L21 – LONGEVITY RISK AND CAPITAL MARKETS SOLUTIONS CONFERENCE

Measuring the Effectiveness of Long-Term Care Integration within Pension Systems

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Agenda

1. Motivation & Research Question
2. A Policy Proposal
3. The Stochastic Model
4. Calibration
5. Measuring Effectiveness: Results
6. Policy Implications & Conclusions

Core question: does forgoing part of a pension annuity deliver LTC protection that is actually adequate – and for whom?

1 Motivation & Research Question

The Long-Term Care Protection Gap

- Life expectancy keeps rising, but **healthy** life expectancy does not rise proportionally.
- A growing share of later life is spent in a state of **LTC dependency** – more markedly for women.
- Ageing puts sustained pressure on public healthcare and pension systems.
- LTC risk remains **one of the least insured risks** in Europe: private supply is thin, public systems are financially constrained.

The Behavioural Barrier: the "Survival Gap"

- Uptake is not just a supply problem – it is also a **perception** problem.
- Apicella & De Giorgi (2024) document a systematic gap between *subjective* and *objective* probabilities of health shocks:
  - Younger individuals **overestimate** their risk;
  - Older individuals **underestimate** it, despite facing it more often.
- The gap widens as **health literacy** falls – precisely where LTC protection is needed most.
- Financial literacy compounds the problem: complex, discretionary LTC premiums are perceived as an avoidable expense.

Empirical Evidence: LTC Risk and Economic Vulnerability

- The protection gap is not evenly distributed: health indicators associated with LTC dependency worsen sharply as **economic difficulty** increases.
- Older individuals **lacking sufficient resources** are systematically less able to manage the onset of disability, and the effect is most severe among **disadvantaged social groups**.

Health / frailty indicator	No difficulty	Some difficulty	Severe difficulty
Self-perceived poor health	7%	15%	29%
Disability (not indep., $\geq$1 ADL)	10%	18%	27%
Frailty (not indep., $\geq$2 IADL)	13%	22%	28%
Depressive symptoms	7%	17%	31%
Falls (past 30 days)	6%	9%	15%
Use of $\geq$4 medications	33%	44%	54%
Social isolation	13%	21%	30%

Every indicator worsens monotonically with economic difficulty – the population most exposed to LTC risk is also the least equipped, financially, to self-insure against it.

Research Question & Contribution

Can a pension-linked LTC option, funded through workers' existing severance-pay contributions, provide **measurably adequate** protection against LTC costs – and can that adequacy be quantified directly?

- We build a **stochastic evaluation framework** linking pension accumulation, annuitisation and LTC coverage through one parameter: the **Benefit Reduction Rate** q.
- We turn "adequacy" into a **measurable, probabilistic** object: the probability that the LTC annuity covers a given cost threshold, as a function of q.
- We apply the framework to Italy's **TFR** (severance pay) as the natural contribution vehicle.

2 A Policy Proposal: TFR-Linked LTC Cover

Why Severance Pay?

- **TFR** (*Trattamento di Fine Rapporto*): Italy's deferred severance-pay scheme.
- Accrues at **6.91%** of gross annual salary per year of service, revalued annually.
- Already earmarked for the future, **not** part of current consumption decisions.
- Can be channelled into a **pension fund with an LTC option** instead of a standard annuity.

6.91%

OF GROSS ANNUAL SALARY

Three Frictions this Design Addresses

Health Literacy

LTC is perceived as unfairly costly. A deferred, already-set-aside contribution bypasses that mental accounting.

Financial Literacy

TFR is illiquid by default; the LTC choice is deferred to retirement and paid for only via foregone pension income.

Data Availability

Broader, near-universal enrolment improves the technical bases, lowering the safety loading on LTC cover.

Open question: are *average* TFR contributions actually enough to fund an adequate LTC annuity? This is exactly what the model below is built to measure.

