# De-Risking Defined Benefit Plans

Yijia Lin
University of Nebraska - Lincoln

Richard D. MacMinn
Illinois State University

Ruilin Tian
North Dakota State University

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# Pension De-Risking Strategies

Table: Cost and Risk Comparisons among Three Pension De-Risking Strategies

| | Longevity Hedging | Buy-In | Buy-Out |
|---|---|---|---|
| Simplicity | Less Complex | More Complex | More Complex |
| Affordability* | More Affordable | More Affordable | Less Affordable |
| Default Risk | Yes | Yes | No |

*For underfunded plans.

- Little is known about whether and the extent to which the near-term costs of de-risking can be offset by the longer-term savings.

# Outline

- We propose an optimization model that minimizes total pension cost while controlling pension funding tail risk.
- We directly link various costs and counter-party risk to a plan's de-risking decision.
  - We examine optimal longevity hedging, buy-in and buy-out strategies.
  - A plan's funding status, transaction cost and counter-party risk are significant determinants of its level of risk management.

# Basic Framework

- Total pension cost $TPC^j$ for de-risking strategy $j$ ($j = LH, BI,$ or $BO$) (Cox et al., 2012)

$$TPC^j = HC^j + \sum_{t=1}^{\infty} \frac{C_t^j(1+\psi_1) - W_t^j(1-\psi_2)}{(1+r)^t}, \qquad (1)$$

where $r$ is the discount rate. The constants $\psi_1$ and $\psi_2$ are penalty factors on contributions $C_t^j$ and withdrawals $W_t^j$ respectively. $HC^j$ is the initial hedge cost.

- The initial hedge cost $HC^j$ ($j = LH, BI,$ or $BO$) depends on
  - Risk premium: $\delta_1^{LH}$, $\delta_1^{BI}$ and $\delta_1^{BO}$
  - Search and monitoring cost: $\delta_2^{LH}$, $\delta_2^{BI}$ and $\delta_2^{BO}$
  - Underfunding cost of the buy-out:

$$\delta_3^{BO} = \gamma \max\left(0, \frac{UL_0}{PL_0}\right) \qquad (2)$$

# Initial Hedge Cost Per Unit Risk Ceded

- Longevity hedging

$$\delta^{LH} = \delta^{LH}_{1+2} = \delta^{LH}_{1} + \delta^{LH}_{2} \tag{3}$$

- Buy-in

$$\delta^{BI} = \delta^{BI}_{1+2} = \delta^{BI}_{1} + \delta^{BI}_{2} \tag{4}$$

- Buy-out

$$\delta^{BO} = \delta^{BO}_{1+2} + \delta^{BO}_{3} = \delta^{BO}_{1} + \delta^{BO}_{2} + \delta^{BO}_{3} \tag{5}$$

# Counter-Party Default Risk of Longevity Hedging and Buy-Ins

Set $X_0 = 1$. For all $t = 1, 2, \cdots$, let

$$X_t = \begin{cases} 0 & \text{with probability } p \\ 1 & \text{with probability } 1 - p \end{cases}, \tag{6}$$

where $p$ is the default probability of the longevity hedging or buy-in risk taker per period.

- The default process

$$\mathbb{I}_t = X_t \cdot X_{t-1}. \tag{7}$$

# Objective Function and Optimization Problem

$$
\begin{aligned}
\underset{w^j, h^j}{\text{Minimize}} \quad & \mathrm{E}\left[TPC^j\right] \\
\text{subject to} \quad & \mathrm{E}(TUL^j) = 0 \\
& \mathrm{CVaR}_\alpha(TUL^j) \leq \tau \\
& \frac{HP^j}{1+\delta^j} \leq PL_0 \\
& HP^j \leq PA_0 \\
& 0 \leq h^j \leq 1 \\
& 0 \leq w_i^j \leq 1, \qquad i = 1, 2, \cdots, n \\
& \sum_{i=1}^{n} w_i^j = 1.
\end{aligned} \tag{8}
$$

where the constant $\tau$ is the pre-specified parameter reflecting the plan's underfunding downside risk tolerance.

