Robust portfolio choice under preference uncertainty: A best-of-many approach

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Introduction

- Investing requires a risk aversion parameter.
- But in practice we never know it exactly
  - Risk preference questionnaire
  - Robo-advisor
  - Pension provider
- all produce

$$\gamma \in [a, b]$$

- rather than one precise value.
- Here γ is the parameter of the CRRA or power utility function.

Introduction

- Need a criterion to handle the uncertainty around γ.
- Range of $\gamma \in [a, b]$ given. Goal is one investment strategy m.
- Robust optimization:
  - Absolute welfare criterion
  - Relative regret criterion
  - Fractional regret criterion
- Imaginary adversary selects the worst-case risk type g in terms of the chosen criterion, while the agent anticipates this and chooses the robust investment strategy m.
- Anunrojwong et al. (2024)
- Avoiding overfitting to one criterion
- Best-of-many criterion

Risk elicitation Eckel and Grossman (2002)

The Eckel and Grossman measure.

Choice (50/50 Gamble)	Low payoff	High payoff
Gamble 1	28	28
Gamble 2	24	36
Gamble 3	20	44
Gamble 4	16	52
Gamble 5	12	60
Gamble 6	2	70

Risk elicitation Eckel and Grossman (2002)

The Eckel and Grossman measure.

Choice (50/50 Gamble)	Low payoff	High payoff	Implied CRRA range
Gamble 1	28	28	$3.46 < \gamma$
Gamble 2	24	36	$1.16 < \gamma < 3.46$
Gamble 3	20	44	$0.71 < \gamma < 1.16$
Gamble 4	16	52	$0.50 < \gamma < 0.71$
Gamble 5	12	60	$0 < \gamma < 0.50$
Gamble 6	2	70	$\gamma < 0$

Risk elicitation Eckel and Grossman (2002)

The Eckel and Grossman measure.

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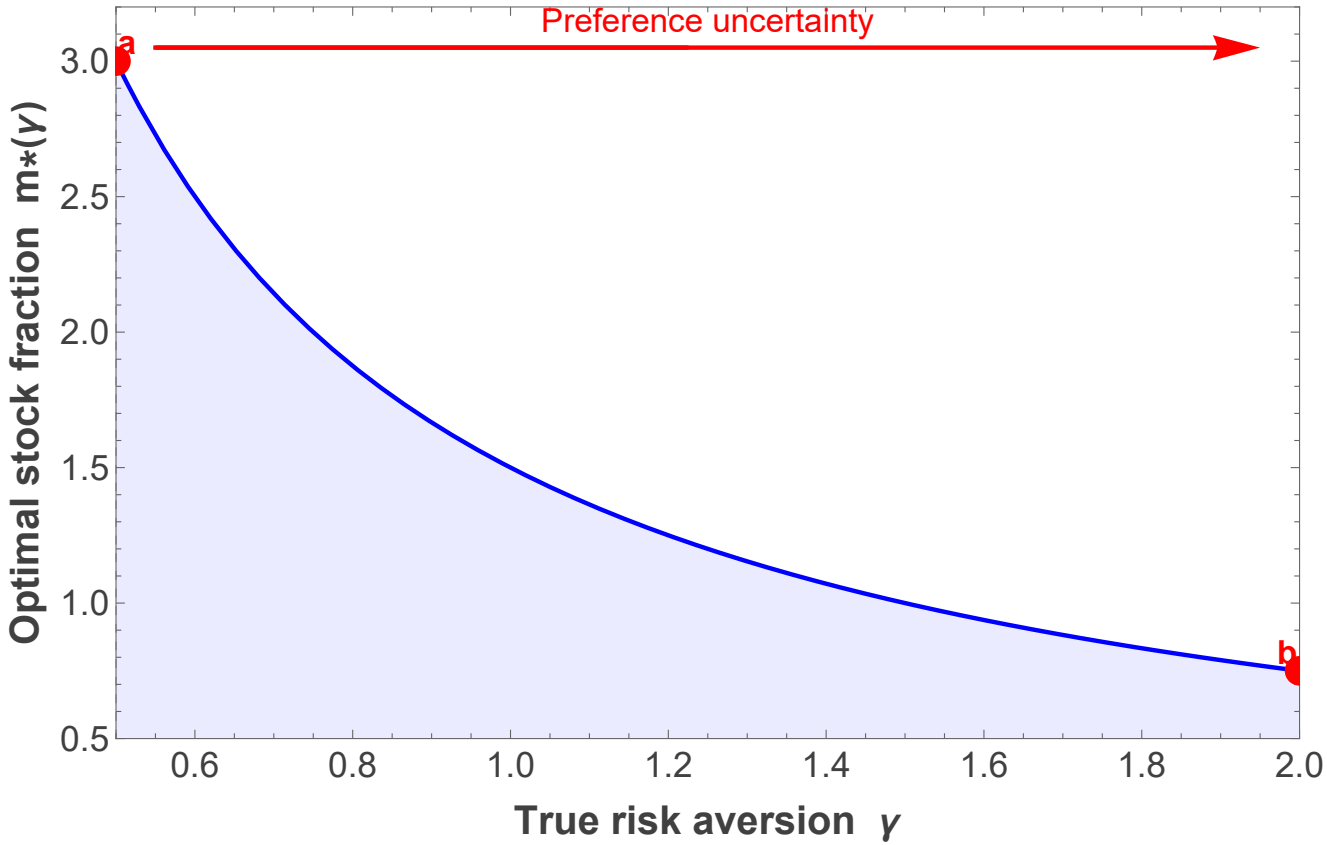
Gamble 5	12	60	$0 < \gamma < 0.50$
Gamble 6	2	70	$\gamma < 0$