3 The Stochastic Model

Modelling Framework – Building Blocks

- **Interest-rate process** – term structure of risk-free yields.
- **Pension fund process** – accumulation from contributions and benchmark returns.
- **Contribution rule** – periodic TFR-based payments during working life.
- **Benchmark process** – market dynamics driving fund performance.
- **Demographic assumptions** – multi-state mortality/morbidity bases.
- **Conversion factors** – turning accumulated capital into a standard or LTC-linked annuity.

All elements are combined via **Monte Carlo simulation** (10,000 trajectories).

Demographic Basis: a Three-State Model

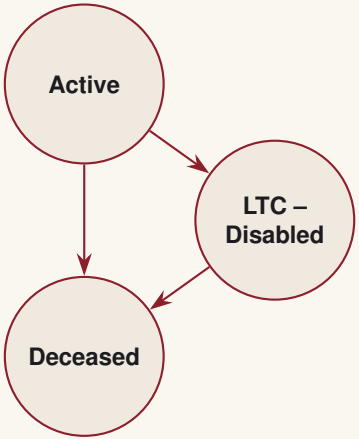

Figure 1 – Multi-state model adopted

- Three states: **Active**, **LTC-Disabled**, **Deceased**.
- No reactivation from LTC-disability – consistent with its chronic nature.
- Transitions drive both the annuity conversion factors and the LTC-need probability.

Interest-Rate Dynamics: the CIR Process

$$dr_t = \kappa(\theta - r_t)\,dt + \sigma\sqrt{r_t}\,dW_t$$

- Cox–Ingersoll–Ross (1985) process: mean-reverting, non-negative short rate.
- κ = speed of mean reversion θ = long-run mean σ = volatility.
- Drives the deterministic yield curve used to discount pension and LTC annuities.

Pension Fund Accumulation

Fund value

$$V_t = \begin{cases} P_0 & t = 0 \\ P_t + V_{t-1}\,(1 + j_{t-1}) & t > 0 \end{cases}$$

Contribution rule

$$P_t = \begin{cases} P_0 & t = 0 \\ P_{t-1}\,(1 + y_t) & t > 0 \end{cases}$$

- j_{t-1}: stochastic annual benchmark return (passive management, e.g. FTSE MIB).
- y_t: contribution growth rate, calibrated on TFR / wage dynamics.
- Defined-contribution logic: no fees or transition costs assumed, for tractability.

From Capital to Annuity: the LTC Option

- **No option exercised:** standard actuarial equivalence

$$S = R\left(a_x^{aa} + a_x^{ai}\right) \qquad \Rightarrow \qquad \tau = \frac{1}{a_x^{aa} + a_x^{ai}}, \qquad R = S\,\tau$$

- **Option exercised:** two distinct annuities, active (R_a) and LTC (R_i)

$$S = R_a\, a_x^{aa} + R_i\, a_x^{ai}$$

a_x^{aa}: annuity while remaining self-sufficient | a_x^{ai}: deferred annuity paid once LTC-dependent

Deriving the LTC Option from Actuarial Equivalence

- Starting from the **actuarial equilibrium** condition, with S the capital accumulated at retirement:

$$R_a\, a_x^{aa} + R_i\, a_x^{ai} = S = \left(a_x^{aa} + a_x^{ai}\right) R \quad \Rightarrow \quad R = \frac{S}{a_x^{aa} + a_x^{ai}} = S\,\tau$$

- Exercising the option means forgoing a fraction $q \in [0,1]$ of the pension benefit R that would otherwise be paid, in exchange for LTC coverage. The active-phase benefit becomes

$$R_a = R(1-q) = S\,\tau(1-q) = S\,\tau_a$$

- Re-injecting R_a into the equilibrium condition yields the LTC-phase benefit and the two conversion factors used throughout the model:

$$R_i = \frac{\left(a_x^{aa} + a_x^{ai}\right) R - R_a\, a_x^{aa}}{a_x^{ai}} = S\,\tau_i, \qquad \tau_i = \frac{a_x^{ai} + q\, a_x^{aa}}{\left(a_x^{aa} + a_x^{ai}\right) a_x^{ai}}, \qquad \tau_a = \frac{1-q}{a_x^{aa} + a_x^{ai}}$$

