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How Persistent is Volatility? An Answer with Markov Regime Switching Stochastic Volatility Models

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How Persistent is Volatility? An Answer with Markov Regime Switching Stochastic Volatility Models

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Abstract

We use stochastic volatility models with Markov regime changing state equation to investigate the important properties of volatility, high persistence and smoothness. With the quasi-maximum likelihood approach, we show that volatility is far less persistent and smooth than the GARCH or stochastic volatility models suggest. Our further analysis shows that volatility is less persistent in high volatility regimes than in low volatility regimes. These results indicate that the extreme persistence we frequently observe from daily index returns is led by structural breaks as well as persistent low volatility regimes.

Keywords: Stochastic Volatility, Markov Switching, Persistence.

JEL Codes: C13

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1 Introduction

Volatility represents the degree of unexpected price movement over time. However, it is not observable and many proxy volatility measures have been introduced. One of the most common methods to obtain a proxy measure of volatility is to fit parametric models such as GARCH or stochastic volatility (SV) models, and others include option implied volatility, the intra-day return volatility (Andersen and Bollerslev, 1998) and range volatility (Parkinson, 1980; Garman and Klass, 1980; Alizadeh, Brandt, and Diebold, 2002). In most cases, the results from these parametric or nonparametric methods show that volatility is highly persistent and smooth, and that ex-post squared returns or absolute returns are too noisy to be used as volatility. These results are consistent with the poor forecasting power of GARCH models on ex-post squared returns or absolute returns.

However, we may ask if the persistence and smoothness represent the true nature of volatility. For example, GARCH conditional volatility reflects only lagged information and is not designed to take account of contemporaneous information. If asset returns follow linear factor models such as Fama and French (1993), then there are multiple factors which are not explained by a conditional mean process but become sources of volatility. Therefore a significant amount of squared returns may not be simply noise but come from heteroskedasticity in factors and factor loadings, which is not explained by conditional volatility.[1]

Another econometric question is that the persistence and smoothness obtained with well known volatility models such as GARCH or SV models may come from the restrictive nature of the models. For example, Lobato and Savin (1998), Granger and Hyung (1999) and Diebold and Inoue (2001) suggest that structural breaks in the mean of volatility may be a source of persistence. As a second example, Bollerslev's

[1]Campbell, Lettau, Malkiel, and Xu (2001), for example, using cross-sectional decomposition on equity volatility, show that market and industry volatilities are important components for the explanation of individual asset volatility. Connor and Linton (2001) and Hwang and Satchell (2004) also suggest that there is common heteroskedasticity in asset-specific returns.

(1986) nonnegativity constraints on coefficients on GARCH model may restrict autocorrelation structure of volatility. Nelson and Cao (1992), He and Teräsvirta (1999), and Hwang and Valls (2004a) show some cases that the nonnegativity constraints may cause serious problems.

In our study we use SV models with Markov regime changing state equations (SVMRS) to investigate the questions on persistence and smoothness of volatility. As in most other studies, we assume that the state equation in our SV model follows an AR(1) process.[2] However, the assumption of volatility process being AR(1) processes seems to be too restrictive, if there are structural breaks. Therefore, we allow the state equation in our SV model to Markov regime switch over time. We first allow the level of volatility to change, which is similar to existing models such as Anderson and Lund (1997), So, Lam, and Li (1998), Smith (2002) and Kalimipalli and Susmel (2004). Like those studies, we find that there are multiple levels of volatility and that volatility becomes far less smooth and persistent than those suggested by SV models.

We also allow the persistence level of volatility (i.e., the AR parameter in the state equation) to Markov regime switch over time. Chu and Hwang (2004) analytically shows that not only changing mean but also changing AR parameter increases persistence level in AR(1) processes. Using a series of Monte Carlo simulations Hwang and Valls (2004b) show that the persistence level of GARCH process may be inflated when it changes over time. However their studies do not provide empirical evidence of changing persistence level in volatility. We find evidence of changing persistence level; there are short exploding periods while volatility is far less persistent during most other periods.

Finally, we generalise our model by allowing regime-dependent volatility levels, persistence levels and volatilities of volatility. We find that the squared returns are better specified with this generalised SVMRS model. The generalised SVMRS

[2]Using ARMA processes in the state equation does not change the persistence and smoothness of the volatility process. See Hwang and Satchell (2000) for example.

model also shows that volatility is far less smooth and persistent. Interestingly persistent short regimes are more likely to happen when volatility is low while far less persistence is likely to be observed in high volatility regimes. Therefore a high level of volatility persistence is led by low volatility regimes not by high volatility regimes. We also find that the 'inlier' problem in SV models could be reduced by allowing regime-dependent volatilities of volatility; larger volatilities of volatility are found in low volatility regimes.

Our results suggest that the conventional SV and GARCH models may be too restrictive for squared returns. In addition, a large proportion of transitory noise in SV models decrease significantly in SVMRS models and many cases in our study show that there is little transitory noise in squared returns when SVMRS models are used.

We also show that when a AR(1) process follows regime changes, under certain conditions in the transition probabilities, autocorrelation coefficients may show more persistence than the AR parameter suggests. These results are consistent with those of Granger and Hyung (1999) and Diebold and Inoue (2001) for example.

In the next section we introduce our model. We derive the autocorrelation function of the regime switching AR(1) process and discuss estimation methods and model selection criteria. In section 3 using S&P500 daily and weekly index returns, we show estimates of our SVMRS models and compare them with those of the conventional SV model. Conclusions follow in section 4.

2 Models

2.1 Stochastic Volatility Models with Markov Regime Switching Equation

Stochastic volatility model was introduced by Taylor (1986) and Hull and White (1987) and has been further developed by Harvey and Shephard (1993, 1996) and

Harvey, Ruiz and Shephard (1994). In the SV model, the log of y_t^2, where y_t is typically asset returns (or residuals from a return process), is modelled as a stochastic process:

$$\begin{aligned} y_t &= \varepsilon_t \sigma \exp(\frac{1}{2} h_t), \\ h_t &= \phi h_{t-1} + \eta_t, \end{aligned} \tag{1}$$

where $\varepsilon_t \sim N(0,1)$ which is independent of $\eta_t \sim N(0, \sigma_\eta^2)$, and σ is a positive scale factor. Squaring (1) and taking logs we have a process

$$\begin{aligned} \ln y_t^2 &= \ln \varepsilon_t^2 + \ln \sigma^2 + h_t \\ &= \mu + h_t + \varphi_t, \end{aligned} \tag{2}$$

where $\mu = E[\ln \varepsilon_t^2] + \ln \sigma^2$ and $\varphi_t = \ln \varepsilon_t^2 - E[\ln \varepsilon_t^2]$ is a martingale difference, but not normal. When we replace $\ln y_t^2$ with z_t and $\mu + h_t$ with x_t, the SV model in equations (1) and (2) can be written as

$$z_t = x_t + \varphi_t, \tag{3}$$

$$x_t - \mu = \phi(x_{t-1} - \mu) + \eta_t. \tag{4}$$

SV models are useful to decompose log-squared returns into transitory noise and permanent innovation. This is because the innovation, η_t, matters over time through the AR(1) process, whilst noise, φ_t, does not. Using this concept, Hwang and Satchell (2000) show that squared daily index returns such as FTSE100 or S&P500 consists of 95% of noise and 5% of unobserved innovation (fundamental volatility). This result is asymptotically consistent with the poor forecasting power of GARCH models (see Andersen and Bollerslev (1998)).

Note that the AR(1) process is commonly used in the state equation (volatility process). However, the assumption of the volatility process being an AR(1) process may be too restrictive when there are structural breaks in the level of volatility (see Lobato and Savin (1998), Granger and Hyung (1999), and Diebold and Inoue

(2001)) or in the persistence level (see Chu and Hwang (2004) and Hwang and Valls (2004b)).

It is clear that if there are structural breaks in volatility, the conventional GARCH or SV models are misspecified. In recent years economic time series have been modelled with the assumption that the distribution of the variables is known conditional on a regime or state occurring. The Markov regime switching models introduced by Hamilton (1989) allow the unobserved regime to follow a first order Markov process. The models have been used extensively in macroeconometrics as a means of capturing the different patterns of expected growth in output, see, for example, Goodwin (1993) and Filardo (1994).

