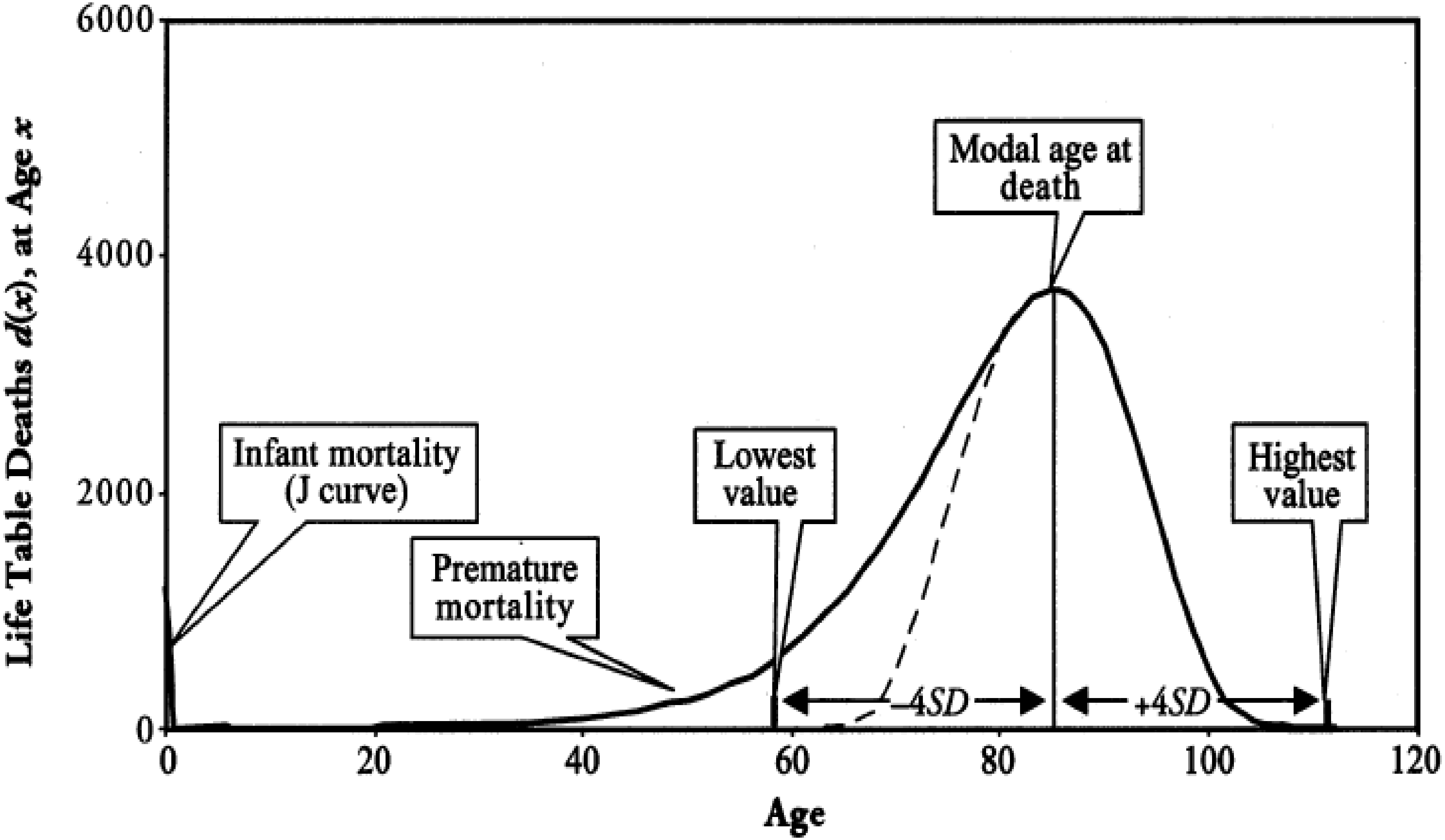
→ A state in which mortality from exogenous causes is eliminated and the remaining variability in the age at death is caused by genetic factors.

- Mortality compression is linked with morbidity compression.



Mortality Compression (Wilmoth and Horiuchi, 1999)

Horizontalization, Longevity Extension, Verticalization



Mortality Compression (Cheung et al., 2005)

Measuring Compression

- Wilmoth and Horiuchi (1999) proposed 10 measurements and they recommended the Interquartile (IQR).
- Kannisto (2000, 2001) calculated percentiles, IQR, shortest age interval (e.g., C50) on numbers of deaths from 22 countries.
- Cheung et al. (2005) computed SD(M+) for Hong Kong data.
- Thatcher et al. (2010) computed SD(M+) for 6 countries from HMD.

Some Questions

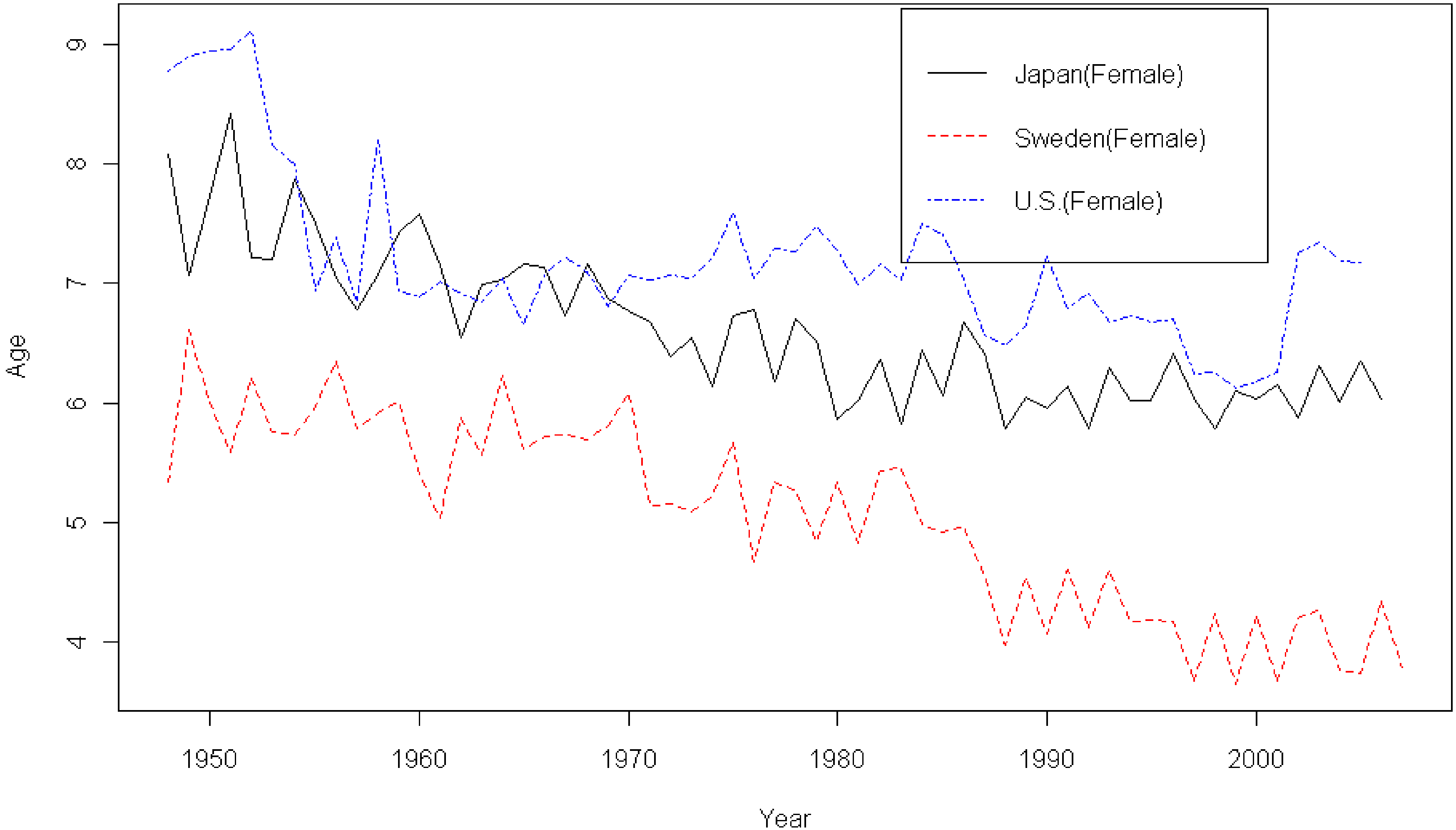
- The calculations rely on values from the life tables, which are being graduated.