The Benefit Reduction Rate q

- $q \in [0,1]$: the share of the active-phase annuity the retiree forgoes to fund LTC cover.

$$R_a = R(1-q) = S\,\tau_a, \qquad \tau_a = \frac{1-q}{a_x^{aa} + a_x^{ai}} \qquad R_i = S\,\tau_i, \qquad \tau_i = \tau\left(1 + q\,\frac{a_x^{aa}}{a_x^{ai}}\right)$$

$q = 0$: no LTC coverage – $R_a = R_i = R$, identical to the base annuity.

$q = 1$: equivalent to a stand-alone LTC policy – $R_a = 0$.

Sensitivity of the Two Annuities to q

$$\frac{dR_a}{dq} = -\tau S < 0 \qquad \frac{dR_i}{dq} = \frac{a_x^{aa}}{a_x^{ai}}\,\tau S > 0$$

- Since $a_x^{aa} > a_x^{ai}$ (staying active is more likely than becoming disabled at any age),
- a marginal increase in q produces a **smaller** drop in the active-phase annuity than the corresponding **gain** in the LTC annuity.
- This asymmetry is precisely what makes even a modest q potentially effective – and it is what we quantify next.

4 Calibration

Calibrating the Interest-Rate Process

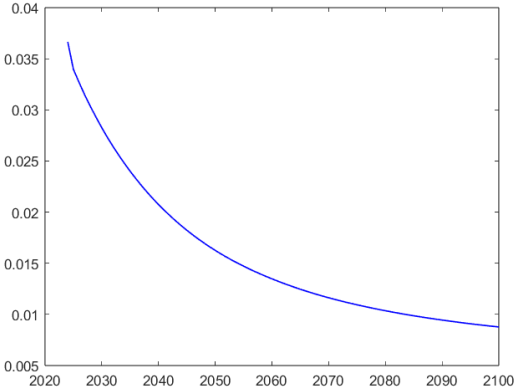

Figure 2 – Simulated yield curve, 2024–2100

Parameter	Value
θ	0.0031^*
κ	0.0833
σ	0.047

1-year Euribor, 1994–2021 (OLS); 10,000 paths, 2024–2100.

Calibrating the Benchmark: GBM on the FTSE MIB

$$dI_t = I_t \, \mu \, dt + \sigma' \, dW_t$$

Parameter	Value
μ	0.0299
σ'	0.2215

- Geometric Brownian Motion: standard, tractable benchmark for equity-linked funds.
- Estimated on FTSE MIB annual levels, 1998–2024.
- Passive management: fund performance replicates the index.

The Demographic Basis: LTC15 Tables

$$p_x^{ij}(2024) = \sum_{k \in S} \alpha_k \, p_x^{ij,k}(2024), \qquad S = \{L, C, H\}$$

- Source: De Angelis & Di Falco (2016), based on INPS disability-benefit data (2002–2012); projected 2013–2043 via an extended Lee–Carter model.
- Three scenarios blended – Low, Central, High mortality/morbidity – weighted **50%** / **30%** / **20%** (extra weight on the High scenario as a post-COVID prudence adjustment).
- Gender-specific tables are combined 50/50 into a **unisex** basis, in line with EU Gender Directive requirements.

Validating the Model Against Market Premiums

- The calibrated CIR model and demographic assumptions build the **fair single premium** of a stand-alone LTC policy (2024 reference year): 10,000 interest-rate paths are simulated, each yielding a single-premium value, and their **expected value** is the model-implied fair premium.