The stochastic volatility model can be generalised by allowing the state equation in (4) to follow Markov regime switching processes. Suppose that there is a state variable, s_t. Stochastic volatility model with Markov regime switching state equations is

$$\begin{aligned} z_t &= x_t + \varphi_t \\ x_t &= \begin{cases} \mu_0 + \phi_0(x_{t-1} - \mu_0) + \eta_{0,t}, & \text{when } s_t = 0, s_{t-1} = 0, \\ \mu_0 + \phi_0(x_{t-1} - \mu_1) + \eta_{0,t}, & \text{when } s_t = 0, s_{t-1} = 1, \\ \mu_1 + \phi_1(x_{t-1} - \mu_0) + \eta_{1,t}, & \text{when } s_t = 1, s_{t-1} = 0, \\ \mu_1 + \phi_1(x_{t-1} - \mu_1) + \eta_{1,t}, & \text{when } s_t = 1, s_{t-1} = 1, \end{cases} . \end{aligned} \tag{5}$$

where $\varphi_t \sim N(0, \sigma_\varphi^2)$, $\eta_{i,t} \sim N(0, \sigma_{\eta_i}^2)$ for $i = 0, 1$ and s_t follows a Markov chain. The transition probabilities are given by

$$p^{(i,j)} = \Pr(s_t = i | s_{t-1} = j) \tag{6}$$

and the transition matrix is given by

$$\mathbf{p} = \begin{bmatrix} p^{(0,0)} & 1 - p^{(1,1)} \\ 1 - p^{(0,0)} & p^{(1,1)} \end{bmatrix}. \tag{7}$$

The above SVMRS model has two unobserved variables; x_t which follows different processes according to an unobserved variable s_t. We may allow more states and lags, but the number of cases we should consider for x_t increases rapidly, i.e., $(number\ of\ states)^{lags+1}$.

As in Hwang and Satchell (2000) we treat φ_t as a transitory noise and η_t as a permanent innovation (or volatility process). The treatment provides intuitively interesting perspective since we can decompose any process into innovation and noise which is not explained by the state equation. As pointed out in Andersen and Bollerslev (1998) among many others, if squared returns are too noisy for a proxy volatility process, we need to take out noise from the squared returns and then investigate the remainder to see if there are structural breaks or persistent.

The SVMRS model above is the generalised version of the SV and the Markov regime switching model. By restricting parameters in appropriate ways, we can derive these models;

- If $\sigma_\varphi^2 = 0$, the SVMRS model becomes Hamilton's Markov regime switching model.
- If $\mu_0 = \mu_1$, $\phi_0 = \phi_1$, $\sigma_{\eta_0} = \sigma_{\eta_1}$, and $\sigma_\varphi^2 \neq 0$, then we have the SV model.
- If $\mu_0 = \mu_1$, $\phi_0 = \phi_1$, $\sigma_{\eta_0} = \sigma_{\eta_1}$, and $\sigma_\varphi^2 = 0$, then we have the AR(1) model.
- If one of σ_{η_i} is zero, then the unobserved process consists of a stochastic process and a deterministic process.

In our study we use three different versions of the SVMRS models as well as SV models. The first SVMRS model we use can be obtained by restricting $\phi_0 = \phi_1$ and $\sigma_0 = \sigma_1$;

$$x_t = \begin{cases} c_0 + \phi x_{t-1} + \eta_t, & \text{when } s_t = 0, \\ c_1 + \phi x_{t-1} + \eta_t, & \text{when } s_t = 1, \end{cases} \tag{8}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$, which we call two regime SVMRS model with changing volatility level ($SVMRS_L^2$). This model is similar to some previous models. For example, in the studies of So, Lam, Li (1998), Smith (2002) and Kalimipalli and Susmel (2004), the level of volatility is allowed to regime-change. So, Lam, Li (1998), using weekly S&P500 index volatility, find that volatility is less persistent than that of SV models. Smith (2002) and Kalimipalli and Susmel (2004) apply their models to explain the behaviour of short-term interest rates. They find that their regime switching models perform better than the GARCH family of models or SV models.

In order to investigate changing persistence levels, we also use a model;

$$x_t = \begin{cases} c + \phi_0 x_{t-1} + \eta_t, & \text{when } s_t = 0, \\ c + \phi_1 x_{t-1} + \eta_t, & \text{when } s_t = 1, \end{cases} \tag{9}$$

which we call two regime SVMRS model with changing persistence level ($SVMRS_P^2$). As in Chu and Hwang (2004) and Hwang and Valls (2004b), an evidence of regime changing persistence could partly explain why we observe an extreme persistence in index volatility.

However, these two models do not explain the two other cases, i.e., $s_t = 0$ and $s_{t-1} = 1$, and $s_t = 1$ and $s_{t-1} = 0$. If the frequency of inter-state changing is small, the effects of disregarding these two cases may be trivial, and this may be appropriate for most macroeconomic variables where the number of structural breaks is usually less than 1%. See Stock and Watson (1996), Ben-David and Papell (1998), McConnell and Perez-Quiros (2000), Hansen (2001), and Bai, Lumsdaine, and Stock (1998) among many. However, as will be shown later, for volatility which may have many structural breaks, models in (8) and (9) may become restrictive. In addition volatility of volatility (σ_η) may not be necessarily the same for different regimes. Therefore we use the generalised SVMRS model in (5), where the level of volatility, persistence, and volatility of volatility regime change over time. We call this model two regime generalised SVMRS model ($SVMRS_G^2$).

2.2 Persistence of Markov Regime Switching Process

In this section we explain the persistence level of volatility by driving the autocorrelation function of the Markov regime switching process for the model in (5).[3] When

[3]In relation to the persistence level of the Markov regime switching process, Francq and Roussignol (1998) and Francq and Zakoian (2001) investigate stationarity conditions for Markov switching

we define ξ_{1t} as a random variable that is equal to unity when $s_t = 1$ and zero otherwise, the AR(1) representation of this state 1 variable is

$$\xi_{1,t} = (1 - p^{(0,0)}) + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1} + v_{1,t}, \tag{10}$$

where $v_{1,t}$ is a martingale difference sequence of state 1 at time t. The unconditional probability that the process will be in regime 1 at any time, $\overline{p}_1$, is;

$$\overline{p}_1 = E(\xi_{1,t}) = \frac{1 - p^{(0,0)}}{2 - p^{(0,0)} - p^{(1,1)}} \tag{11}$$

Theorem 1 *The autocorrelation function with lag τ, $\rho(\tau)$, of the state equation in (5) is*

$$\rho(\tau) = E\left[\prod_{l=1}^{\tau} (\phi_0 + (\phi_1 - \phi_0)s_{t-l+1})\right].$$

Proof. See the Appendix. ■

Therefore, Theorem 1 shows that generally $\rho(\tau) \neq \rho(1)^\tau$, unless either $\phi_1 = \phi_0$ or $\overline{p}_1 = 0$ or 1, i.e., there is only one state. Therefore, the autocorrelations of the Markov regime changing AR(1) process may not show the persistence level that the value of $\rho(1)$ suggests, and not surprisingly, a regime-dependent AR(1) process is not unconditionally AR(1) process.

Remark 1 *Equation (11) suggests that when $p^{(1,1)} > \overline{p}_1$ and $\phi_1 \neq \phi_0$, we have $\rho(2) > \rho(1)^2$.*

Proof. See the Appendix. ■

Therefore even though (5) has an AR(1) representation, the process does not show the same exponential decay rate as the conventional AR(1) process because of the probability of states. In addition, the difference between $\rho(2)$ and $\rho(1)^2$ is a positive function of persistence difference $\phi_1 - \phi_0$, $\overline{p}_1$, and $p^{(1,1)} - \overline{p}_1$.[4]

ARMA processes.

[4]We only show the cases of $\rho(1)$ and $\rho(2)$. The autocorrelations with larger lags are complicted and we do not discuss them further in this study.

2.3 Estimation Procedure

Estimating SVMRS models is not trivial. In this study we use a Quasi-Maximum Likelihood (QML) estimation method using the Kalman filter. Harvey, Ruiz and Shephard (1994) adopt a procedure based on the Kalman filter to estimate simple SV models such as (3) and (4). They treat φ_t as though it were $N(0, \pi^2/2)$, and maximise the resulting quasi-likelihood function. Ruiz (1994) suggests that for the kind of data typically encountered in empirical finance, the QML for the SV model has good finite sample properties.

We could use other methods such as Markov Chain Monte Carlo (MCMC), the generalised method of moments (GMM), the efficient method of moments (EMM), or Particle filter and Sequential Monte Carlo algorithms by Pitt and Shephard (1999). These methods are less straightforward to apply and more computationally intensive than the QML with the updating procedure proposed in the Appendix. However, the estimates obtained from the QML may not be as efficient as these Bayesian methods because of the assumption of normality of error terms (i.e., φ_t and $\eta_{i,t}$) in equation (5).