→ The elderly mortality rates (and ex) are influenced the most.

- Like in the Gompertz model, the estimation of parameter can be modified.

→ Yue (2002) considered 3 estimation methods for the parameter C, where $\mu_x = BC^x$.

Standard Deviation of Number of Deaths (Raw Data)



Measures of Compression (Kannisto, 2000)

- Modal Age, M, or the age with the maximal number of deaths.

$$\rightarrow \quad M^{*} = x + \frac{f(x) - f(x-1)}{[f(x) - f(x-1)] + [f(x) - f(x+1)]}$$

- Standard deviation (σ) of the age at death above the mode, SD(M+).

$$SD(M+) = \sqrt{\frac{\sum_{M}^{\omega} f(x)(x-M)^{2}}{\sum_{M}^{\omega} f(x)}} \quad or \quad \sqrt{\frac{\int_{M^{*}}^{\omega} f(x)(x-M^{*})^{2}}{\int_{M^{*}}^{\omega} f(x)}}$$

Proposed Approaches

- Three Optimization methods:

Weighted Least Squares (WLS)

$$\min \sum_{x=M-k}^{M+k} w_x \{\log(\frac{d_{x+1}}{d_x}) - a - bx\}^2$$

Non-linear Maximiazation (NM)

$$\arg\min \sum_{x=M-k}^{M+k} d_x \{\frac{1}{\sqrt{2\pi}\sigma} \exp[\frac{-1}{2\sigma^2}(x-M)^2] - \frac{d_x}{l_0}\}^2$$

Maximal Likelihood Estimation (MLE)

$$\arg\max \sum_{x=M-k}^{M+k} (n_x - d_x) \log\{1 - \frac{1}{\sqrt{2\pi\sigma^2}} \exp[\frac{-1}{2\sigma^2}(x-\mu^*)^2]\}$$

$$+ d_x \log\{\frac{1}{\sqrt{2\pi\sigma^2}} \exp[\frac{-1}{2\sigma^2}(x-\mu^*)^2]\}$$

About the Estimation

- In addition to the estimation methods, the results of parameter estimates are also influenced by the number of observations used, or the data range *k*.

  →The estimation results using M~M+2*k* and M–*k*~M+*k* are similar, we will show only the results of M~M+2*k*.

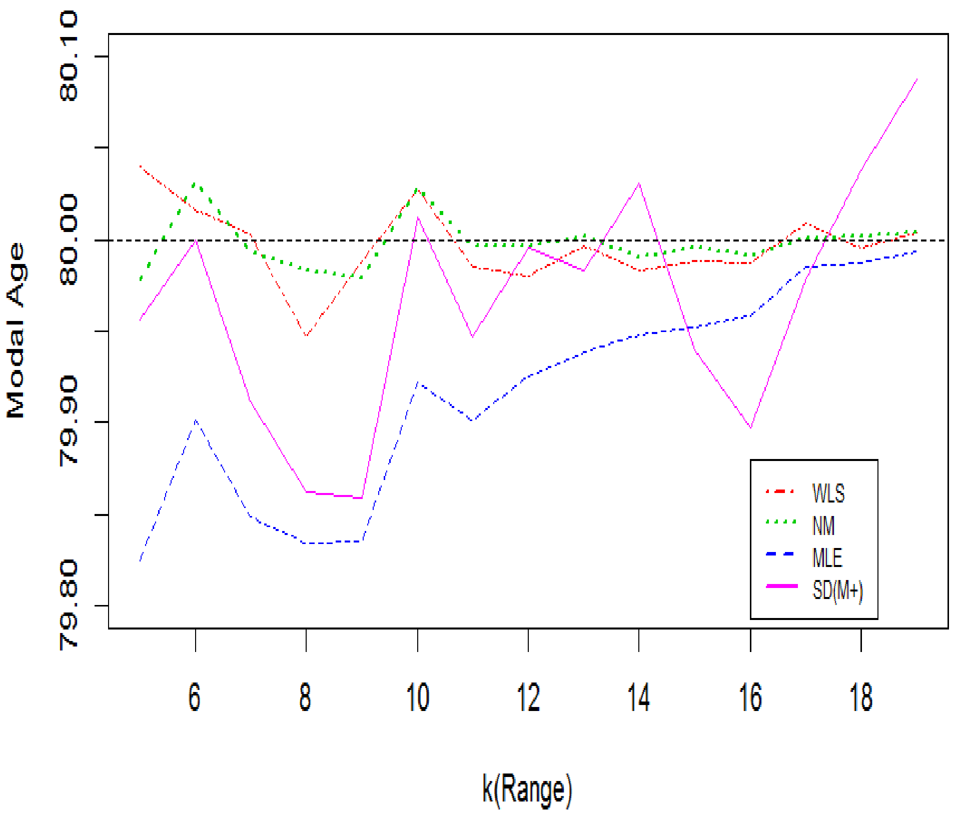
  →The data format is "age-last-birthday" and the estimation of M shall be modified.

Evaluating the Proposed Approaches

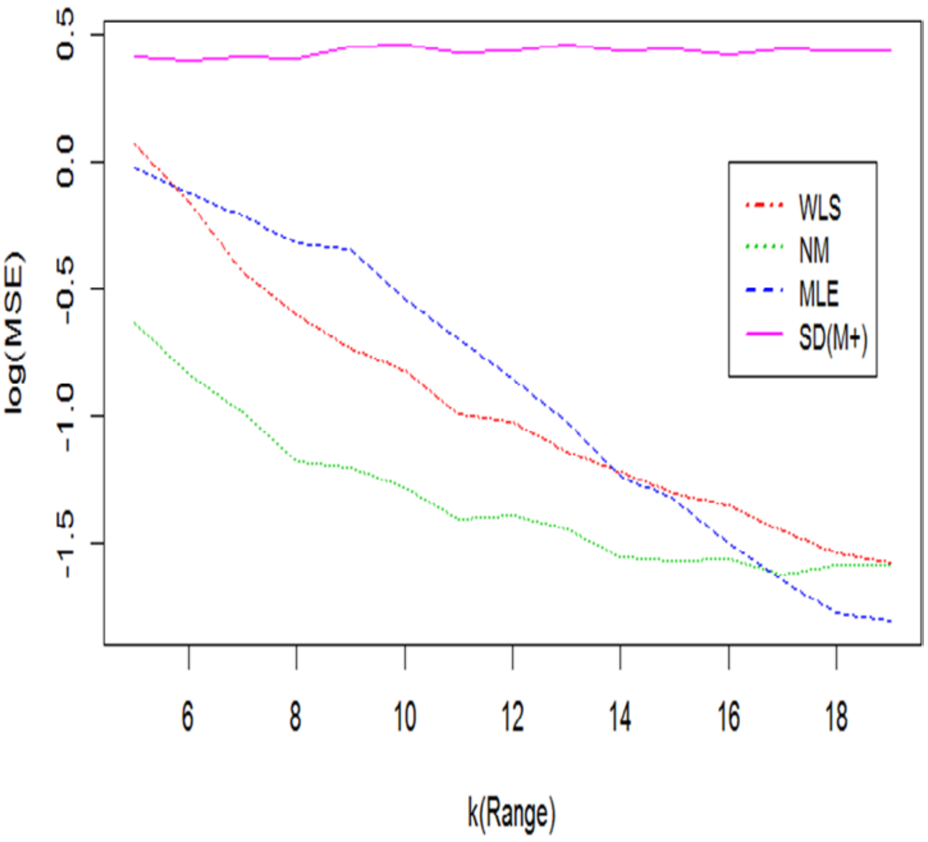
- We use the computer simulation to evaluate the proposed approaches and that by Kannisto.
- Suppose the modal age M is 80 and standard deviation σ is 10. Randomly generate 100,000 death from $N(M, \sigma^2)$. The comparison criteria:
  →Bias and variance, or Mean Squares Error (MSE), *Loss function (MSE) = Bias*^{2} + *Variance*.
  →Coverage probability, the probability of confidence interval covering true parameter.