# Example Assumptions

- A cohort with all retirees at age $x_0 = 65$ at $t = 0$.
- Annual retirement benefit of $B = \$10$ million
- The pension funds are invested in three assets:
  - S&P 500 index;
  - Merrill Lynch corporate bond index;
  - 3-month T-bill.
- Pension discount rate $r = 0.05$
- Penalty factors on contributions and withdrawals are both equal to $\psi_1 = \psi_2 = 0.2$

# Total Pension Cost: Longevity Hedging

Table: Optimal Longevity Hedging Strategies Given $UL_0/PL_0 = 13.2\%$, $PA_0 = 95$ and Different $\delta_{1+2}^{LH}$ and $p^{LH}$

| $p^{LH}$ | 0.1% | | | 0.5% | | |
|---|---|---|---|---|---|---|
| $\delta_{1+2}^{LH}$ | 0.02 | 0.03 | 0.05 | 0.02 | 0.03 | 0.05 |
| $w_1^{LH}$ | 9% | 9% | 8% | 9% | 9% | 8% |
| $w_2^{LH}$ | 50% | 50% | 49% | 50% | 50% | 49% |
| $w_3^{LH}$ | 41% | 41% | 43% | 41% | 41% | 43% |
| $h^{LH}$ | 100% | 100% | 37% | 100% | 100% | 34% |
| $\mathrm{E}(TPC^{LH})$ | 7.06 | 7.11 | 7.20 | 7.07 | 7.12 | 7.20 |

# Total Pension Cost: Buy-Ins

Table: Optimal Buy-In Strategies Given $UL_0/PL_0 = 13.2\%$, $PA_0 = 95$ and Different $\delta^{BI}_{1+2}$ and $p^{BI}$

| $p^{BI}$ | 0.1% | | | 0.5% | | |
|---|---|---|---|---|---|---|
| $\delta^{BI}_{1+2}$ | 0.02 | 0.03 | 0.04 | 0.02 | 0.03 | 0.04 |
| $w^{BI}_1$ | 7% | 12% | 8% | 7% | 10% | 8% |
| $w^{BI}_2$ | 93% | 71% | 49% | 60% | 61% | 49% |
| $w^{BI}_3$ | 0.0% | 17% | 43% | 33% | 29% | 43% |
| $h^{BI}$ | 66% | 50% | 0% | 39% | 39% | 0% |
| $\mathrm{E}(TPC^{BI})$ | 6.10 | 6.90 | 7.20 | 6.47 | 6.94 | 7.20 |

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- The expected total pension cost increases with the default probability.

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| $w_1^{LH}$ | 9% | 9% | 8% | 9% | 9% | 8% |
| $w_2^{LH}$ | 50% | 50% | 49% | 50% | 50% | 49% |
| $w_3^{LH}$ | 41% | 41% | 43% | 41% | 41% | 43% |
| $h^{LH}$ | 100% | 100% | 37% | 100% | 100% | 34% |
| $\mathrm{E}(TPC^{LH})$ | **7.06** | 7.11 | 7.20 | **7.07** | 7.12 | 7.20 |

- The impact of the default risk on the longevity hedging is much lower.

# Total Pension Cost: Buy-Outs

Table: Optimal Buy-Out Strategies Given $UL_0/PL_0 = 13.2\%$, $PA_0 = 95$ and Different $\delta_{1+2}^{BO}$ and $\gamma$

| $\gamma$ | 0.05 | | | 0.1 | |
|---|---|---|---|---|---|
| $\delta_{1+2}^{BO}$ | 0.02 | 0.03 | 0.04 | 0.02 | 0.03 |
| $w_1^{BO}$ | 15% | 9% | 8% | 10% | 8% |
| $w_2^{BO}$ | 82% | 52% | 49% | 59% | 49% |
| $w_3^{BO}$ | 3% | 39% | 43% | 31% | 43% |
| $h^{BO}$ | 59% | 12% | 0% | 31% | 0% |
| $\mathrm{E}(TPC^{BO})$ | 6.66 | 7.19 | 7.20 | 7.08 | 7.20 |

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| $h^{BO}$ | 59% | 12% | 0% | 31% | 0% |
| $\mathrm{E}(TPC^{BO})$ | **6.66** | 7.19 | 7.20 | **7.08** | 7.20 |

- A higher underfunding cost makes the buy-out more expensive.