The basic concept of the QML is that both x_t and conditional probability are unobserved processes, which can be obtained through predicting and updating procedure proposed by Smith (2002). That is, we have $(number\ of\ states)^{lags+1}$ state equations, each of which is updated and predicted in the same way as for the standard Kalman filter. We use the method suggested in Hamilton (1989) to update the conditional probability with a transition probability matrix. A detailed explanation on estimation procedure can be found in the Appendix.[5]

The log-likelihood function of the model in (5) is

$$\pounds(\mathbf{z}|\boldsymbol{\theta}) = \sum_{t=1}^{T} \log[f(z_t|I_{t-1})]$$

where $\boldsymbol{\theta} = \{\mu_0, \mu_1, \phi_0, \phi_1, \sigma_0^2, \sigma_1^2, \Pr(s_t = 0|s_{t-1} = 0), \Pr(s_t = 1|s_{t-1} = 1)\}$, I_{t-1} includes all available information at time $t-1$, and $f(\cdot)$ is the likelihood function

[5]We refer to Smith (2002) for further explanation on the smoothing procedure.

defined in the Appendix. Many different initial value sets of $\boldsymbol{\theta}$ were tried to find the global maximum likelihood value, since the likelihood surface in SVMRS models has many local maxima.

An important question is which model is preferred among the three different versions of SVMRS models in (5), (8) and (9). The two simple SVMRS models in (8) and (9) are nested in (5), and thus we can use the likelihood ratio test to investigate if there is a significant difference between the models. However, when $\sigma_{\eta_i} = 0$ in (5) or $\sigma_\eta = 0$ in (8) and (9), the AR parameters are not identifiable, and the likelihood ratio test statistics are not valid since the information matrix is singular under the null. Monte Carlo simulations could be used to get the empirical distribution of the test statistics, but because of the many local maxima of SVMRS models it is not a feasible option. See Smith (2002) for more discussion on model selection methods for SV models, Markov regime switching models and SVMRS models.

In our study we use the relative magnitude of σ_{η_i} to σ_φ (the signal-to-noise ratio) besides maximum likelihood values.[6] Using the signal-to-noise ratio is based on intuition rather than econometric arguments. If a model is well specified, then the proportion that is not explained by the model, i.e., the transitory noise in SVMRS models, should be minimised. Since the true volatility process and thus the amount of transitory noise included in squared returns are not known, a model that explains squared returns as much as possible may be better than a model that does not. We also use statistics of standardised residuals to see how the models work. As in (5) we expect standardised residuals to be distributed as normal. However when the signal-to-noise is extremely large (or the transitory noise (i.e., σ^2_φ) becomes close to zero), standardised residuals are either -1 or 1. In this situation analysing standardised

[6]However, this criterion may be controversial. We may use the Bayesian analysis, but again we need some knowledge of the true volatility process. Empirical results in the next section show that there are not significant differences in model selection between ML values and signal-to-noise ratio.

residuals is not helpful.

3 Empirical Tests

We use a total number of 2606 S&P500 daily index log-returns from 27 February 1992 to 27 February 2002, and 522 weekly log-returns from 26 February 1992 to 27 February 2002.[7] To calculate residuals, we simply take the sample mean return from the log-returns.[8]

Table 1 reports the property of the index returns. For the sample period, daily returns are negatively skewed and leptokurtic, suggesting non-normality. In addition, autocorrelation coefficients are not significant. For the weekly returns, we also find similar properties, but unsurprisingly the magnitude of non-normality is much smaller than that of the daily returns. On the other hand, the log-squared returns, as reported in many other studies, are negatively skewed and fat-tailed, and display persistence. One noticeable difference between the daily and weekly log-squared returns is that daily log-squared returns are more persistent than weekly log-squared returns. The temporal aggregation affects the level of persistence, i.e., the autocorrelation structure.

The large negative skewness in the log-squared returns results from the so-called 'inlier' problem in stochastic volatility models. For the daily returns used in this study, for example, the largest log-squared residual of 3.935 is within three standard deviations. However, the lowest log-squared residual is -16.237, which is outside five standard deviations.

Various methods have been proposed for inlier adjustment for the log-squared returns. Harvey and Shephard (1993) set an arbitrary critical value and trim all

[7]We also used daily and weekly FTSE100 index log-returns for the same period, but the results are similar to those of the S&P500 index log-returns.

[8]Since daily and weekly expected returns are very small, taking the mean returns from the daily and weekly returns does not have significant effects but does reduce the inlier problem discussed earlier. In the following we use 'log-squared returns' for the logs of squared de-meaned returns.

values less than the critical value to the arbitrary critical value. These trimmed estimates behave better than the untrimmed estimates in their simulations. However, these kinds of inlier adjustments are criticised to be 'profoundly suspicious' by Nelson (1994). In this study, we use the following Breidt and Carriquiry (BC) (1996) transformation:

$$\widehat{\ln y_t^2} = \ln(y_t^2 + \kappa\sigma_y^2) - \kappa\sigma_y^2/(y_t^2 + \kappa\sigma_y^2) \tag{12}$$

The idea behind the BC transformation is as follows. For zero or extremely small y_t^2, $\ln(y_t^2 + \delta)$, where δ is a small increment, is evaluated. Then, the transformed $\ln(y_t^2)$ can be obtained by the linear extrapolation from the point $(y_t^2 + \delta, \ln(y_t^2 + \delta))$ using the slope of the tangent line, $(y_t^2 + \delta)^{-1}$; in the above equation, δ is set to $\kappa\sigma_y^2$.

Table 1 reports the property of log-volatility changes for the three different parameter values of κ, i.e., 0.02, 0.05, 0.1. For different time series, different values of κ are required. Daily log-squared returns show that when $\kappa = 0.02$, we have the smallest Jarque-Bera statistic, whilst $\kappa = 0.05$ gives the smallest Jarque-Bera statistic for weekly log-squared returns. Note that when κ is too large, we may lose information included in the original data. For example, the autocorrelation coefficients increase as κ increases. On the other hand when κ is too small, we still have the inlier problem. In many case, the choice of κ is arbitrary and needs econometrician's subjective decision. In this study, we choose $\kappa = 0.02$ to examine if the inlier problem has significant effects on our models.

3.1 SV Model

We first estimate the SV model in (3) and (4) for daily data and report the results in the first column of Table 2. As in many other previous studies, we find that the unobserved volatility process is highly persistent for daily log-squared returns. Figure 1A shows the absolute values of residuals and the smoothed standard deviation obtained for daily returns from the SV model. As in most empirical results on SV models, it shows that the volatility is smooth. However, the autocorrelation coefficients presented in Table 1 do not suggest such a high level of persistence. The difference between the two is usually attributed to high level of noise in squared returns (see Andersen and Bollerslev, 1998), which is supported by the small signal-to-noise (SN) ratio, $\sigma_\eta/\sigma_\varphi$ (i.e., 0.018 for the daily squared returns). This means that SV models (or asymptotically GARCH models) explain only a small proportion of squared returns.[9]

Standardised residuals are still negatively skewed and leptokurtic. However, when these statistics are compared with those in Table 1, we find that they are much smaller in the SV model, suggesting that SV models explain fat-tails and skewness to some degree.

3.2 Two Regime SVMRS Model

We estimate the three two regime SVMRS models in (8), (9), and (5), and report the results in columns 2, 3, and 4 in Table 2.

3.2.1 Regime Switching in the Level of Volatility

We first discuss SVMRS models with changing volatility level ($SVMRS^2_L$). The ML values of the $SVMRS^2_L$ model are larger than those of the SV model, suggesting that the $SVMRS^2_L$ model better specifies the log-squared returns. The likelihood ratio statistics are 137.4 for daily log-squared returns and 68 for weekly log-squared returns, both of which are highly significant for $\chi^2(3)$.[10] The significant improvement from the SV model is also supported by the close-to-zero transitory noise and the large SN ratios of the $SVMRS^2_L$ model.[11]

[9]See Hwang and Pedro (2004b) for how the signal-to-noise ratio of SV models is related to their explanatory power for squared residuals.

[10]Note that $\sigma_\eta \neq 0$, and thus these likelihood ratio test statistics will be asymptotically chi-squared.