Estimation of Modal Age (M=80)

Bias



MSE

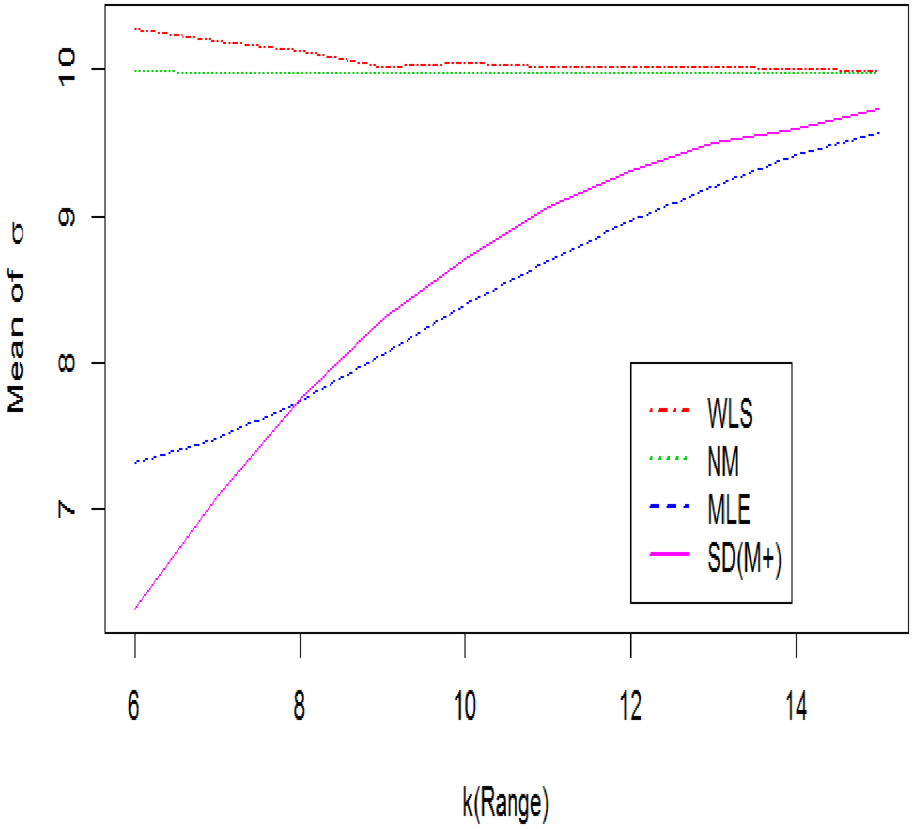


Coverage Probability of M=80

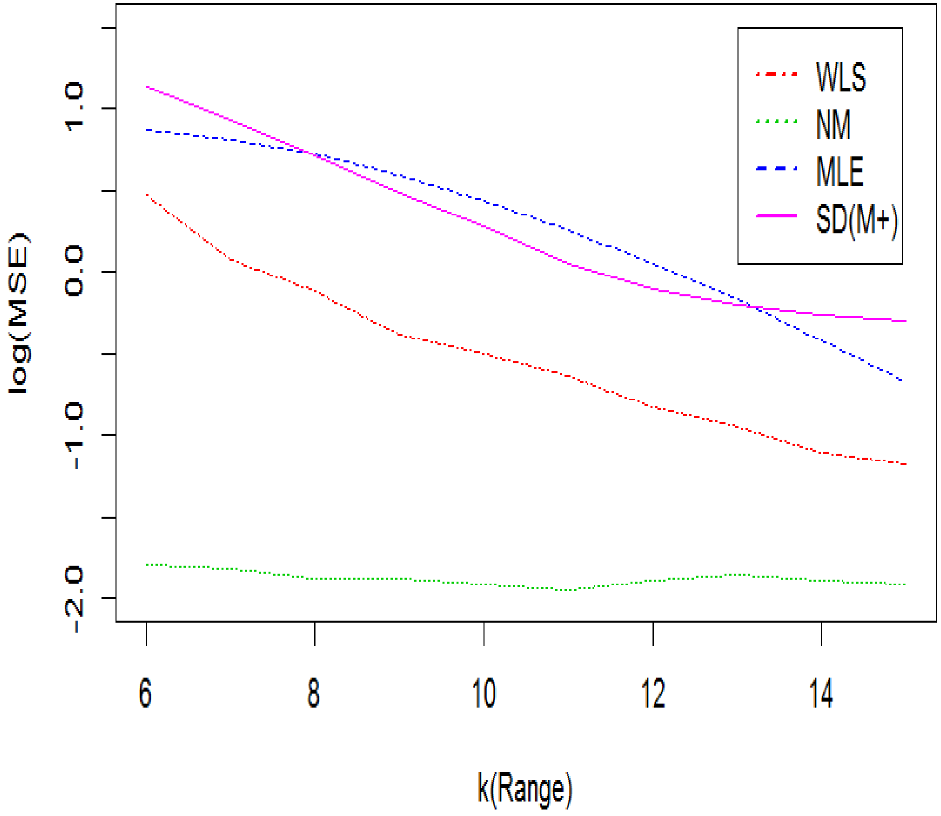
Covering Probability	Estimation Methods			
k	WLS	NM	MLE	SD(M+)
6	0.961	0.951	0.953	0.954
8	0.941	0.947	0.937	0.951
10	0.957	0.952	0.940	0.960
12	0.963	0.955	0.943	0.967
14	0.953	0.944	0.917	0.969

Estimation of Standard Deviation (σ=10)

Bias



MSE



Coverage Probability of σ=10

Coverage Probability	Estimation Methods			
k	WLS	NM	MLE	SD(M+)
6	0.951	0.939	0	0
8	0.950	0.955	0.001	0
10	0.956	0.948	0.003	0.234
12	0.956	0.951	0.018	0.735
14	0.961	0.952	0.115	0.899

Empirical Studies

- We use the data of 8 countries from Human Mortality Database (HMD) to evaluate the mortality compression.