Range of optimal investment strategies

- If $\gamma \in [0.5, 2]$
- Let $\mu = 0.07, r = 0.01, \sigma = 0.2$



Set-up

- Financial market
  - Risky asset dynamics

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

  - Risk-free bank account

$$dB_t = rB_t dt$$

  - Wealth process

$$\begin{aligned} dW_t &= W_t \left(m \frac{dS_t}{S_t} + (1-m) \frac{dB_t}{B_t} \right) \\ &= \left(r + m(\mu - r) \right) W_t dt + m\sigma W_t dZ_t \end{aligned}$$

- Preferences
  - CRRA utility function

$$u(x) = \begin{cases} \frac{x^{1-\gamma}-1}{1-\gamma} & \gamma > 0, \gamma \neq 1 \\ \ln x & \gamma = 1. \end{cases}$$

Optimum when knowing γ

- Solving the agent's problem while knowing his true γ coincides with Merton's portfolio problem

$$\max_m CE(m,\gamma) = \max_m u^{-1}\left(E\left[u(W_T)\right]\right) = \max_m W_0 e^{rT+(\mu-r)mT-\frac{1}{2}\gamma\sigma^2 m^2 T}$$

- The individual optimal investment strategy is

$$m^*(\gamma) = \frac{\mu - r}{\gamma\sigma^2}$$

CE growth rate

- The certainty equivalent growth rate is defined as

$$CE(m,\gamma) = W_0 e^{\kappa(m,\gamma)T}$$

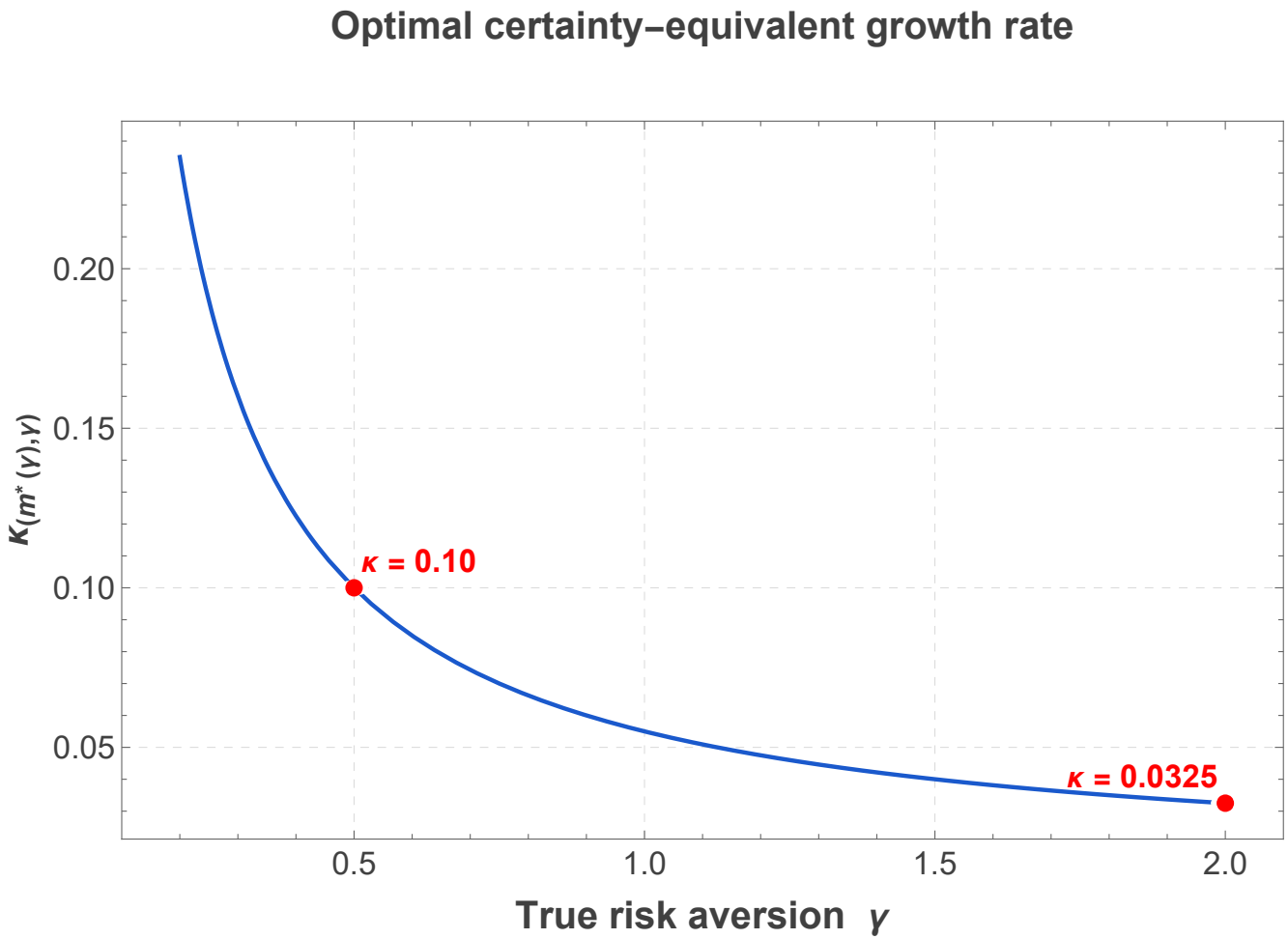
- The agent with risk aversion γ is indifferent between obtaining a certain rate of $\kappa(m,\gamma)$ per year and investing m in the risky stock.
- I.e.,