[11]However, when the transitory noise becomes zero, analysing standardised residuals may not give us much information on the fitness of models because they are either -1 or 1.

Note that the unconditional probability that $s_t = 1$, $E(s_t = 1)$, is

$$E\left(s_t = 1\right) = \frac{1 - p^{(0,0)}}{2 - p^{(0,0)} - p^{(1,1)}} \tag{13}$$

from (11). Using this equation we calculate that the unconditional probabilities of the high volatility regimes ($s_t = 1$) are 0.870 and 0.888 for the daily and weekly log-squared returns respectively. This means that for approximately 87% of cases, daily volatility is in a high volatility regime with an unconditional log-volatility of -1.148 ($= \frac{c_1}{1-\phi}$).

The level of volatility persistence drops significantly from 0.992 for the SV model to 0.086 for the $SVMRS_L^2$ model for daily log-squared returns and from 0.998 to 0.081 for weekly log-squared returns. These results are consistent with recent studies such as Lobato and Savin (1998), Granger and Teräsvirta (1999), Granger and Hyung (1999), and Diebold and Inoue (2001). However, the persistence of the $SVMRS_L^2$ model is far less than the 0.48 and 0.47 reported by Hamilton and Susmel (1994) and So, Lam, Li (1998). The difference could be attributed to different estimation methods, indices, or sample periods. In addition, regimes do not have memory. That is, the probability of state 1 has the AR coefficient of $\xi_{1,t}$ ($=-1 + p^{(0,0)} + p^{(1,1)}$) in equation (10) which is close to zero when estimated with $\widehat{p}^{(0,0)}$ and $\widehat{p}^{(1,1)}$.

3.2.2 Regime Switching in the Level of Persistence

The third column in Table 2 reports the estimates of the SVMRS model whose persistence level is dependent on different regimes over time ($SVMRS_P^2$). We find evidence of regime switching in persistence level. One regime shows a low level of persistence and the other shows explosive behaviour; in daily log-squared returns, we have AR parameters of 0.120 with an unconditional probability of 0.860 and 3.008 with an unconditional probability of 0.140. Figure 1C shows that the high level of persistence is more frequent when the level of volatility was low during 1992 to 1995, but it became less likely since 1997 when the level of volatility was high.

When we calculate the persistence level of the two regimes using the unconditional probabilities as weights, we have 0.524 (=$0.86 \times 0.120 + 0.14 \times 3.008$) which is far less than the value 0.99 of the SV model in the first column. This is consistent with Chu and Hwang (2004) who shows that when an AR(1) process has structural breaks in the AR parameter, the ML estimate of the AR parameter obtained without considering the structural breaks always tends toward the largest sub-AR parameter in the process. However, for weekly log-squared returns we cannot discuss persistence level since σ_η is not different from zero (t-statistic is 1.56). As explained earlier when $\sigma_\eta = 0$, the AR parameters are not identifiable.[12]

The significance of regime switching in the level of persistence can be tested with likelihood ratio statistics. In the daily data case the likelihood ratio statistic with respect to the SV model is 55.2 which is highly significant for $\chi^2(3)$. The properties of standardised residuals show that allowing regime switching in the level of persistence can give better results than the SV model. The skewness and kurtosis of the standardised residuals with regime switching in the level of persistence are smaller than those of the SV model and thus the Jarque-Bera statistic of the $SVMRS_P^2$ model becomes smaller.

Figure 1C shows an interesting pattern in smoothed volatility. Although the $SVMRS_P^2$ model explains volatility better than the SV model, it does not explain the changing level of volatility well. But Figure 1C shows that high persistence is more likely to be observed when the level of volatility is low. As in the $SVMRS_L^2$ case, regimes are not persistent at all. The AR coefficients of $\xi_{1,t}$ are all small and negative. The structural breaks do not have memory.

[12]Therefore we do not discuss likelihood ratio statistics in the weekly log-squared returns case in the below.

3.2.3 Regime Switching in the Level of Volatility, Persistence, and Volatility of Volatility

When we compare the results of the $SVMRS_L^2$ and $SVMRS_P^2$ models, we find that the changing level of volatility explains volatility better since the signal-to-noise ratios and maximum likelihood values of the $SVMRS_L^2$ model are larger than those of the $SVMRS_P^2$ model. However, the results of the $SVMRS_P^2$ model suggest that high volatility is related with less persistence, and thus both persistence and level of volatility needs to switch to better explain volatility. In addition, the volatility of volatility is not necessarily the same for different regimes. By allowing volatility of volatility (σ_η) to regime switch we could also evaluate the effects of the inlier problem discussed earlier. Inliers could be captured by a larger value of σ_η in the low volatility regime. Therefore we estimate the generalised SVMRS model ($SVMRS_G^2$) in (5) where the level of volatility, the level of persistence, and volatility of volatility regime change.

The ML, AIC and BIC values reported in the fourth column of Table 2 show that the $SVMRS_G^2$ model specifies the log-squared returns better than the SV and the two SVMRS Models in columns 1 to 3. This is also supported by the large SN ratios, e.g., 7167.3 ($s_t = 0$) and 3890.8 ($s_t = 1$) in the daily data case, and by the fact that the transitory noises from the $SVMRS_G^2$ model are close to zero.[13] Figures 1D and 2D show that most squared returns are explained by the SVMRS model. The large amount of transitory noise unexplained by the SV model is explained by switching regimes.

More importantly, the high persistence found with the SV model disappears in the $SVMRS_G^2$ model. For example, in the daily data case the estimates of the $SVMRS_G^2$ model show that the AR parameters are 0.295 ($s_t = 0$) and 0.146 ($s_t = 1$) which are far from that of the SV model. The different levels of volatility and persistence in states 0 and 1 suggest that these structural breaks are sources

[13] This means that analysing standardised residuals is not helpful for model selection in this context.

of high persistence. Using the results in Theorem 1 we calculate the first order autocorrelation coefficient of the Markov regime switching state equation for the daily data. Since the unconditional probability of $s_t = 1$ for daily log-squared returns is 0.719, we have

$$\widehat{\phi}_0 + (\widehat{\phi}_1 - \widehat{\phi}_0)\widehat{\overline{p}}_1 = 0.1922$$

from Theorem 1. Thus when we remove the transitory noise and allow regimes, the AR coefficients estimated are much smaller than those with the SV model which show extreme persistence. In addition, the estimated transition probabilities in Table 2 show that $p^{(1,1)} < 1 - p^{(0,0)}$, suggesting $\rho(2) < \rho(1)^2$. Therefore, at the second lag the autocorrelation coefficient decays faster than the ordinary AR(1) process whose autocorrelation coefficient at lag 2 is equivalent to $\rho(1)^2$. These results are not different in the weekly data case reported in panel B of Table 2.

The AR coefficients of $\xi_{0,t}$ in equation (10) with the estimates of $p^{(0,0)}$ and $p^{(1,1)}$ are all close to zero. Again structural breaks do not have memory. These results seem to be inconsistent with So, Lam, Lee (1998), and in particular Granger and Hyung (1999) who found a small probability (less than 1%) of structural breaks in squared returns in a long memory volatility model. However, the difference between our approach and other previous studies including Granger and Hyung (1999) is that in our model, all three components, i.e., the level of volatility, AR coefficient and the volatility of permanent error, are allowed to change. The difference also could be attributed to allowing transitory noise in our model.

Asymptotically SV models are equivalent to GARCH models (see Nelson and Foster (1994) and Nelson (1996)), and thus our results may also be applicable to GARCH models; when we allow structural breaks, the persistence level is reduced and the explanatory power of the model will increase. However, to our knowledge there is no direct analytical comparison on the effects of Markov switching process (or structural breaks) on the persistence of GARCH and SV models that has been published.

As expected, the results in Table 2 show that the low volatility regime is more volatile. The large volatility of volatility when $s_t = 0$ may be explained by inliers. We investigate the effects of inliers on the estimates of $SVMRS_G^2$ models.[14] To evaluate the inlier problem, we use the BC method in (12) with $\kappa = 0.02$. The last column of Table 2 reports that we have a much smaller AR parameter for $s_t = 0$ with the modified log-squared volatility. However, for the regime $s_t = 0$, $\sigma_{\eta 0}$ is zero, suggesting that lower volatility becomes constant. When we compare this result with the $SVMRS_G^2$ model in column 4, we find that transforming the data using the BC method trims inliers so that the volatility of the low volatility regime becomes constant. These results indicate that the volatility of the low volatility regime is inflated by inliers, and that we could reduce the inlier problem using regime switching models.