→Compare NM & SD(M+), k=10

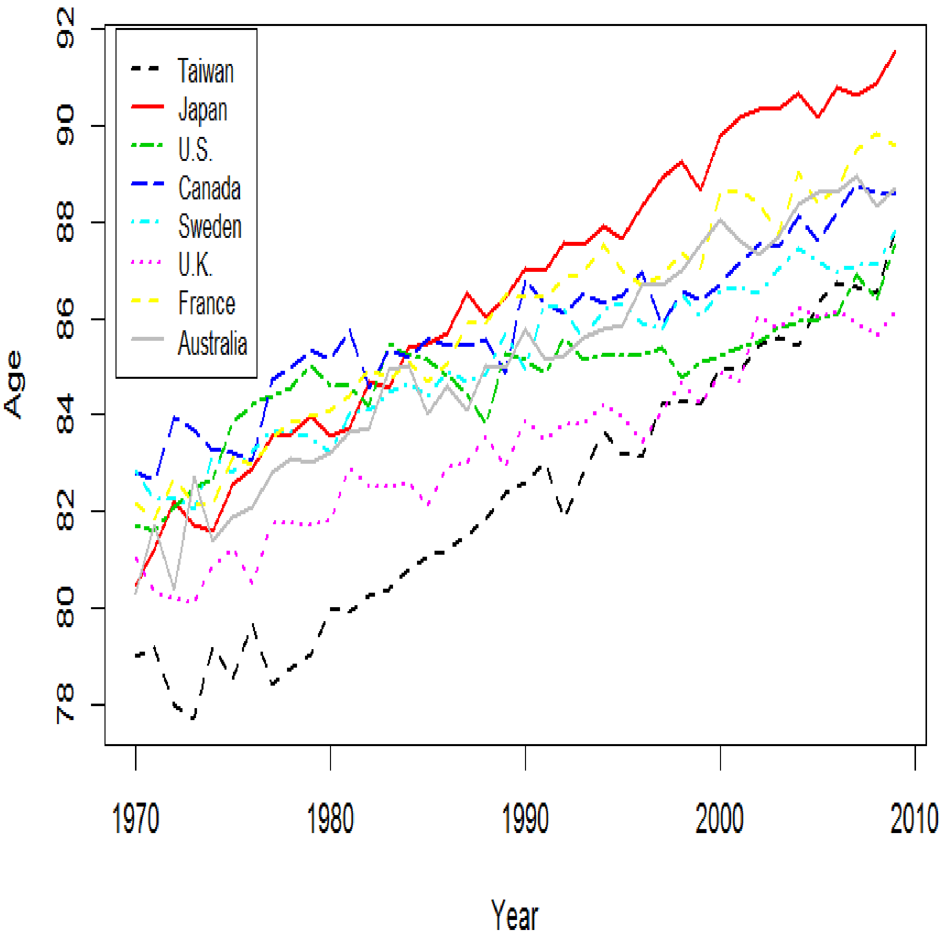
→Modal age M

→Standard deviation σ

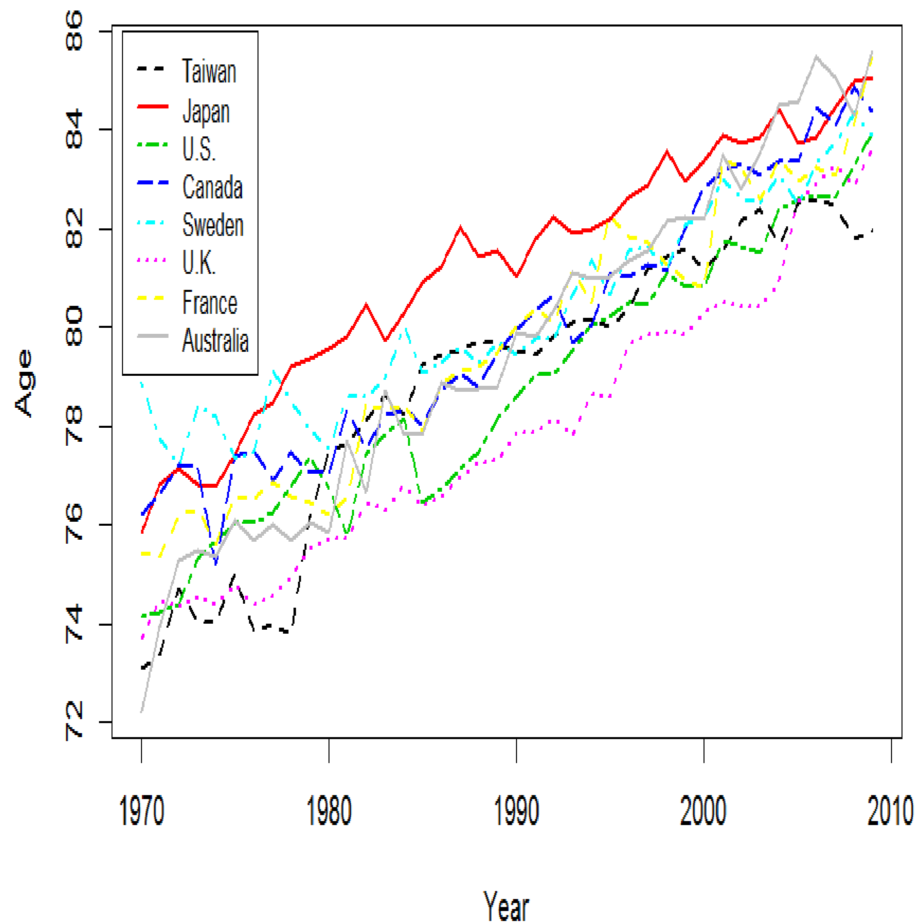
→95-percentile (P95), or $\Phi(1.645) = 0.95$

The Estimate of M

Female

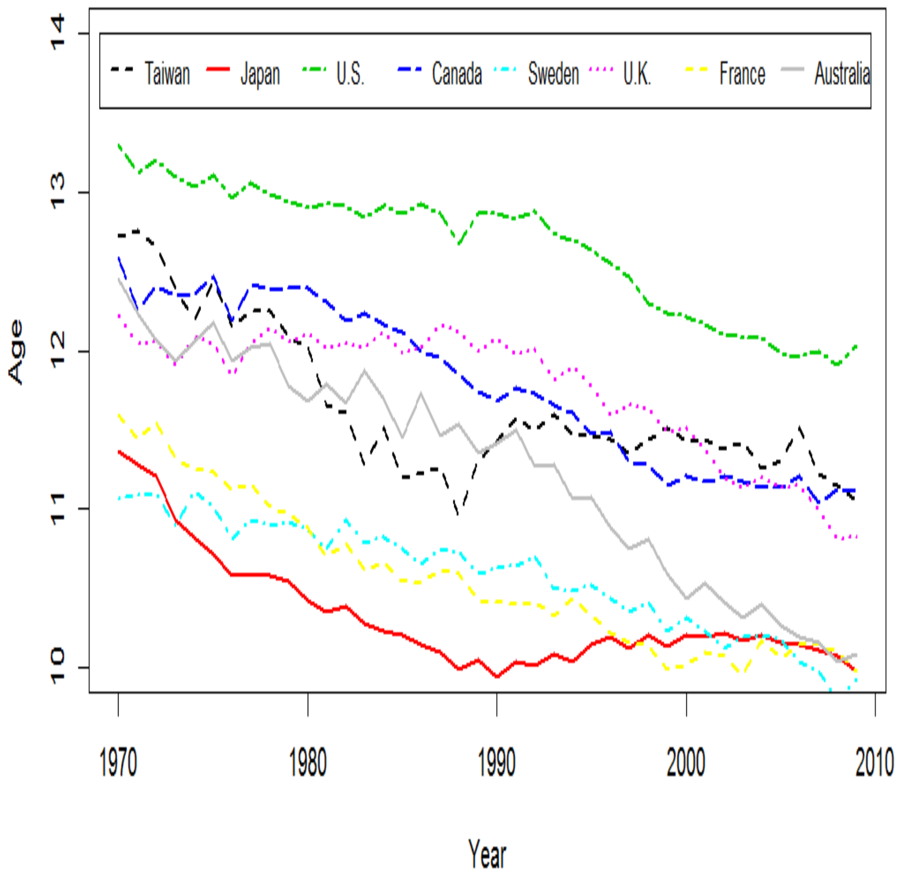


Male

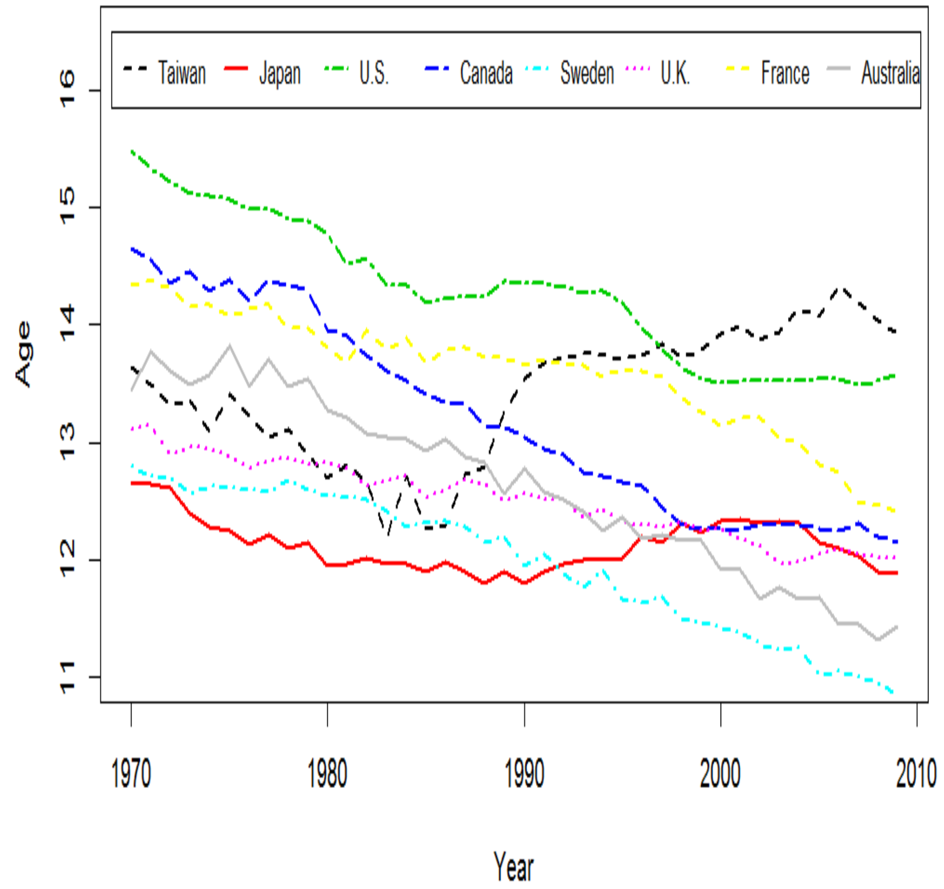


The Estimate of σ (NM)