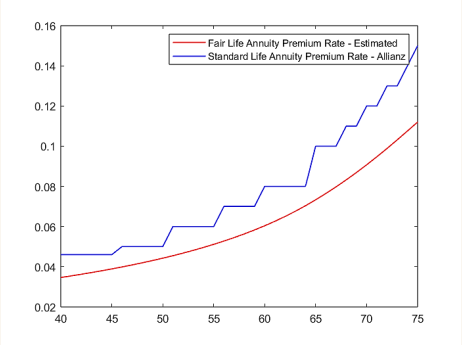

Figure – Model-implied fair premium rate vs. a standard market rate (Allianz), by age at entry

- Our fair-value estimate sits consistently **below** the observed commercial rate, as expected once expense and profit loadings are stripped out.
- The two curves share the same **shape and slope** across ages 40–75, supporting the calibration before it is used downstream.

TFR Contribution Dynamics

$$g_x = \prod_t \left(1 + g_x^{(t)}\right)^{1/n} - 1$$

Age-related wage growth

(geometric mean across observation years)

$$g_t = \prod_x \left(1 + g_t^{(x)}\right)^{1/n} - 1$$

Time-related wage growth

(geometric mean across ages)

- ISTAT hourly-wage data by age band, 2014–2021, interpolated to single ages.
- Combined growth rate applied to project TFR contributions (y_t in the fund equation):

$g_x = 0.636\%$	$g_t = 0.69\%$	$g = 1.33\%$
AGE EFFECT	TIME EFFECT	COMBINED GROWTH

5 Measuring Effectiveness: Results

Case Study: a 46-Year-Old Worker in 2024

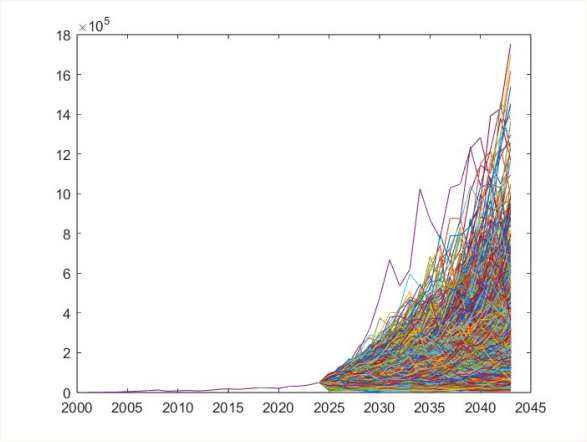

Figure 3 – Simulated fund paths, 2001–2043 (10,000 trajectories)

- Career: 2001 (age 23) to retirement in 2043 – set by the demographic-data horizon.
- Full-time career; TFR = 6.91% of gross salary, revalued via g.
- 10,000 simulated trajectories.
- Terminal fund value is, as expected, approximately **log-normal**.

LTC Cost Benchmarks

Home caregiver

€ 1,500 /mo

CCNL Lavoro Domestico wage bands, incl. board & lodging

Nursing home

€ 3,500 /mo

Hospitality component; healthcare part largely covered by NHS

These two thresholds ($\bar{R}_i$ = €1,500 and €3,500) anchor the effectiveness measure that follows.

Turning "Adequacy" into a Measurable Quantity

$$\mathbb{P}\left(R_i < \bar{R}_i\right) = \mathbb{P}\left(S\,\tau\left(1 + q\,\frac{a_x^{aa}}{a_x^{ai}}\right) < \bar{R}_i\right) = \mathbb{P}\left(S < \frac{\bar{R}_i\,a_x^{ai}}{\tau\left(a_x^{ai} + q\,a_x^{aa}\right)}\right)$$

- Given the simulated distribution of S, for any target percentile α we solve for the q satisfying

$$\mathbb{P}\left(S < \frac{\bar{R}_i\,a_x^{ai}}{\tau\left(a_x^{ai} + q\,a_x^{aa}\right)}\right) = \alpha, \qquad q \in [0,1]$$

- This is our **effectiveness metric**: the smallest sacrifice q needed for a given confidence level that the LTC annuity will cover the cost threshold $\bar{R}_i$.