3.3 Three Regime SVMRS Models

We also use three regime SVMRS models to examine how robust our results with the two regimes are. The two regimes assumed so far may be too restrictive if the true volatility process has three regimes. As in the two regime cases, we use three three-regime SVMRS models; regime switching in 1) the level of volatility ($SVMRS_L^3$), 2) the level of persistence ($SVMRS_P^3$), and 3) the level of volatility, persistence, and volatility of volatility ($SVMRS_G^3$). Table 3 shows that the last case is not identifiable for both daily and weekly data. In addition $\widehat{\sigma}_{\eta 0}$'s for weekly data are not significant for all three models and thus the models are not identifiable. The large number of unidentifiable models indicates that three regimes may be over-parameterised.[15]

The first case ($SVMRS_L^3$) in Table 3 shows that we now have an AR parameter -0.036 which is not significantly different from zero. The results in Tables 2 and 3

[14]We have also considered various other cases; for example, BC transformed data for the SVMRS models in (8) and (9). These results can be obtained by request to the authors.

[15]Determining the number of regimes in stochastic volatility Markov regime switching models is not trivial. In our study we rely on informal discussion, and leave this issue for future study.

suggest that when we move from SV models to two and three regime models, we find that the AR parameters become smaller and in the three regime case there is no persistence at all!

Inliers are explained by the regime corresponding to $s_t = 2$ whose parameter $\mu_2 = -11.762$ with a small probability of occurrence. Therefore the remaining two regimes $s_t = 0$ and $s_t = 1$ represent high and low volatility regimes. The likelihood ratio statistics of the $SVMRS_L^3$ model with respect to the $SVMRS_L^2$ model is 25.28 which is significant with $\chi^2(5)$. The AIC and BIC statistics also show that the three regime model performs better than the two regime model. In particular the Jarque-Bera statistic for the standardised residuals in the $SVMRS_L^3$ model suggests that the standardised residuals are not different from normal.

The three regime model in the level of persistence shows a big improvement from its equivalent two regime model in Table 2. The likelihood ratio statistic (88.25) is significant and the AIC and BIC values of the three regime model are smaller than those of the two regime model. The Jarque-bera statistic for the standardised residuals is also much smaller with the three regime model. As in the two regime model, we find that one regime explodes with an AR parameter of 5.616. The other two regimes have AR parameters not different from zero and are not persistent.

4 Conclusions

This paper has presented a SV model with regime-dependent volatility levels, persistence levels, and volatilities of volatility that generalises existing SV regime-dependent models. We estimate our model using generalisations of the Kalman filter methods of Harvey, Ruiz, and Shephard (1994). Our results show that squared returns are better specified by our SVMRS models. A broad pattern we have found is that the regime-dependent estimates are far less persistent (and more volatile) than SV models. High volatility persistence we obtain with SV models come from structural breaks in the level of volatility as well as in the level of persistence. We also find that high volatility regimes are less persistent than low volatility regimes. By allowing more regimes we could reduce the inlier problem which is one of the major econometric problems in SV models.

Appendix

Proof of Theorem 1

The state equation in (5) can be represented as

$$\begin{aligned} x_t - [\mu_0 + (\mu_1 - \mu_0)s_t] &= (\phi_0 + (\phi_1 - \phi_0)s_t)\,[x_{t-1} - [\mu_0 + (\mu_1 - \mu_0)s_{t-1}]] \\ &\quad +(\sigma_{\eta_0} + (\sigma_{\eta_1} - \sigma_{\eta_0})s_t)\xi_t, \end{aligned}$$

where $\xi_t\tilde{}N(0,1)$. Note that

$$\begin{aligned} E\,[x_t - [\mu_0 + (\mu_1 - \mu_0)s_t]] &= E(x_t) - E\,[\mu_0 + (\mu_1 - \mu_0)s_t] \\ &= E\left[x_t\xi_{1,t} + x_t(1-\xi_{1,t})\right] - E\,[\mu_0 + (\mu_1 - \mu_0)s_t] \\ &= \mu_1\overline{p}_1 + \mu_0(1-\overline{p}_1) - \mu_0 - (\mu_1 - \mu_0)\overline{p}_0 \\ &= 0. \end{aligned}$$

Substituting $w_t = x_t - [\mu_1 + (\mu_0 - \mu_1)s_t]$, we have the following mean zero AR(1) process;

$$w_t = (\phi_0 + (\phi_1 - \phi_0)s_t)w_{t-1} + (\sigma_{\eta_0} + (\sigma_{\eta_1} - \sigma_{\eta_0})s_t)\xi_t,$$

and thus

$$\rho(\tau) = E\left[\prod_{l=1}^{\tau}(\phi_0 + (\phi_1 - \phi_0)s_{t-l+1})\right].$$

Proof of Remark 1

Note that when $\tau = 1$, $\rho(1) = \phi_0 + (\phi_1 - \phi_0)\overline{p}_1$, since $E(s_t) = \overline{p}_1$. However, when $\tau = 2$, we have

$$\begin{aligned}
\rho(2) &= E\left[(\phi_0 + (\phi_1 - \phi_0)s_t)(\phi_0 + (\phi_1 - \phi_0)s_{t-1})\right] \\
&= E\left[\phi_0^2 + (\phi_1 - \phi_0)\phi_0 s_t + (\phi_1 - \phi_0)\phi_0 s_{t-1} + (\phi_0 - \phi_1)^2 s_t s_{t-1}\right] \\
&= \phi_0^2 + 2(\phi_1 - \phi_0)\phi_0\overline{p}_1 + (\phi_1 - \phi_0)^2 E[s_t s_{t-1}].
\end{aligned}$$

Since

$$\begin{aligned}
E[s_t s_{t-1}] &= E[\xi_{1,t}\xi_{1,t-1}] \\
&= E[((1 - p^{(0,0)}) + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1} + v_{1,t})\xi_{1,t-1}] \\
&= E[((1 - p^{(0,0)})\xi_{1,t-1} + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1}^2] \\
&= (1 - p^{(0,0)})\overline{p}_1 + (-1 + p^{(0,0)} + p^{(1,1)})\overline{p}_1 \\
&= p^{(1,1)}\overline{p}_1,
\end{aligned}$$

we have

$$\begin{aligned}
\rho(2) &= \phi_0^2 + 2(\phi_1 - \phi_0)\phi_0\overline{p}_1 + (\phi_1 - \phi_0)^2 p^{(1,1)}\overline{p}_1 \\
&= [\phi_0 + (\phi_1 - \phi_0)\overline{p}_1]^2 + (\phi_1 - \phi_0)^2[p^{(1,1)}\overline{p}_1 - (\overline{p}_1)^2] \\
&= \rho(1)^2 + (\phi_1 - \phi_0)^2\overline{p}_1[p^{(1,1)} - \overline{p}_1].
\end{aligned}$$

Estimation Procedure for SVMRS Models

We need some notation for the prediction of the state vector and also for its variance depending upon which regime is being used in the conditional set. The prediction equation ($x_{t|t-1}^{(i,j)}$), mean squared error associated with $x_{t|t-1}^{(i,j)}$ ($M_{t|t-1}^{(i,j)}$), prediction error ($v_t^{(i,j)}$), prediction variance ($f_t^{(i,j)}$) and updating equations ($x_{t|t}^{(i,j)}, M_{t|t}^{(i,j)}$), can be obtained using the following procedure; For $s_t = i$ and $s_{t-1} = j$, $i, j = 0, 1$, we have

$$\begin{aligned}
x_{t|t-1}^{(i,j)} &= E[x_t | s_t = i, s_{t-1} = j, I_{t-1}] \\
&= (\mu_i - \mu_j \phi_i) + \phi_i x_{t-1|t-1}^{(j)}, \\
M_{t|t-1}^{(i,j)} &= E[(x_t - x_{t|t-1}^{(i,j)})^2 | s_t = i, s_{t-1} = j, I_{t-1}] \\
&= \phi_i^2 M_{t-1|t-1}^{(j)} + \sigma_i^2, \\
v_t^{(i,j)} &= y_t - x_{t|t-1}^{(i,j)} \\
&= x_t - x_{t|t-1}^{(i,j)} + \varphi_t \\
f_t^{(i,j)} &= M_{t|t-1}^{(i,j)} + \sigma_\varphi^2 \\
x_{t|t}^{(i,j)} &= x_{t|t-1}^{(i,j)} + M_{t|t-1}^{(i,j)} \left(f_t^{(i,j)} \right)^{-1} v_t^{(i,j)} \\
M_{t|t}^{(i,j)} &= M_{t|t-1}^{(i,j)} - M_{t|t-1}^{(i,j)} \left(f_t^{(i,j)} \right)^{-1} M_{t|t-1}^{(i,j)}
\end{aligned}$$