Female

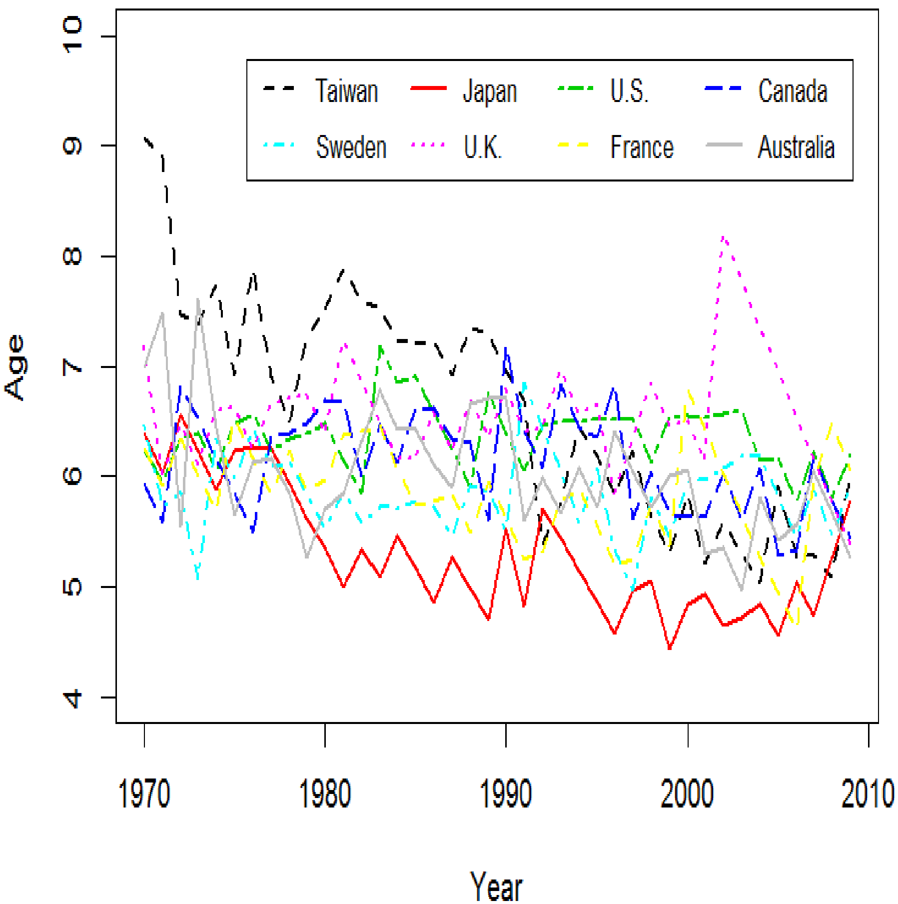


Male

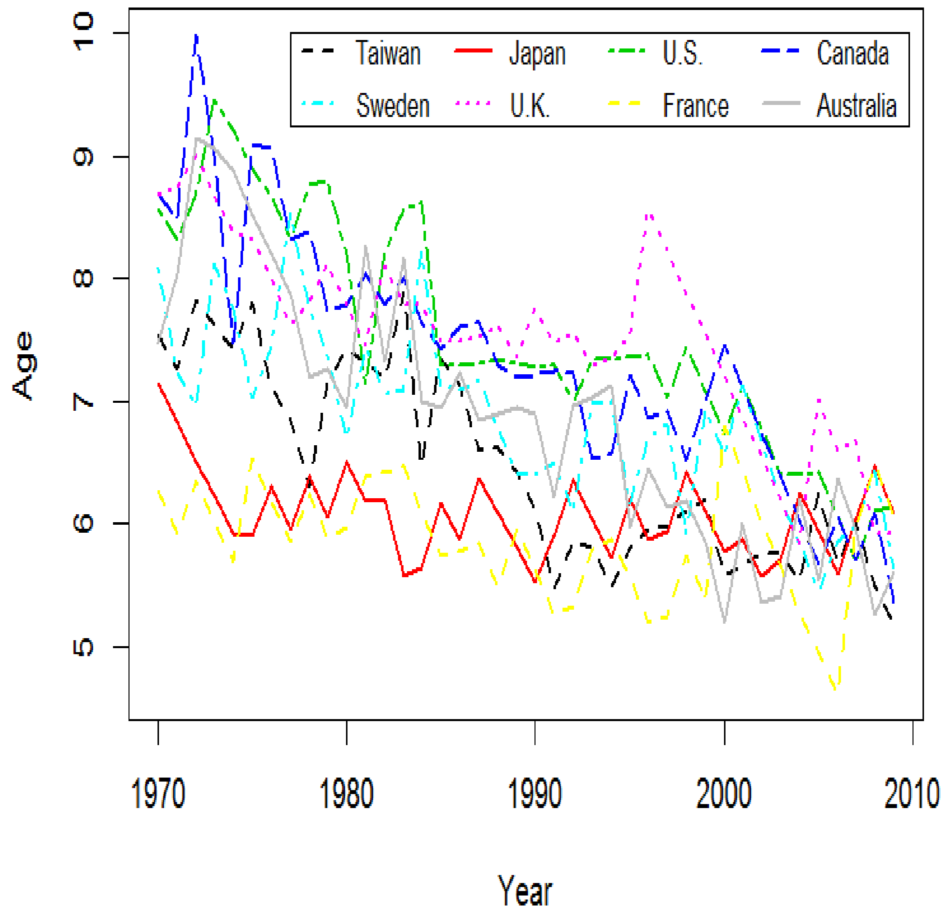


The Estimate of σ (SD(M+))