# Impact of Initial Underfunding Ratio $UL_0/PL_0$ on Total Pension Cost: Buy-Ins vs. Buy-Outs

Table: Optimal Buy-Ins and Buy-Outs Given $\delta^{BI}_{1+2} = \delta^{BO}_{1+2} = 0.02$

| $UL_0/PL_0$ | 17.7% | | 13.2% | | 8.6% | |
|---|---|---|---|---|---|---|
| $p^{BI}$ | 0.1% | 0.5% | 0.1% | 0.5% | 0.1% | 0.5% |
| $\mathrm{E}(TPC^{BI})$ | 7.29 | 7.40 | 6.10 | 6.47 | 5.04 | 5.62 |
| $\gamma$ | 0.05 | 0.1 | 0.05 | 0.1 | 0.05 | 0.1 |
| $\mathrm{E}(TPC^{BO})$ | 7.88 | 8.07 | 6.66 | 7.08 | 5.36 | 5.76 |

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| $p^{BI}$ | 0.1% | 0.5% | 0.1% | 0.5% | 0.1% | 0.5% |
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| $\mathrm{E}(TPC^{BI})$ | 7.29 | 7.40 | 6.10 | 6.47 | 5.04 | **5.62** |
| $\gamma$ | 0.05 | 0.1 | 0.05 | 0.1 | **0.05** | 0.1 |
| $\mathrm{E}(TPC^{BO})$ | 7.88 | 8.07 | 6.66 | 7.08 | **5.36** | 5.76 |

- The buy-in is more appropriate for an underfunded plan with a low counter-party default risk.

# Impact of Initial Underfunding Ratio $UL_0/PL_0$ on Total Pension Cost: Buy-Ins vs. Longevity Hedging

Table: Optimal Buy-Ins and Longevity Hedging Given $\delta_{1+2}^{BI} = \delta_{1+2}^{BO} = 0.02$

| $UL_0/PL_0$ | 17.7% | | 13.2% | | 8.6% | |
|---|---|---|---|---|---|---|
| $p^{BI}$ | 0.1% | 0.5% | 0.1% | 0.5% | 0.1% | 0.5% |
| $\mathrm{E}(TPC^{BI})$ | 7.29 | 7.40 | 6.10 | 6.47 | 5.04 | 5.62 |
| $p^{LH}$ | 0.1% | 0.5% | 0.1% | 0.5% | 0.1% | 0.5% |
| $\mathrm{E}(TPC^{LH})$ | 7.95 | 7.96 | 7.06 | 7.07 | 6.23 | 6.24 |

# Impact of Initial Underfunding Ratio $UL_0/PL_0$ on Total Pension Cost: Buy-Ins vs. Longevity Hedging

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| $p^{LH}$ | 0.1% | 0.5% | 0.1% | 0.5% | 0.1% | 0.5% |
| $\mathrm{E}(TPC^{LH})$ | **7.95** | **7.96** | **7.06** | **7.07** | **6.23** | **6.24** |

- The expected total pension cost of the longevity hedging is always higher than that of the buy-in.

# Initial Cash Requirements of Three De-Risking Strategies

Table: Hedge Prices of Three De-Risking Strategies Given Different $UL_0/PL_0$ and $PA_0$ at $\delta^{LH}_{1+2} = \delta^{BI}_{1+2} = \delta^{BO}_{1+2} = 0.02$

| | $UL_0/PL_0$ | 17.7% | 13.2% | 8.6% |
|---|---|---|---|---|
| Longevity Hedging | $p^{LH} = 0.1\%$ | 4.59 | 4.59 | 4.59 |
| | $p^{LH} = 0.5\%$ | 4.23 | 4.23 | 4.23 |
| Buy-In | $p^{BI} = 0.1\%$ | 61.35 | 72.67 | 83.95 |
| | $p^{BI} = 0.5\%$ | 44.19 | 40.84 | 38.78 |
| Buy-Out | $\gamma = 0.05$ | 44.67 | 65.83 | 79.93 |
| | $\gamma = 0.1$ | No Hedge | 35.22 | 75.18 |

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| | $UL_0/PL_0$ | 17.7% | 13.2% | 8.6% |
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| Longevity Hedging | $p^{LH} = 0.1\%$ | **4.59** | **4.59** | **4.59** |
| | $p^{LH} = 0.5\%$ | **4.23** | **4.23** | **4.23** |
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| Buy-Out | $\gamma = 0.05$ | 44.67 | 65.83 | 79.93 |
| | $\gamma = 0.1$ | No Hedge | 35.22 | 75.18 |

# Conclusions

- This paper proposes a model to identify the optimal asset allocation and pension de-risking strategy subject to an underfunding CVaR constraint for a DB pension plan.
- The pension de-risking strategies are sensitive to various costs and counter-party risk.
  - If the counter-party risk is low, it is more desirable to implement the longevity hedging or buy-in strategies for an excessively underfunded pension plan.
  - The buy-in strategy is more sensitive to the counter-party risk than the longevity hedging.