Note that as in Hamilton (1989), the filtered transition probability, $\Pr(s_t = i, s_{t-1} = j | I_{t-1})$, is updated with transition probability, $\Pr(s_t = i | s_{t-1} = j)$, and conditional probability, $\Pr(s_{t-1} = j | I_{t-1})$ as follows;

$$\Pr(s_t = i, s_{t-1} = j | I_{t-1}) = \Pr(s_t = i | s_{t-1} = j) \Pr(s_{t-1} = j | I_{t-1}).$$

The conditional probability updated with information at time t are given by:

$$\begin{aligned}
\Pr(s_t &= i, s_{t-1} = j | I_t) = \frac{f(y_t, s_t = i, s_{t-1} = j | I_{t-1})}{f(y_t | I_{t-1})} \\
&= \frac{f(y_t | s_t = i, s_{t-1} = j, I_{t-1}) \Pr(s_t = i, s_{t-1} = j | I_{t-1})}{\sum_{i=0}^{1} \sum_{j=0}^{1} f(y_t | s_t = i, s_{t-1} = j, I_{t-1}) \Pr(s_t = i, s_{t-1} = j | I_{t-1})}.
\end{aligned}$$

Assuming normality the density for y_t conditional on s_t, s_{t-1} and I_{t-1} is given by:

$$f(y_t | s_t = i, s_{t-1} = j, I_{t-1}) = \frac{1}{\sqrt{2\pi f_t^{(i,j)}}} \exp\left\{ -\frac{\left(v_t^{(i,j)} \right)^2}{2 f_t^{(i,j)}} \right\}.$$

and

$$\Pr(s_t = i | I_t) = \sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j | I_t).$$

Note that

$$\begin{aligned}
\Pr(s_t &= 0|s_{t-1} = 1) = 1 - \Pr(s_t = 1|s_{t-1} = 1) \\
\Pr(s_t &= 1|s_{t-1} = 0) = 1 - \Pr(s_t = 0|s_{t-1} = 0).
\end{aligned}$$

We also obtain

$$\begin{aligned}
x_{t|t}^{(i)} &= E[x_t|s_t = i, I_t] \\
&= \frac{\sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j|I_t) x_{t|t}^{(i,j)}}{\Pr(s_t = i|I_t)}, \\
M_{t|t}^{(i)} &= E[(x_t - x_{t|t}^{(j)})^2|s_t = i, I_t] \\
&= \frac{\sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j|I_t) M_{t|t}^{(i,j)}}{\Pr(s_t = i|I_t)}.
\end{aligned}$$

References

Alizadeh, S., M. Brandt, and Diebold, F. X., 2002. Range-Based Estimation of Stochastic Volatility Models, *Journal of Finance,* 57, 2002, 1047-1091.

Andersen, T. and Lund, J., 1997. Estimating continuous time stochastic volatility models of the short-term interest rates, *Journal of Econometrics*, 77, 343-377.

Andersen, T. and Bollerslev, T., 1998. Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts, *International Economic Review*, 39(4), 885-905.

Bai, J., Lumsdaine, R. L., and Stock, J. H., 1998. Testing for and Dating Common Breaks in Multivariate Time Series, *Review of Economic Studies*, 63, 395-432.

Ben-David, D. and Papell, D. H., 1998. Slowdowns and Meltdowns: Postwar Growth Evidence from 74 Countries, *Review of Economics and Statistics*, 80, 561-571.

Bollerslev, T., 1986, Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics,* 31, 307-327.

Breidt, F., and Carriquiry, A., 1996. Improved Quasi-Maximum Likelihood Estimation for Stochastic Volatility Models, *Modelling and Prediction*: Honoring Seymour Geisser, J. Lee, W. Johnson, and A. Zellner (eds), Springer-Verlag.

Campbell, J. Y., Lettau, M., Malkiel, B. G., and Xu, Y., 2001. Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk, *Journal of Finance,* 56(1), 1-43.

Chu, B., and Hwang, S., 2004. Structural Breaks in AR(1) Process and Persistence. Mimeo. (Faculty of Finance, Cass Business School, London).

Connor, G. and Linton, O., 2001. The Common and Specific Components of Dynamic Volatility, Mimeo, London School of Economics and Political Science.

Diebold, F. X., and Inoue, A., 2001. Long Memory and Regime Switching, *Journal of Econometrics*, 105, 131-159.

Fama, E. and French, K. R., 1993. Common Risk factors in the Returns on Stocks and Bonds, *Journal of Financial Economics*, 33, 3-56.

Filardo, A. J., 1994. Business-cycle phases and their transitional dynamics, *Journal of Business and Economic Statistics*, 12(3), 299-308.

Francq, C. and Roussignol, M., 1998. Ergodicity of autoregressive processes with Markov-switching and consistency of the maximum-likelihood estimator, *Statistics*, 32, 151-173.

Francq, C., and Zakoian, J. M., 2001. Stationarity of multivariate Markov-switching ARMA models, *Journal of Econometrics,* 102, 339-364.

Goodwin, T. H., 1993. Business-cycle analysis with a Markov switching model, *Journal of Business and Economic Statistics,* 11(3), 331-339.

Granger, C. W. J. and Hyung, N., 1999. Occasional Structural Breaks and Long Memory. Discussion Paper 99-14, Department of Economics, University of California, San Diego.

Granger, C. W. J. and Teräsvirta, T., 1999, A Simple Nonlinear Time Series Model with Misleading Linear Properties. *Economics Letters,* 62, 161-165.

Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time-Series and the Business Cycle, *Econometrica,* 57, 357-384.

Hamilton, J. D. and Susmel, R., 1994. Autoregressive Conditional Heteroscedasticy and Changes in Regime, *Journal of Econometrics,* 64, 307-333.

Hansen, B. E., 2001. The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity, mimeo, University of Wisconsin.

Harvey, A. C. and Shephard, N., 1993. Estimation and testing of stochastic variance models. Econometrics discussion paper EM/93/268, London School of Economics.

Harvey, A. C. and Shephard, N., 1996. Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics,* 14(4), 429-34.

Harvey, A. C., Ruiz, E. and Shephard, N., 1994, Multivariate stochastic variance models. *Review of Economic Studies,* 61, 247-264.

He, C and T. Teräsvirta, 1999. Properties of the Autocorrelation Function of Squared Observations for Second-order GARCH Processes under Two Sets of Parameter Constraints, *Journal of Time Series Analysis,* 20, 23–30.

Hwang, S. and Satchell, S. E., 2000. Market risk and the concept of fundamental volatility: measuring volatility across asset and derivative markets and testing for the impact of derivatives markets on financial markets. *Journal of Banking and Finance*, 24(5), 759-785.

Hwang, S. and Satchell, S. E., 2004. GARCH Model with Cross-Sectional Volatility: GARCHX Models, forthcoming in *Applied Financial Economics.*

Hwang, S. and Valls, P., 2004a. Small Sample Properties of GARCH Estimates and Persistence. Mimeo. (Faculty of Finance, Cass Business School, London)

Hwang, S. and Valls, P., 2004b. Structural Breaks in ARCH and GARCH Parameters and Persistence of Volatility. Mimeo. (Faculty of Finance, Cass Business School, London)

Hull, J., and A. White, 1987. The pricing of options on assets with stochastic volatilities, *Journal of Finance,* 42, 281-300.

Kalimipalli, M. and Susmel, R., 2004. Regime-Switching Stochastic Volatility and Short-Term Interest Rates, forthcoming in the *Journal of Empirical Finance.*

Lobato, I. N. and Savin, N. E., 1998. Real and spurious long-memory properties of stock-market data. *Journal of Business and Economic Statistics,* 16(3), 261-268.

McConnell, M. M. and Perez-Quiros, G., 2000. Output Fluctuations in the United States: What Has Changed Since the Early 1980's? *American Economic Review,* 90(5), 1464-1476.

Nelson, D. B., 1994. Comment on Jacquier, Polson, and Rossi's "Beyesian Analysis of Stochastic Volatility Models", *Journal of Business and Economic Statistics,* 12, 403-406.

Nelson, D. B., 1996, Asymptotically Optimal Smoothing with ARCH Models, *Econometrica* 64, 561-573.