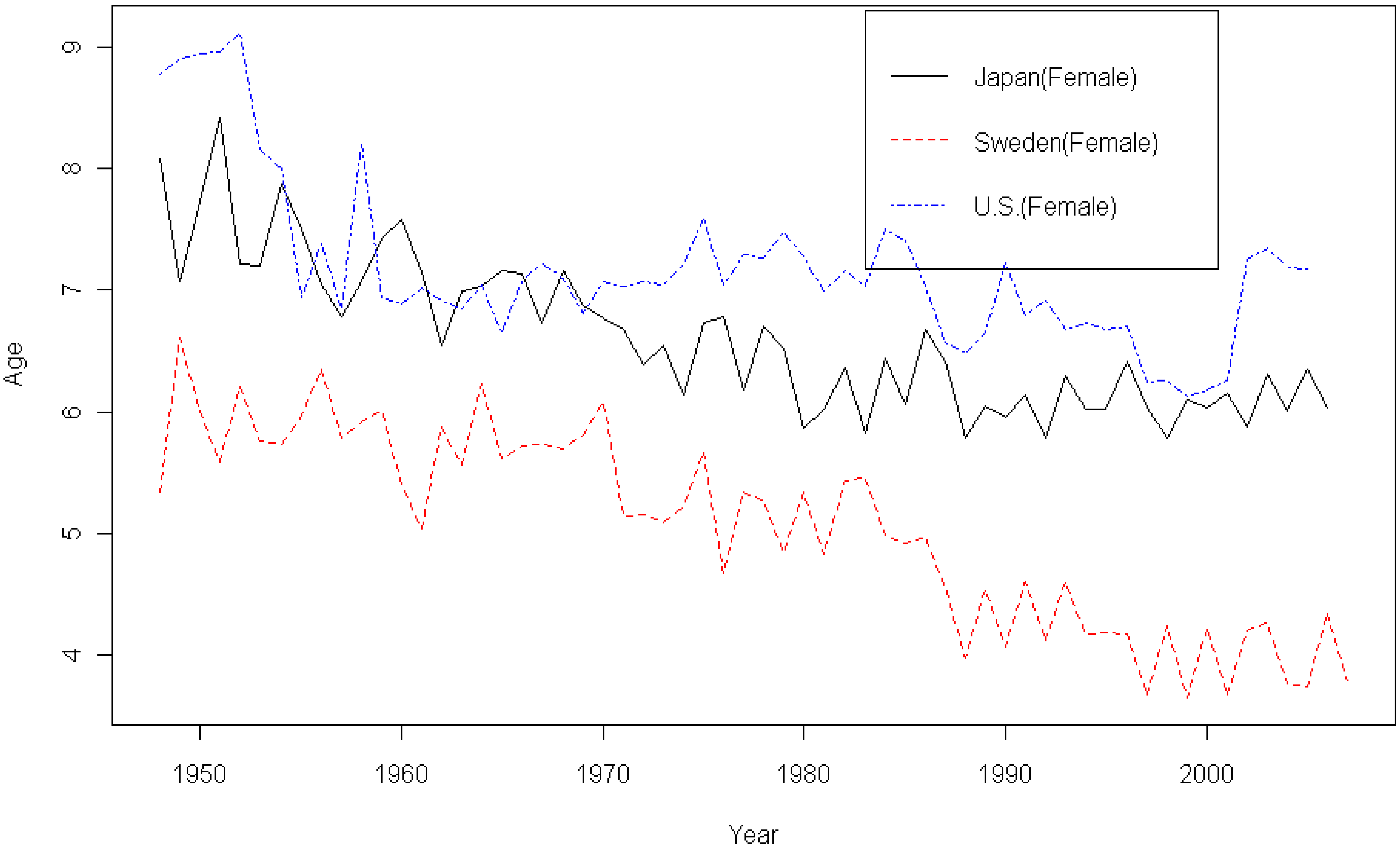
Female



Male

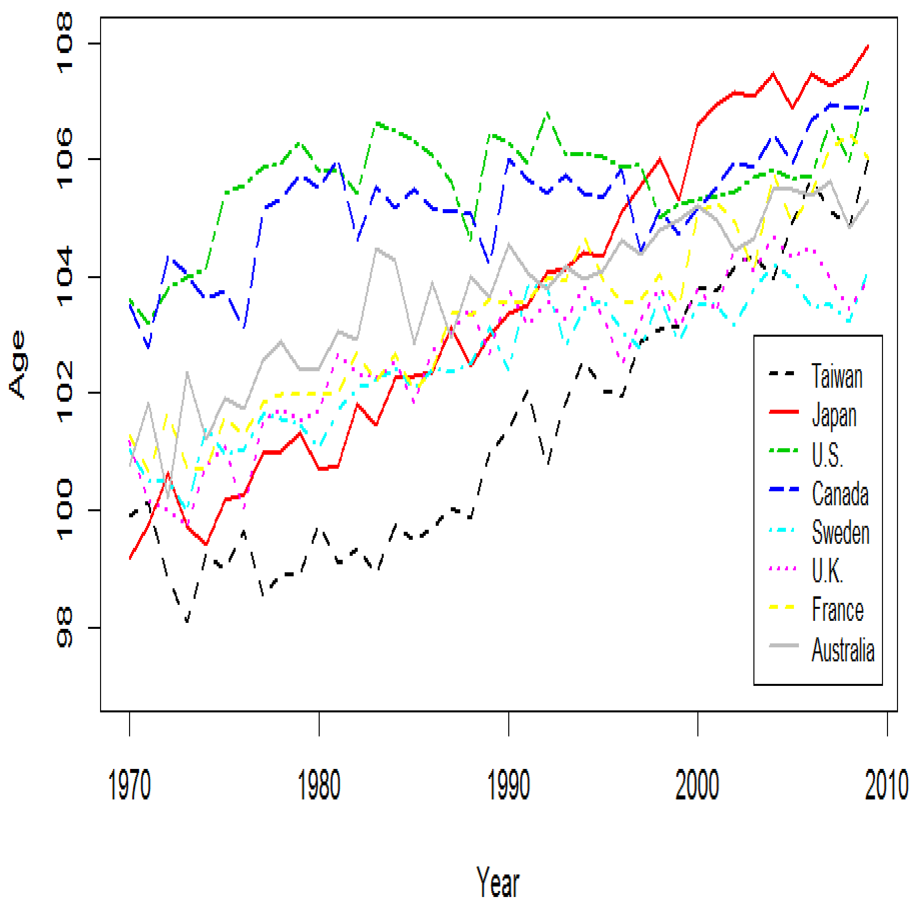


The Estimate of σ (WLS; Yue, 2012)

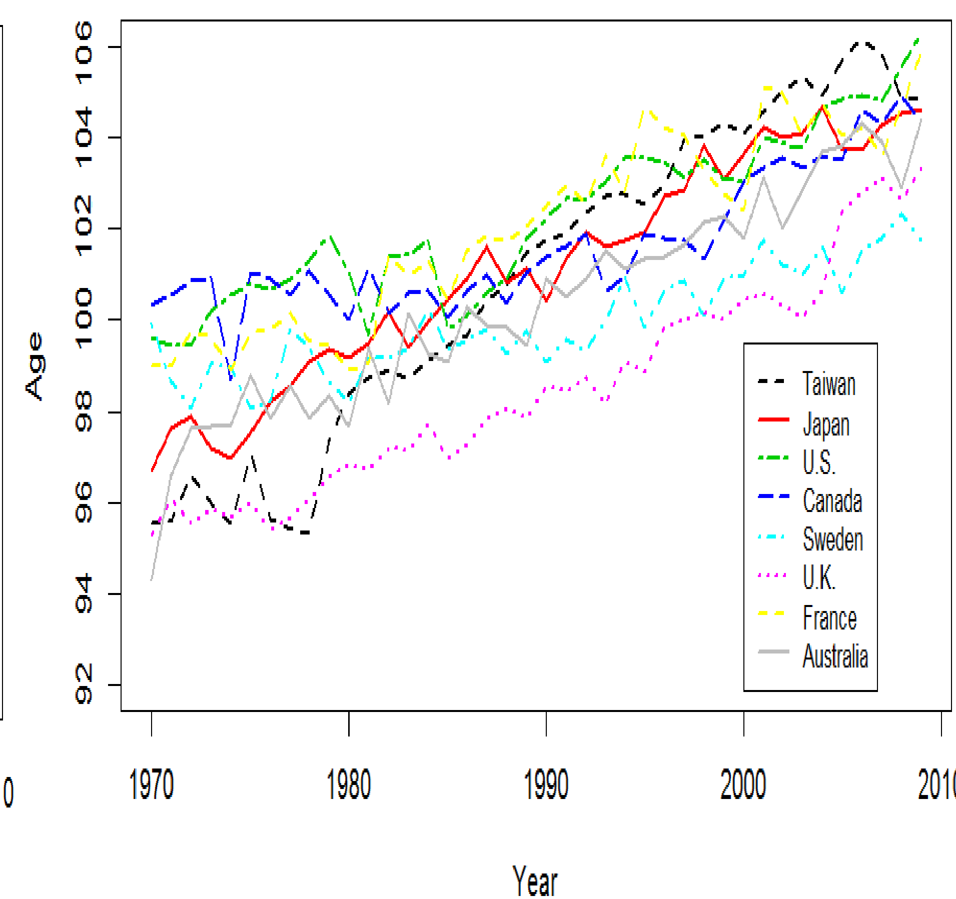


The Estimate of P95 (NM)