$$\kappa(m,\gamma) \quad = \quad r + (\mu - r)m - \frac{1}{2}m^2\sigma^2\gamma$$

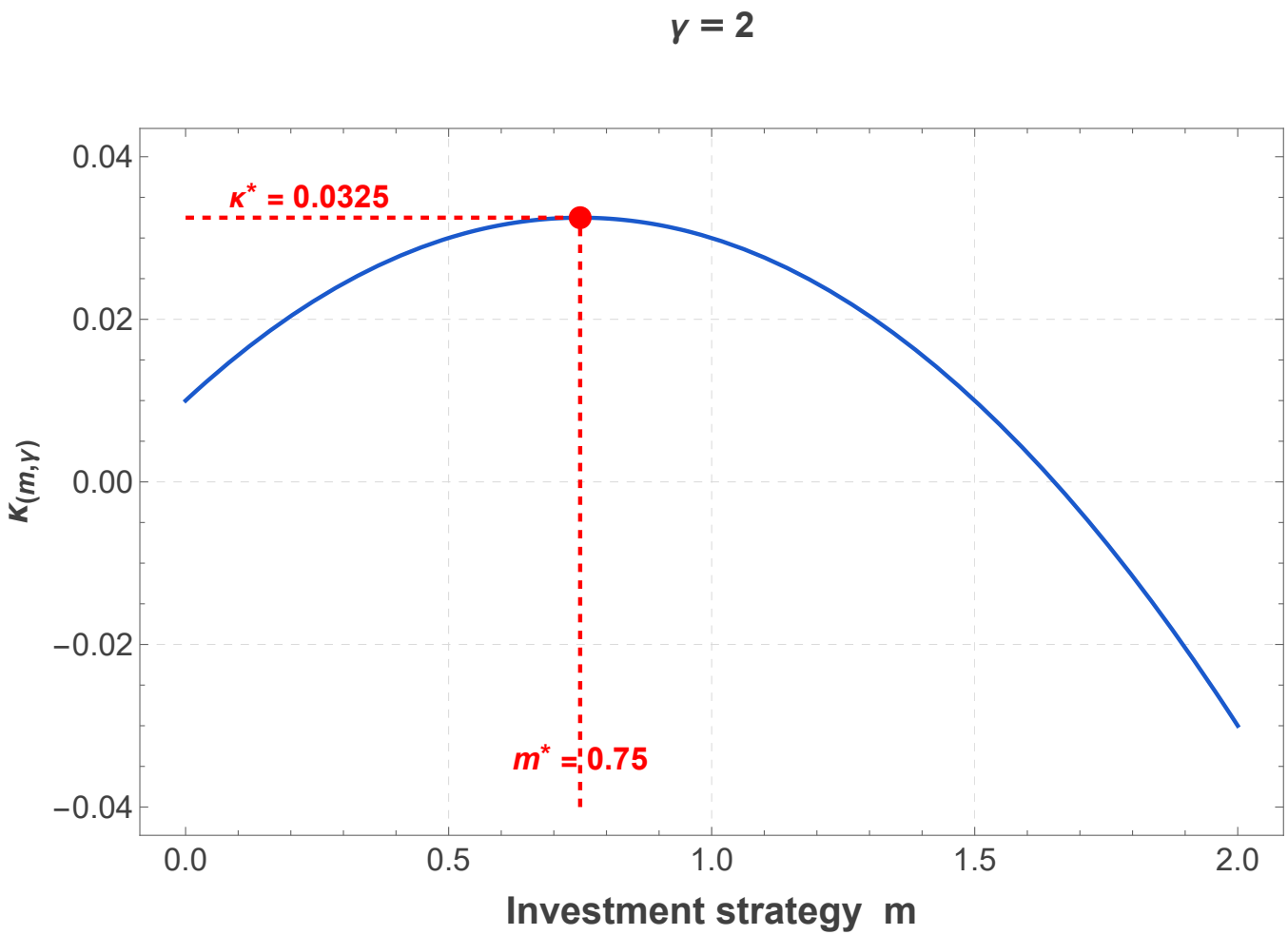
- The optimal growth rate for an agent with risk type γ is