Nelson, D. B. and Cao, C. Q., 1992. Inequality Constraints in the Univariate GARCH Model, *Journal of Business and Economic Statistics,* 10, 229–235.

Nelson, D. B. and D. P. Foster, 1994, Asymptotic Filtering Theory for Univariate ARCH Models, *Econometrica,* 62, 1-41.

Pitt, M. and Shephard, N., 1999. Filtering via simulation: Auxiliary Particle Filters, *Journal of the American Statistical Association,* 94(446), 590-599.

Smith, D. R., 2002. Markov-Switching and Stochastic Volatility Diffusion Models of Short-Term Interest Rates, *Journal of Business and Economic Statistics*, 20, 183–197.

So, M. K. P., Lam, K., and Li, W. K., 1998. A stochastic volatility model with Markov switching, *Journal of Business and Economic Statistics*, 16, 244-253.

Stock, J. H. and Watson, M. W., 1996. Evidence on structural instability in macroeconomic time series relations, *Journal of Business and Economic Statistics*, 14, 11-30.

Taylor, S. J., 1986. *Modelling Financial Time Series.* Chichester: John Wiley.

Table 1 Statistical Properties of the S&P500 Index Volatility

		Returns (%)	Log- Squared Residuals (Returns-Mean)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.02)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.05)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.10)
Daily Returns (A total number of 2606 returns from 27 February 1992 to 27 February 2002)	Mean	0.046	-1.945	-1.624	-1.459	-1.290
	Standard Deviation	0.983	2.606	1.956	1.739	1.551
	Skewness	-0.307	-1.120	-0.051	0.208	0.434
	Excess Kurtosis	4.937	2.101	-0.868	-0.867	-0.708
	Jarque and Bera Statistics	2687.326	1024.458	82.962	100.341	136.093
	Ljung and Box (10)	21.335	304.897	545.184	630.323	697.345
	Ljung and Box (50)	95.914	1371.533	2328.800	2606.691	2807.545
	Autocorrelation (Lag 1)	0.005	0.092	0.117	0.126	0.133
	Autocorrelation (Lag 5)	-0.034	0.127	0.178	0.190	0.199
	Autocorrelation (Lag 10)	0.043	0.133	0.171	0.183	0.191
	Autocorrelation (Lag 15)	0.016	0.099	0.137	0.143	0.147
	Autocorrelation (Lag 20)	-0.008	0.094	0.122	0.126	0.130
	Autocorrelation (Lag 30)	-0.010	0.109	0.142	0.150	0.154
	Autocorrelation (Lag 40)	0.012	0.084	0.121	0.131	0.138
	Autocorrelation (Lag 50)	-0.013	0.088	0.110	0.114	0.115
Weekly Returns (A total number of 522 returns from 26 February 1992 to 27 February 2002)	Mean	0.227	-0.110	0.130	0.259	0.397
	Standard Deviation	2.090	2.395	1.856	1.664	1.492
	Skewness	-0.421	-1.237	-0.251	0.010	0.244
	Excess Kurtosis	1.743	2.207	-0.701	-0.829	-0.786
	Jarque and Bera Statistics	81.463	239.005	16.166	14.957	18.610
	Ljung and Box (10)	16.357	51.645	76.889	88.010	98.154
	Ljung and Box (50)	41.056	190.159	296.735	332.606	361.150
	Autocorrelation (Lag 1)	-0.094	0.099	0.117	0.123	0.128
	Autocorrelation (Lag 5)	-0.015	0.035	0.067	0.083	0.098
	Autocorrelation (Lag 10)	0.031	0.051	0.069	0.079	0.087
	Autocorrelation (Lag 15)	0.017	0.098	0.121	0.119	0.115
	Autocorrelation (Lag 20)	0.043	0.064	0.128	0.146	0.158
	Autocorrelation (Lag 30)	0.032	0.101	0.123	0.128	0.132
	Autocorrelation (Lag 40)	-0.020	0.006	0.037	0.047	0.055
	Autocorrelation (Lag 50)	0.045	0.097	0.121	0.126	0.130

Table 2 Estimates of SV, Unrestricted and Restricted Markov Regime Switching SV Models

A. Daily Volatility

	Stochastic Volatility Model		Regime Switching Stochastic Volatility Models (2 States)							
	Log-Squared Residuals		Regime Switching in the Level of Volatility		Regime Switching in the Level of Persistence		Regime Switching in the Level of Persistence and Volatility		Regime Switching in the Level of Persistence and Volatility for Log-Squared Residuals Modified with Breidt and Carriquiry (1996) Method (k=0.02)	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-1.976	0.455	-6.546	0.229	-1.148	0.088	-4.543	0.261	-4.748	0.026
μ_1			-1.075	0.064			-0.940	0.071	-1.208	0.044
ϕ_0	0.999	0.001	0.082	0.019	0.120	0.034	0.295	0.123	0.009	0.014
ϕ_1					3.008	0.112	0.146	0.039	0.179	0.027
$\sigma_{\eta 0}$	0.044	0.016	1.828	0.051	0.801	0.077	2.867	0.114	0.225	0.518
$\sigma_{\eta 1}$							1.556	0.042	1.666	0.082
σ_{φ}	2.465	0.054	0.000	0.002	1.819	0.047	0.000	0.003	0.001	102.272
$p^{(0,0)}$			0.135	0.028	0.842	0.018	0.255	0.044	0.147	0.025
$p^{(1,1)}$			0.871	0.013	0.028	0.013	0.709	0.037	0.887	0.010
$\sigma_{\eta 0}/\sigma_{\varphi}$	0.0180		9142.0		0.4406		7167.3		204.5	
$\sigma_{\eta 1}/\sigma_{\varphi}$							3890.8		1514.5	
ML Values	-6071.2		-6002.5		-6043.6		-5949.3		-5149.1	
AIC	12150.3		12018.9		12101.1		11916.7		10316.2	
BIC	12173.8		12060.0		12142.2		11969.4		10369.0	
STD	2.241		0.997		1.646		0.998		0.932	
Skewness	-0.238		0.059		-0.120		0.059		0.027	
Excess Kurtosis	1.870		-1.996		1.732		-1.996		-1.902	
Jarque-Bera	404.267		434.2		332.2		434.2		393.1	

Notes: A total number of 2606 returns from 27 February 1992 to 27 February 2002 is used.

B. Weekly Volatility

	Stochastic Volatility Model		Regime Switching Stochastic Volatility Models (2 States)							
	Log-Squared Residuals		Regime Switching in the Level of Volatility		Regime Switching in the Level of Persistence		Regime Switching in the Level of Persistence and Volatility		Regime Switching in the Level of Persistence and Volatility for Log-Squared Residuals Modified with Breidt and Carriquiry (1996) Method (k=0.02)	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-0.0498	0.4022	-4.8999	0.6261	0.0006	0.0040	-2.5821	0.3487	-3.0606	0.0811
μ_1			0.5028	0.1331			0.8033	0.1226	0.5584	0.0963
ϕ_0	0.9922	0.0066	0.0864	0.0376	0.7034	0.0718	0.3856	0.2288	0.0344	0.0394
ϕ_1					2.9946	0.5359	0.1090	0.0536	0.1536	0.0639
$\sigma_{\eta 0}$	0.0875	0.0356	1.6661	0.1038	0.1797	0.1151	2.6882	0.2115	0.0000	0.0083
$\sigma_{\eta 1}$							1.3667	0.0791	1.4698	0.0634
σ_{φ}	2.2798	0.1089	0.0034	0.0015	2.2253	0.1156	0.0005	0.0030	0.3151	0.0475
$p^{(0,0)}$			0.1070	0.0543	0.8295	0.0366	0.2618	0.0855	0.1164	0.0616
$p^{(1,1)}$			0.8878	0.0326	0.1047	0.1784	0.7263	0.0494	0.8790	0.0203
$\sigma_{\eta 0}/\sigma_{\varphi}$	0.0384		490.0		0.0808		5376.4		0.0000	
$\sigma_{\eta 1}/\sigma_{\varphi}$							2733.4		4.6646	
ML Values	-1179.4		-1145.4		-1176.9		-1133.6		-1015.5	
AIC	2366.8		2304.8		2367.7		2285.2		2049.0	
BIC	2383.8		2334.6		2397.6		2323.5		2087.4	
STD	1.968		1.000		1.907		1.000		0.955	
Skewness	-0.431		-0.133		-0.416		-0.133		-0.157	
Excess Kurtosis	0.850		-1.977		0.716		-1.977		-1.902	
Jarque-Bera	31.879		86.6		26.2		86.6		80.8	

Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.