Female



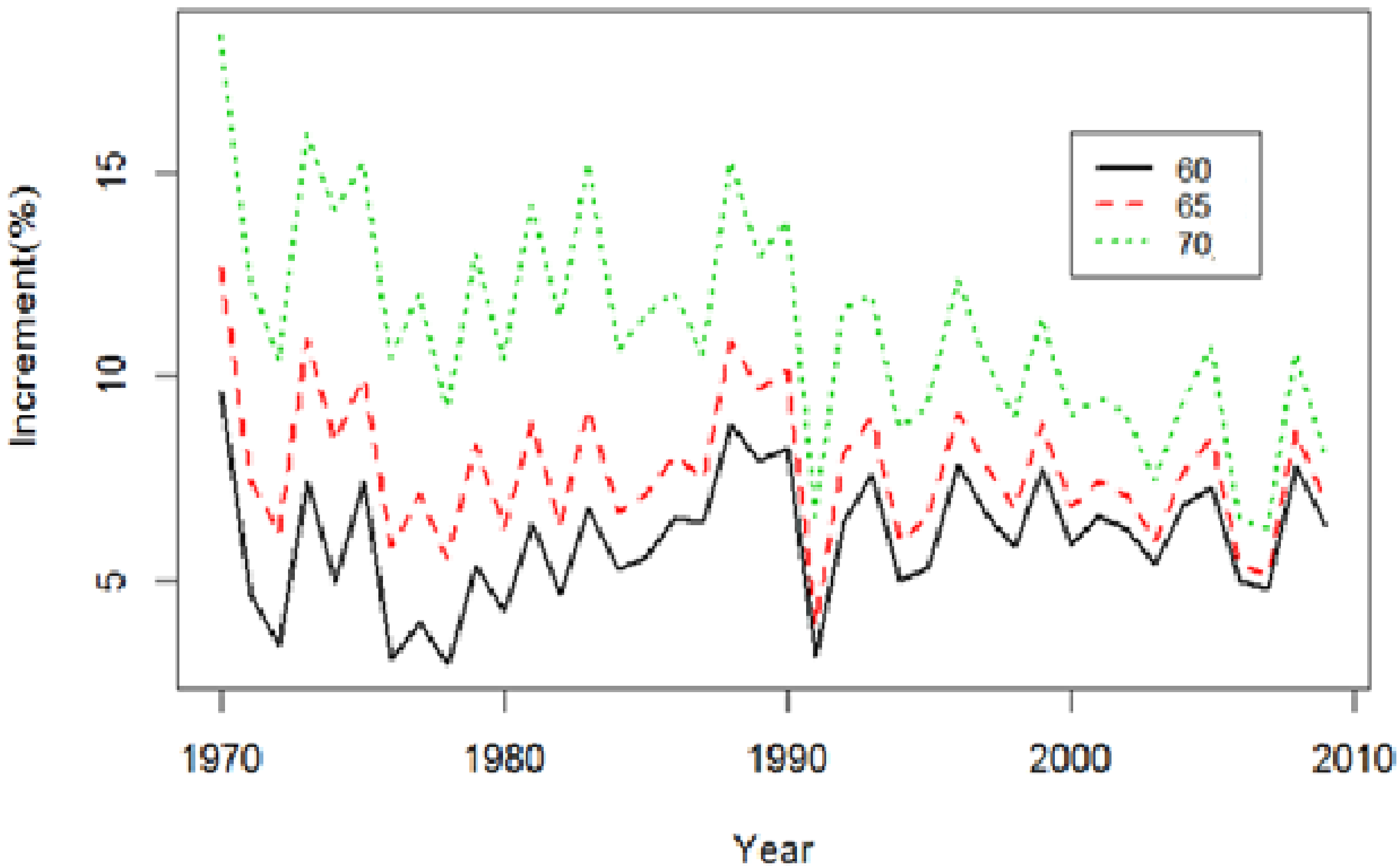
Male



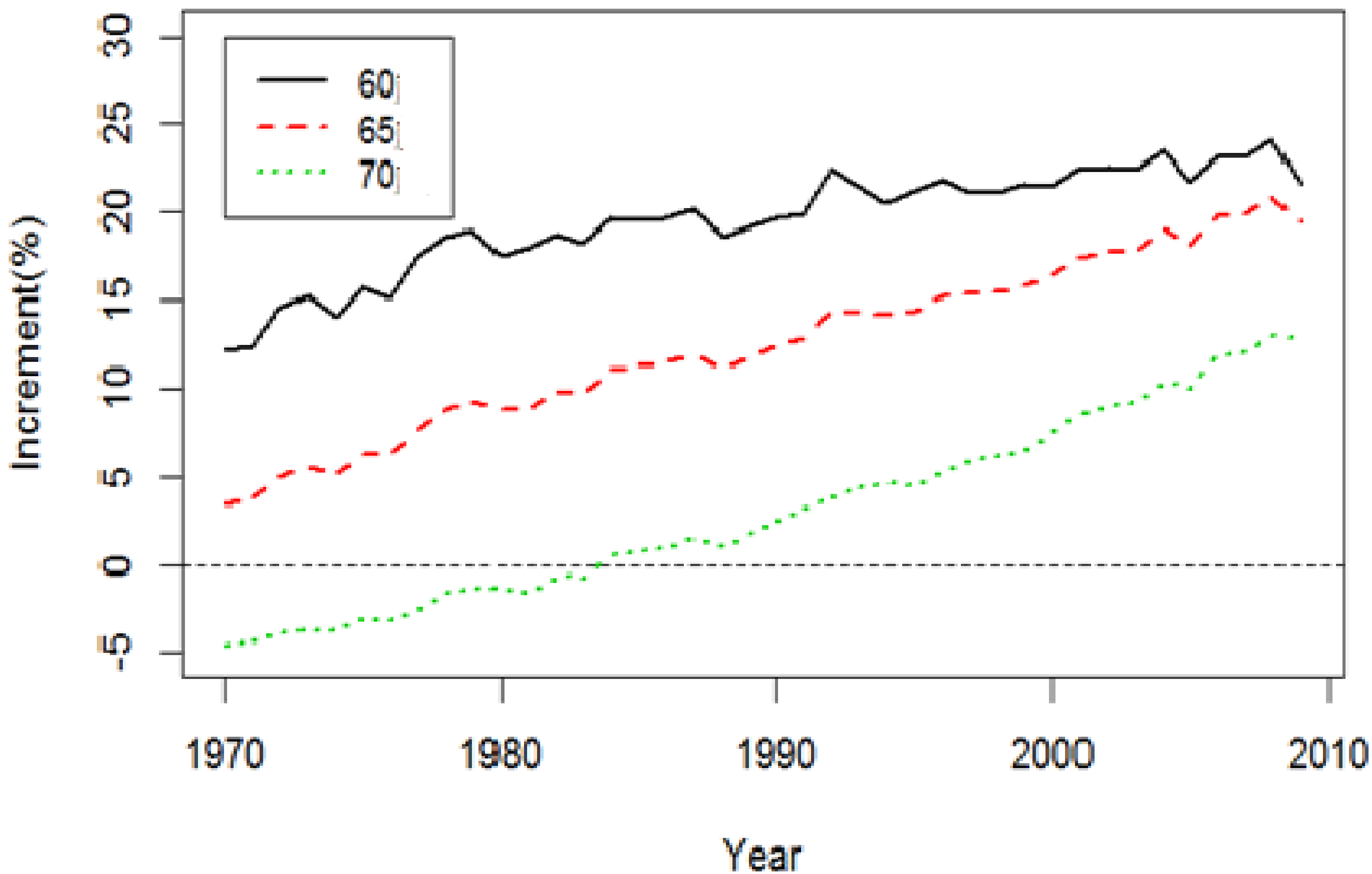
Applications

- We use the normal distribution N(M, σ^2) to approximate the pure premium of whole life annuity products.
  →Issue ages: 60, 65, and 70
  →Interest rate 2.5%
  →Compare the differences of pure premium and its variance, using the raw data and the mortality rates derived via normal approximation.
  Note: Estimate M and σ by the raw data.