$$\kappa(m^*(\gamma),\gamma) = r + \frac{1}{2}\frac{(\mu - r)^2}{\sigma^2\gamma}$$

Optimal CE growth rate



Suboptimal CE growth rate



Robust objectives

- Need a criterion to handle the uncertainty around γ.
- Range of $\gamma \in [a, b]$ given. Goal is one investment strategy m.
- Robust optimization:
  - Absolute welfare criterion
  - Relative regret criterion
  - Fractional regret criterion
- Imaginary adversary selects the worst-case risk type g in terms of the chosen criterion, while the agent anticipates this and chooses the robust investment strategy m.

Robust objectives

- Agent plays a simultaneous-move zero-sum game against an imaginary adversary: $\max_m \min_{g \in [a,b]}$
- Absolute criterion

$$\begin{aligned} \mathcal{A}(m,g) &= \kappa(m,g) \\ m^*(b) &= \frac{\mu - r}{\sigma^2 b} \\ g^* &= b \end{aligned}$$

- Relative criterion (Savage, 1972, Balter and Schweizer, 2024)

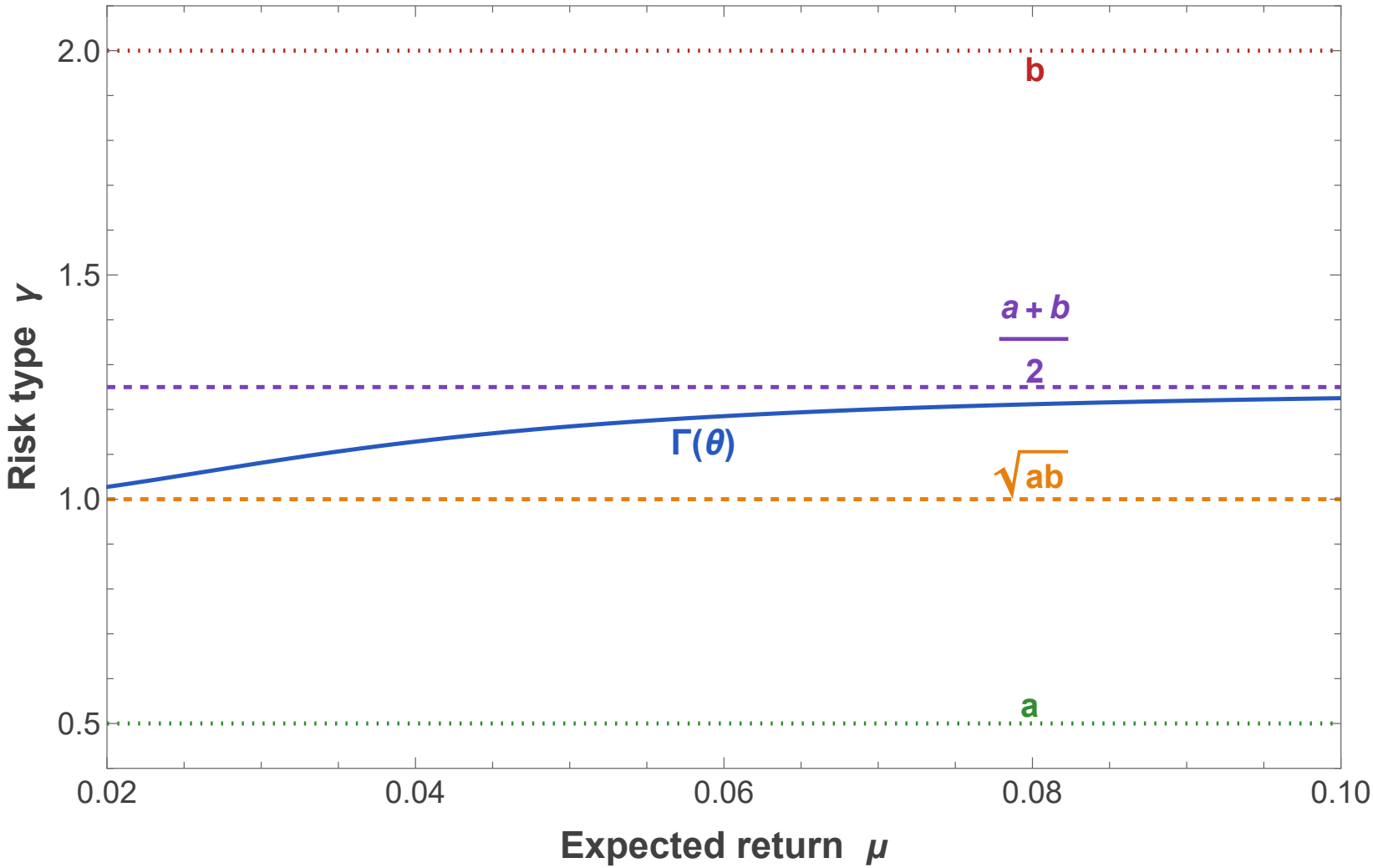
$$\begin{aligned} \mathcal{R}(m,g) &= \kappa(m,g) - \kappa(m^*(g),g) \\ m^*(\sqrt{ab}) &= \frac{\mu - r}{\sigma^2 \sqrt{ab}} \\ g^* &= (a, q; b, 1-q) \end{aligned}$$