Table 3 Estimates of Unrestricted and Restricted 3 State Markov Regime Switching SV Models

A. Daily Volatility

	Regime Switching Stochastic Volatility Models (3 States)					
	Regime Switching in the Level of Volatility		Regime Switching in the Level of Persistence		Regime Switching in the Level of Persistence and Volatility	
	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-2.714	0.230	-2.952	0.238	-3.156	0.149
μ_1	-4.249	0.345			-1.251	0.184
μ_2	-11.762	0.641			-11.805	0.573
ϕ_0	-0.036	0.065	0.056	0.047	-0.138	0.041
ϕ_1			5.616	0.487	-0.353	0.065
ϕ_2			0.038	0.107	-0.604	0.218
$\sigma_{\eta 0}$	1.892	0.120	2.132	0.126	2.180	0.092
$\sigma_{\eta 1}$					0.000	-
$\sigma_{\eta 2}$					1.042	0.407
σ_{φ}	1.374	0.044	1.268	0.053	1.242	0.046
$p^{(0,0)}$	0.856	0.046	0.696	0.049	0.775	0.024
$p^{(0,1)}$	0.224	0.214	0.199	0.112	0.392	0.071
$p^{(1,0)}$	0.129	0.046	0.013	0.004	0.215	0.024
$p^{(1,1)}$	0.776	0.027	0.801	0.058	0.608	0.042
$p^{(1,2)}$	0.035	0.119	0.139	0.065	0.024	0.306
$p^{(2,2)}$	0.000	0.207	0.190	0.119	0.000	0.303
$\sigma_{\eta 0}/\sigma_{\varphi}$	1.377		1.681		1.754	
$\sigma_{\eta 1}/\sigma_{\varphi}$					0.000	
$\sigma_{\eta 2}/\sigma_{\varphi}$					0.839	
ML Values	-5989.860		-5999.474		-5968.7	
AIC	12003.719		12022.948		11969.5	
SIC	12074.106		12093.334		12063.3	
STD	1.169		1.129		1.148	
Skewness	-0.070		-0.014		-0.014	
Excess Kurtosis	0.126		-0.450		-0.310	
Jarque-Bera	3.854		22.1		10.5	

Notes: A total number of 2606 returns from 27 February 1992 to 27 February 2002 is used.

B. Weekly Volatility

	Regime Switching Stochastic Volatility Models (3 States)					
	Regime Switching in the Level of Volatility		Regime Switching in the Level of Persistence		Regime Switching in the Level of Persistence and Volatility	
	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-7.830	0.560	-5.870	0.760	-4.067	2.607
μ_1	1.069	0.301			1.059	0.728
μ_2	-4.080	0.349			-5.463	1.268
ϕ_0	0.074	0.130	-1.148	0.503	-0.555	0.973
ϕ_1			-1.234	1.328	-0.012	0.178
ϕ_2			0.098	0.249	0.392	0.151
$\sigma_{\eta 0}$	0.000	-	0.411	1.891	2.022	1.749
$\sigma_{\eta 1}$					0.000	-
$\sigma_{\eta 2}$					0.363	1.721
σ_{φ}	1.566	0.074	1.857	0.090	1.549	0.087
$p^{(0,0)}$	0.043	0.017	0.275	0.252	0.066	0.049
$p^{(0,1)}$	0.835	0.191	0.060	0.102	0.911	0.175
$p^{(1,0)}$	0.742	0.052	0.312	0.227	0.743	0.055
$p^{(1,1)}$	0.166	1.285	0.940	1.731	0.089	3.522
$p^{(1,2)}$	0.260	1.321	1.000	2.189	0.131	3.531
$p^{(2,2)}$	0.000	1.451	0.000	2.219	0.000	3.687
$\sigma_{\eta 0}/\sigma_{\varphi}$	0.000		0.221		1.305	
$\sigma_{\eta 1}/\sigma_{\varphi}$					0.000	
$\sigma_{\eta 2}/\sigma_{\varphi}$					0.234	
ML Values	-1144.188		-1155.840		-1141.1	
AIC	2312.376		2335.681		2314.2	
SIC	2363.468		2386.773		2382.3	
STD	1.603		2.092		1.601	
Skewness	-0.331		-0.410		-0.296	
Excess Kurtosis	0.984		0.724		0.054	
Jarque-Bera	30.551		79.3		31.8	

Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.

Figure 1 Smoothed Volatility and State Probabillity for S&P500 Index Daily Volatility

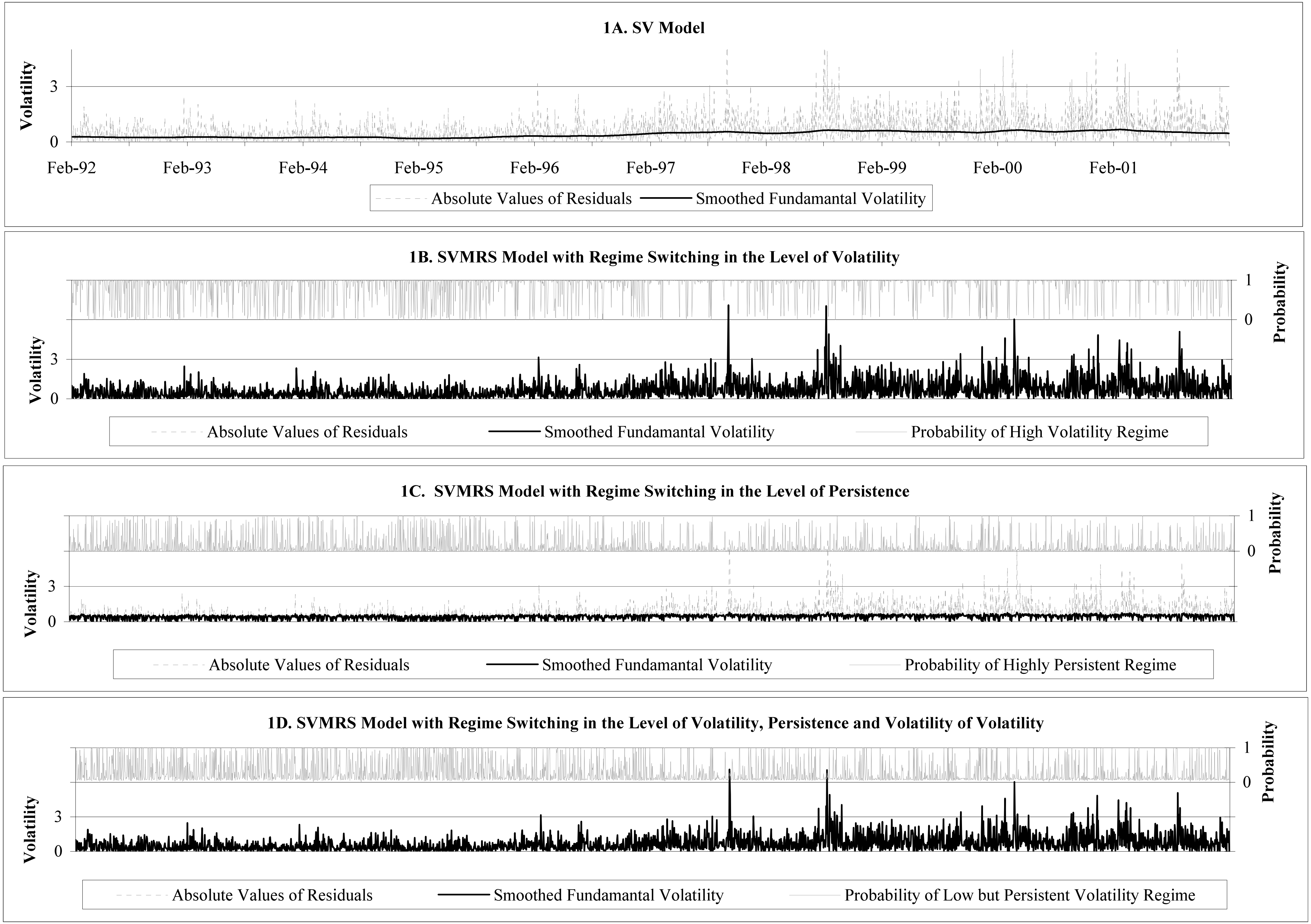

Figure 2 Smoothed Volatility and State Probabillity for S&P500 Index Weekly Volatility