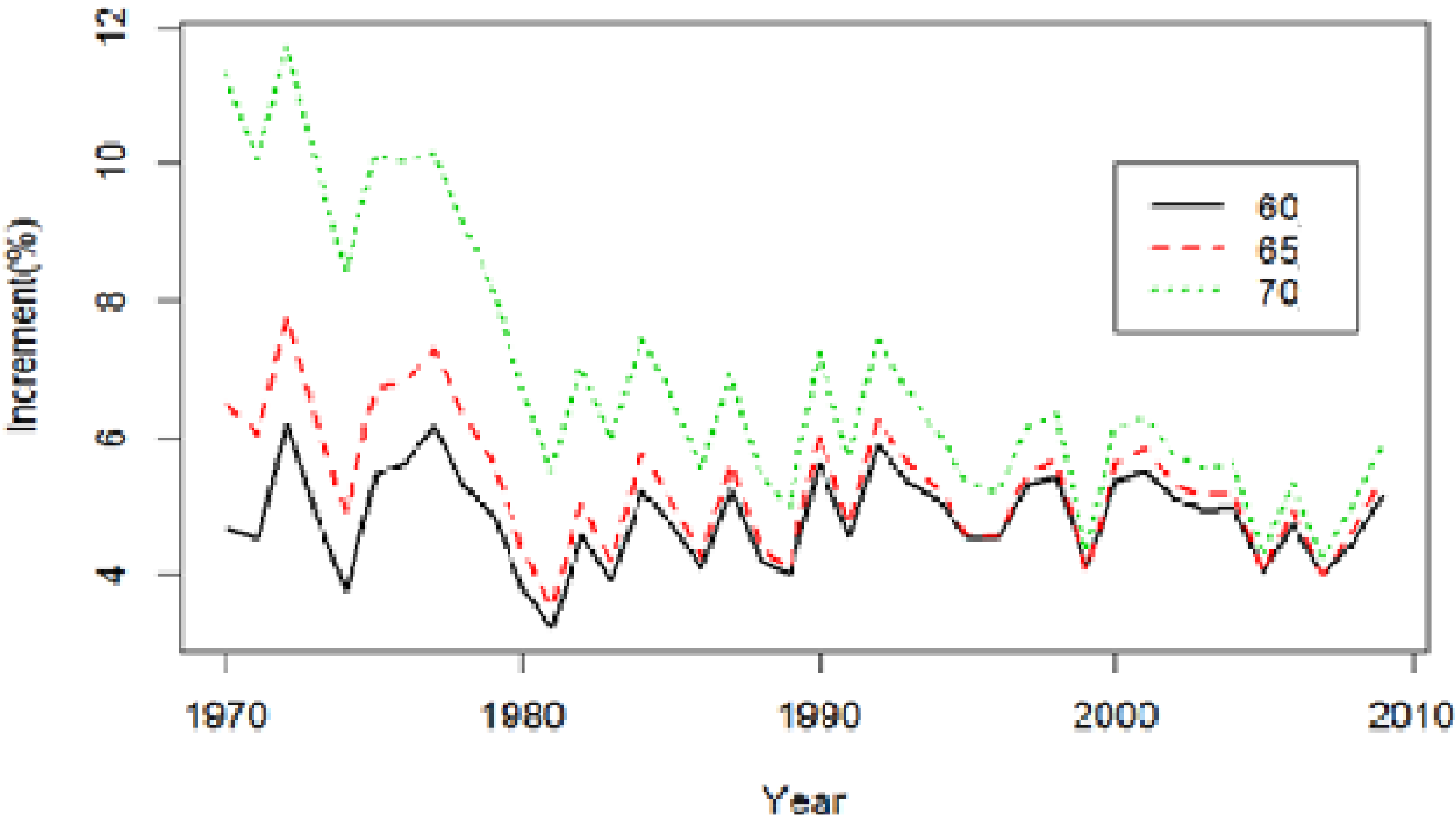
Differences in Pure Premium (Taiwan Female)



Variance of Pure Premium (Taiwan Female)



Differences in Pure Premium (Japan Female)



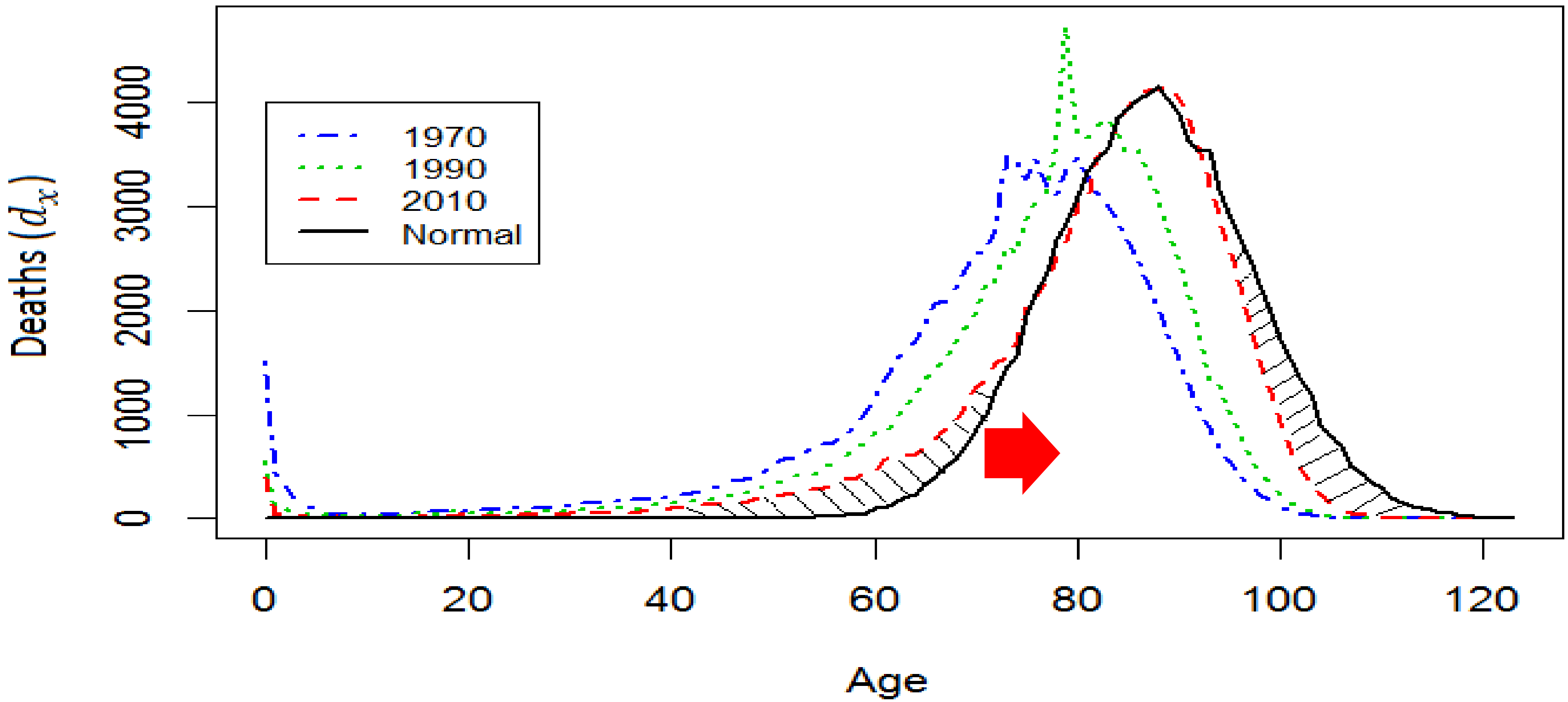
Normality Test (Permutation Test)

Group	1	2	3	4	5	6	7	8
Ei	11504	12690	10262	11560	15778	10786	13856	13563

Note: The expected numbers are based on 10,000 simulation runs.

Normality Test (Taiwan Female)

Year	1	2	3	4	5	6	7	8
2007	24232	11093	11378	13046	13570	11771	10116	4792
2008	23977	11085	11321	13176	13576	11859	10197	4814
2009	23104	10682	11062	13111	13757	12140	10744	5399



Conclusions and Discussions

- The proposed approaches are more reliable in estimating the modal age M and σ.

→NM and WLS are preferred.

→The mortality compression is still not clear, using the NM or SD(M+) method.

- Applying the normality approximation to compute the pure premiums of annuities.

→Normal approximation is over-biased and can serve as an upper bound.

Conclusions and Discussions (Conti.)

- Judging from the results of empirical studies, the normal assumption is questionable.

→ The estimation methods are influenced by the distribution assumption, and there is still no conclusion about the mortality compression.

- Using the standard deviation as the measure of mortality compression is not feasible.

→ Are there alternative measures, other than the proposed measures, such as the IQR and percentile.

Conclusions and Discussions (Conti.)

- The probability of surviving to very high age cannot be ignored and we are not sure if the life expectancy has a limit. (Longevity Risk!)

→The normal approximation might serve as an upper bound for pricing annuity products.

- Some possible future study topics:

→Modify the idea of mortality compression and apply it in dealing with longevity risk.

→Combing the extreme value theory into the study of mortality compression.


Thank you!