Robust objectives

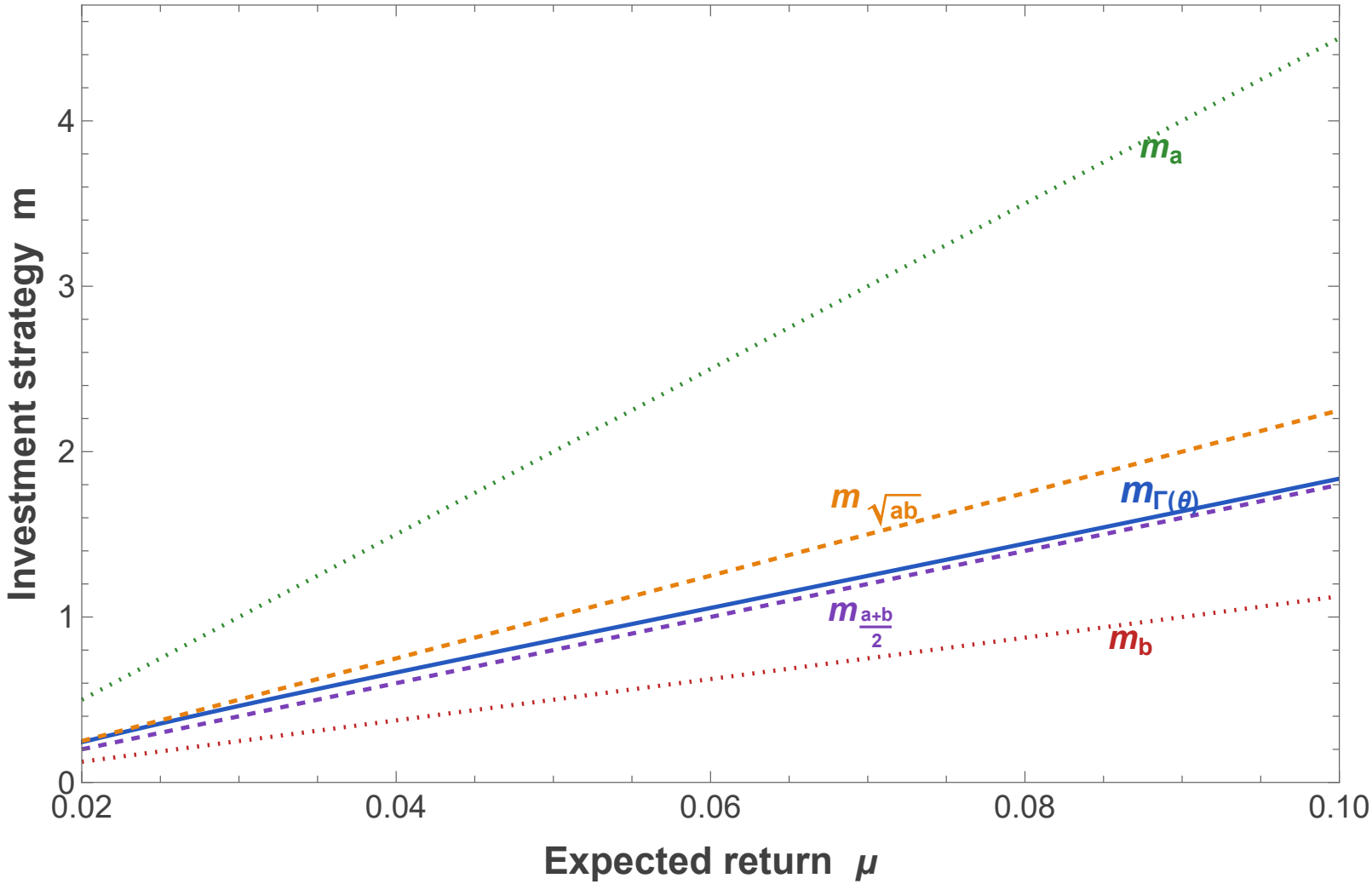
- Fractional criterion

$$
\begin{aligned}
\mathcal{F}(m,g) &= \frac{\kappa(m,g)}{\kappa(m^*(g),g)} \\
m^*(\Gamma(\theta)) &= \frac{\mu - r}{\sigma^2 \Gamma(\theta)} \\
\Gamma(\theta) &= \sqrt{(\theta + a)(\theta + b)} - \theta \\
\theta &= \frac{(\mu - r)^2}{2r\sigma^2} \\
g^* &= (a, p; b, 1-p)
\end{aligned}
$$