Effectiveness in Pictures

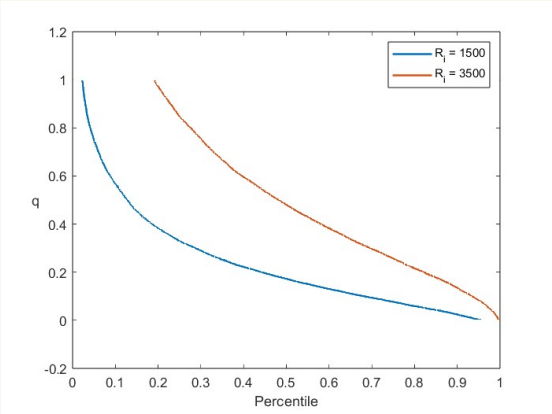

Figure 4 – Benefit Reduction Rate *q* vs. probability of *not* covering LTC costs

Reading the chart: for any confidence level on the horizontal axis, the vertical axis gives the *q* required to cover home-care costs (blue line) or residential-care costs (orange line).

Effectiveness in Numbers

Confidence level α	q (home care, €1,500)	q (nursing home, €3,500)
0.5% – 1%	n.a.	n.a.
5%	0.7439	n.a.
10%	0.5530	n.a.
25%	0.3281	0.8440
50%	0.1669	0.4748

- Forgoing **74.4%** of the annuity caps home-care shortfall risk at 5%; **84.4%** does the same for residential care at 25%.
- At full commitment ($q = 1$): 2.1% residual shortfall risk for home care, **18.9%** for RSA care.
- A “moderate” choice ($q = 0.3$) still leaves **28.1%** / **68.9%** shortfall risk – effectiveness is highly sensitive to how much is sacrificed.

6 Policy Implications & Conclusions

Key Findings

- Home-care costs can be covered with a **reasonable confidence** by forgoing a large – but not extreme – share of the complementary pension annuity.
- **Residential care** is materially harder to cover: even full commitment ($q = 1$) leaves close to a 1-in-5 shortfall probability.
- The asymmetric sensitivity of R_a and R_i to q means the trade-off is **non-linear** – early increments in q buy more protection per euro forgone than later ones.

The Equity Concern: an Average Is Not a Guarantee

The wage data underlying TFR contributions are **population averages**. Adequacy "on average" does not imply adequacy for everyone.

- Workers in the **lower wage tail** accumulate proportionally smaller TFR funds – the model's conclusions do not extend automatically to them.
- Lower wages correlate with **lower education**, which in turn correlates with lower health literacy and less preventive care.
- Result: those most exposed to LTC risk are also the **least well served** by a purely individual, income-linked mechanism.

Policy Implications

Solidarity

A welfare layer built on mutuality is needed to rebalance coverage for lower-income, higher-risk workers.

EFL

Economic and Financial Literacy programmes to make the q trade-off legible to the average worker.

Health Literacy

Closing the survival gap is a precondition for rational uptake of the LTC option.

Limitations & Future Research

- Deterministic (rather than jointly stochastic) interest-rate/benchmark dependence – a copula-based extension would allow a fully stochastic, fair-value analysis.
- Single representative worker profile; a natural next step is to run the effectiveness metric across the **full wage distribution**, not just the average.
- Demographic bases predate COVID-19 and are only partially adjusted for it.
- Extending q to a **dynamic, life-cycle** choice rather than a one-off retirement decision.

Conclusions

- We proposed and quantified a **TFR-funded, pension-linked LTC option**, parameterised by a single, interpretable Benefit Reduction Rate q.
- The probabilistic effectiveness metric shows coverage is **achievable but costly** for home care, and **harder** to guarantee for residential care.
- Average adequacy masks **distributional** risk: the workers most exposed to LTC risk are the least protected by an income-linked design.
- A robust solution needs the model's actuarial precision **plus** a solidarity mechanism and stronger financial/health literacy.

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Thank you

Questions & discussion welcome

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