Strategies


2.0
1.5
1.0
0.5
Risk type γ
b
a + b
2
Γ(θ)
√ab
a
0.02
0.04
0.06
0.08
0.10
Expected return μ

Strategies


Investment strategy m
Expected return μ
0
1
2
3
4
0.02
0.04
0.06
0.08
0.10
m_a
$m_{\sqrt{ab}}$
$m_{\Gamma(\theta)}$
$m_{\frac{a+b}{2}}$
m_b

Best-of-many

- Anunrojwong et al. (2024)
- Avoiding overfitting to one criterion
- Best-of-many

$$\begin{aligned}\mathcal{B}(m,g) &= \min\left\{\mathcal{P}^{\mathcal{A}}(m,g),\, \mathcal{P}^{\mathcal{R}}(m,g),\, \mathcal{P}^{\mathcal{F}}(m,g)\right\} \\ &= \min\left\{\frac{\mathcal{A}(m,g)}{\mathcal{A}(m^*(b),b)}, 2-\frac{\mathcal{R}(m,g)}{\mathcal{R}(m^*(\sqrt{ab}),\{a,q;\ b,1-q\})},\right. \\ &\qquad \left.\frac{\mathcal{F}(m,g)}{\mathcal{F}(m^*(\Gamma(\theta)),\{a,p;\ b,1-p\})}\right\}\end{aligned}$$

Best-of-many

- Solution to best-of-many

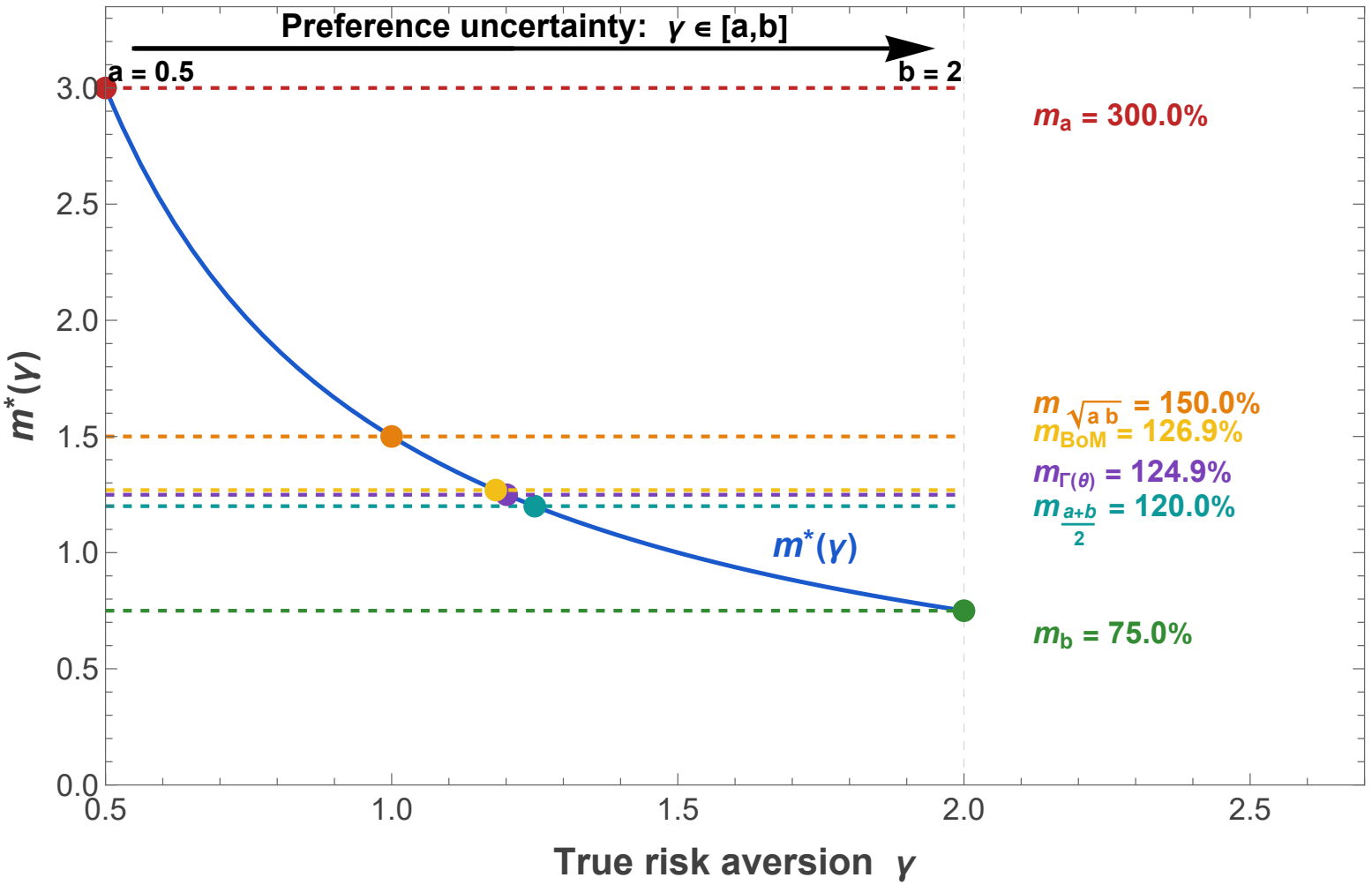
$$
\begin{aligned}
m^*(\gamma^{BoM}) &= \frac{\mu - r}{\sigma^2 \gamma^{BoM}} \\
\gamma^{BoM} &= \min\{\gamma^{AR}, \gamma^{AF}\}
\end{aligned}
$$

where γ^{AR} and γ^{AF} are determined by

$$
\begin{aligned}
\mathcal{P}^{\mathcal{A}}(m(\gamma^{AR}), b) &= \mathcal{P}^{\mathcal{R}}(m(\gamma^{AR}), a) \\
\mathcal{P}^{\mathcal{A}}(m(\gamma^{AF}), b) &= \mathcal{P}^{\mathcal{F}}(m(\gamma^{AF}), a)
\end{aligned}
$$

- When risk preferences are only known up to an interval, the dominant trade-off is between sacrificing **CE growth for the most risk-averse type** and creating **regret for the least risk-averse type**.

Let $a = 0.5$ and $b = 2$



CE growth rates

Strategy	$\gamma_a = 0.5$ κ	Δ	$\gamma_b = 2$ κ	Δ
$m_a = 300\%$	**10.0%**		-17.0%	
$m_b = 75\%$	4.9%		**3.3%**	
$m_{\sqrt{ab}} = 150\%$	7.7%		1.0%	
$m_{\text{BoM}} = 127\%$	7.0%		2.2%	

CE growth rates

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$m_a = 300\%$	$\mathbf{10.0\%}$	0.0%	-17.0%	-20.3%
$m_b = 75\%$	4.9%	-5.1%	$\mathbf{3.3\%}$	0.0%
$m_{\sqrt{ab}} = 150\%$	7.7%	$\mathbf{-2.3\%}$	1.0%	$\mathbf{-2.3\%}$
$m_{\text{BoM}} = 127\%$	7.0%	-3.0%	2.2%	-1.1%

Best-of-many

Strategy	$\gamma_a = 0.5$ κ	Δ	$\gamma_b = 2$ κ	Δ
$m_a = 300\%$	**10.0%**	0.0%	−17.0%	−20.3%
$m_b = 75\%$	4.9%	−5.1%	**3.3%**	0.0%
$m_{\sqrt{ab}} = 150\%$	7.7%	**−2.3%**	1.0%	**−2.3%**
$m_{\mathrm{BoM}} = 127\%$	7.0%	**−3.0%**	**2.2%**	−1.1%

$$\mathcal{P}^{\mathcal{A}}(m_{BoM}, b) = \frac{\mathcal{A}(m_{BoM}, b)}{\mathcal{A}(m^*(b), b)} = \frac{2.2\%}{3.3\%} = 0.668$$

$$\mathcal{P}^{\mathcal{R}}(m_{BoM}, a) = 2 - \frac{\mathcal{R}(m_{BoM}, a)}{\mathcal{R}(m^*(\sqrt{ab}), \{a, q;\ b, 1-q\})} = 2 - \frac{-3.0\%}{-2.3\%} = 0.668$$

Performance measures for $a = 0.5, b = 2$

		Normalized performance		
Criterion	Optimal risk type	$\mathcal{P}^{\mathcal{A}}$	$\mathcal{P}^{\mathcal{R}}$	$\mathcal{P}^{\mathcal{F}}$
Absolute	b	1	-0.25	0.7120
Relative	$\sqrt{ab}$	0.3077	1	0.4437
Fractional	$\Gamma(\theta)$	0.6934	0.6374	1
Best-of-many	γ^{BoM}	**0.6683**	**0.6683**	0.9640

Main result

- The binding trade-off is between the absolute criterion $\mathcal{A}$ for the most risk-averse type b and one of the two regret criteria for the least risk-averse type a.
  - The fractional-regret criterion $\mathcal{F}$ determines the solution for sufficiently small a.
  - The additive-regret criterion $\mathcal{R}$ determines the solution otherwise.

Near risk neutrality: $a = 0.01$ and $b = 2$

Strategy	$\gamma_a = 0.01$ κ	Ratio	$\gamma_b = 2$ κ	Ratio
$m_a = 15000\%$	**451.0%**		-89099.0%	
$m_b = 75\%$	5.5%		**3.3%**	

Near risk neutrality: $a = 0.01$ and $b = 2$

	$\gamma_a = 0.01$		$\gamma_b = 2$	
Strategy	κ	Ratio	κ	Ratio
$m_a = 15000\%$	**451.0%**	100.0%	−89099.0%	−2741508.1%
$m_b = 75\%$	5.5%	1.2%	**3.3%**	100.0%
$m_{\Gamma(\theta)} = 164\%$	10.8%	**2.4%**	0.1%	**2.4%**

Near risk neutrality: $a = 0.01$ and $b = 2$

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$m_a = 15000\%$	**451.0%**	100.0%	−89099.0%	−2741508.1%
$m_b = 75\%$	5.5%	1.2%	**3.3%**	100.0%
$m_{\Gamma(\theta)} = 164\%$	10.8%	**2.4%**	0.1%	**2.4%**
$m_{\text{BoM}} = 120\%$	8.1%	**1.8%**	**2.5%**	75.5%

$$\mathcal{P}^{\mathcal{F}}(m_{BoM}, a) = \frac{\mathcal{F}(m_{BoM}, a)}{\mathcal{F}(m^*(\Gamma(\theta)), \{a, p;\ b, 1-p\})} = \frac{1.8\%}{2.4\%} = 0.76$$

and

$$\mathcal{P}^{\mathcal{A}}(m_{BoM}, b) = \frac{\mathcal{A}(m_{BoM}, b)}{\mathcal{A}(m^*(b), b)} = \frac{2.5\%}{3.3\%} = 0.76$$

Performance measures

		Normalized performance		
Criterion	Optimal risk type	$\mathcal{P}^{\mathcal{A}}$	$\mathcal{P}^{\mathcal{R}}$	$\mathcal{P}^{\mathcal{F}}$
Absolute	b	1	0.8536	0.5087
Relative	$\sqrt{ab}$	-118.6	1	**-4956.4**
Fractional	$\Gamma(\theta)$	**0.0239**	0.8672	1
Best-of-many	γ^{BoM}	**0.7552**	0.8604	**0.7552**

Scenario-based portfolio choice

- Use previous investment decisions as rule of thumb for aggregating uncertainty about γ when underlying model varies from standard Black-Scholes world.
- E.g., stock returns are not log-normally distributed but follow a GARCH process or come from the KNW-model.
- Performance close to optimal when underlying process deviates from BS.

Conclusion

- We derive robust investment rules while acknowledging preference uncertainty
- The fractional regret criterion leads to effective risk preferences that are sensitive to the market parameters
- To avoid overfitting to one criterion, we introduce the best-of-many criterion.
- Numerically, we find that the best-of-many never leads to lower performance than 51% of what could be achieved under each criterion separately.
- The investment rules also work well for different data generating processes.

Thank you!

References

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Eckel, C. C. and Grossman, P. J. (2002). Sex differences and statistical stereotyping in attitudes toward financial risk. *Evolution and Human Behavior*, 23(4):281–295.

Savage, L. J. (1972). *The Foundations of Statistics*. Courier Corporation.

	$\gamma_a = 0.5$			$\gamma_b = 2$		
Strategy	κ	Δ	Ratio	κ	Δ	Ratio
$m_a = 300\%$	**10.0%**	0.0%	100.0%	−17.0%	−20.3%	−523.1%
$m_b = 75\%$	4.9%	−5.1%	49.4%	**3.3%**	0.0%	100.0%
$m_{\sqrt{ab}} = 150\%$	7.7%	**−2.3%**	77.5%	1.0%	**−2.3%**	30.8%
$m_{\Gamma(\theta)} = 124.9\%$	6.9%	−3.1%	**69.3%**	2.3%	−1.0%	**69.3%**
$m_{\mathrm{BoM}} = 126.9\%$	7.0%	**−3.0%**	70.0%	**2.2%**	−1.1%	66.8%

	$\gamma_a = 0.01$			$\gamma_b = 2$		
Strategy	κ	Δ	Ratio	κ	Δ	Ratio
$m_a = 15000\%$	**451**%	0%	100%	−89099%	−89102%	−2741508%
$m_b = 75\%$	5.5%	−446%	1.2%	**3.3**%	0%	100%
$m_{\sqrt{ab}} = 1061\%$	62.4%	−**389**%	13.8%	−385%	−**389**%	−11857%
$m_{\Gamma(\theta)} = 164\%$	10.8%	−440%	**2.4**%	0.1%	−3.2%	**2.4**%
$m_{\mathrm{BoM}} = 120\%$	8.1%	−443%	**1.8%**	**2.5%**	−0.8%	75.5%