Modeling Multi-Country Mortality Dependence and Its Application in Pricing Survivor Swaps: A Dynamic Copula Approach

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Outline

- Introduction
- Stochastic Mortality under a Multi-Country Setting
  - Lee-Carter Model
  - Copula Functions
- Examining and Selecting the Copula Model for Modeling Mortality Dependence across Countries
- Valuation Framework for Survivor Swaps
- Analysis of Fair Charges for Survivor Swaps

Introduction

- Many researches focused on design & pricing of mortality linked securities
  - Survivor Swaps(Dawson, 2002; Blake, 2003; Lin and Cox, 2005; Dowd et al., 2006)
  - Longevity Bonds

Introduction

- Increase Hedge effectiveness
  - The underlying combined mortality index of a mortality linked security plays a important role
  - Cannot ignore basis risk in mortality linked securities

- Wang & Yang (2013) capture multi-country morality dynamics applying co-integration analysis
- Li and Hardy (2011) examine basis risk by four approaches

Introduction

- However, short-term catastrophe mortality shocks
  - Jump Effect

- Wang et al. (2011) demonstrate heavy-tailed distribution appear long-term mortality data

Introduction

- Our purpose
  - Dealing with the heavy-tailed distribution for long-term mortality data
  - Modeling mortality dependence (structure) across countries using a dynamic copula approach
  - Determining the dependence structure of mortality rate using actual mortality data
  - Application: Pricing survivor swaps

Introduction

- Copula model
  - Non-standard multivariate distributions
  - Capture the symmetric or asymmetric dependence
  - Time varying dependence
- Using time varying copula for modeling long-term multi-country mortality

Introduction

- Morality model
  - Lee-Carter with jump diffusion (JD) & Generalized Hyperbolic (GH) innovation

  - GH innovation can capture
    - Skewness
    - Leptokurtosis
    - Tail-thickness
    - Nests many distribution (Normal; T; Hyperbolic; Variance Gamma (VG); Normal Inverse Gaussian (NIG); GH skewed T (GHST))

Stochastic Mortality under a Multi-Country Setting

- Multi-country mortality with dependence
  - Lee-Carter & time-varying copula
  - Marginal Distribution : JD & GH distribution

- Lee Carter Model under multi-country

$$\ln m_{x,t}^{j} = a_x^{j} + b_x^{j} k_t^{j} + e_{x,t}^{j} \quad j = 1, \ldots, 2N$$

$$k_t^{j} - k_{t-1}^{j} = \gamma^{j} + \varepsilon_t^{j} \quad j = 1, \ldots, 2N$$

Copula Function

- Symmetric dependence: Gaussian & Student-T copula
  - Gaussian:

$$C_N(u_1,\ldots,u_{2N};\Sigma) = \Phi_{\Sigma}(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_{2N}))$$

  - Student-T:

$$C_T(u_1,\ldots,u_{2N};\Sigma,v) = T_{\Sigma,v}(T_v^{-1}(u_1),\ldots,T_v^{-1}(u_{2N}))$$

Stochastic Mortality under a Multi-Country Setting

- Copula Function
  - Asymmetric dependence: Gumbel & Clayton copula
    - Gumbel (upper tail dependence):

$$C_G(u_1,\ldots,u_N;\theta)=\exp\left\{-\left[\left(\sum_{i=1}^{N}(-\ln u_i)^{\theta}\right)^{1/\theta}\right]\right\}$$

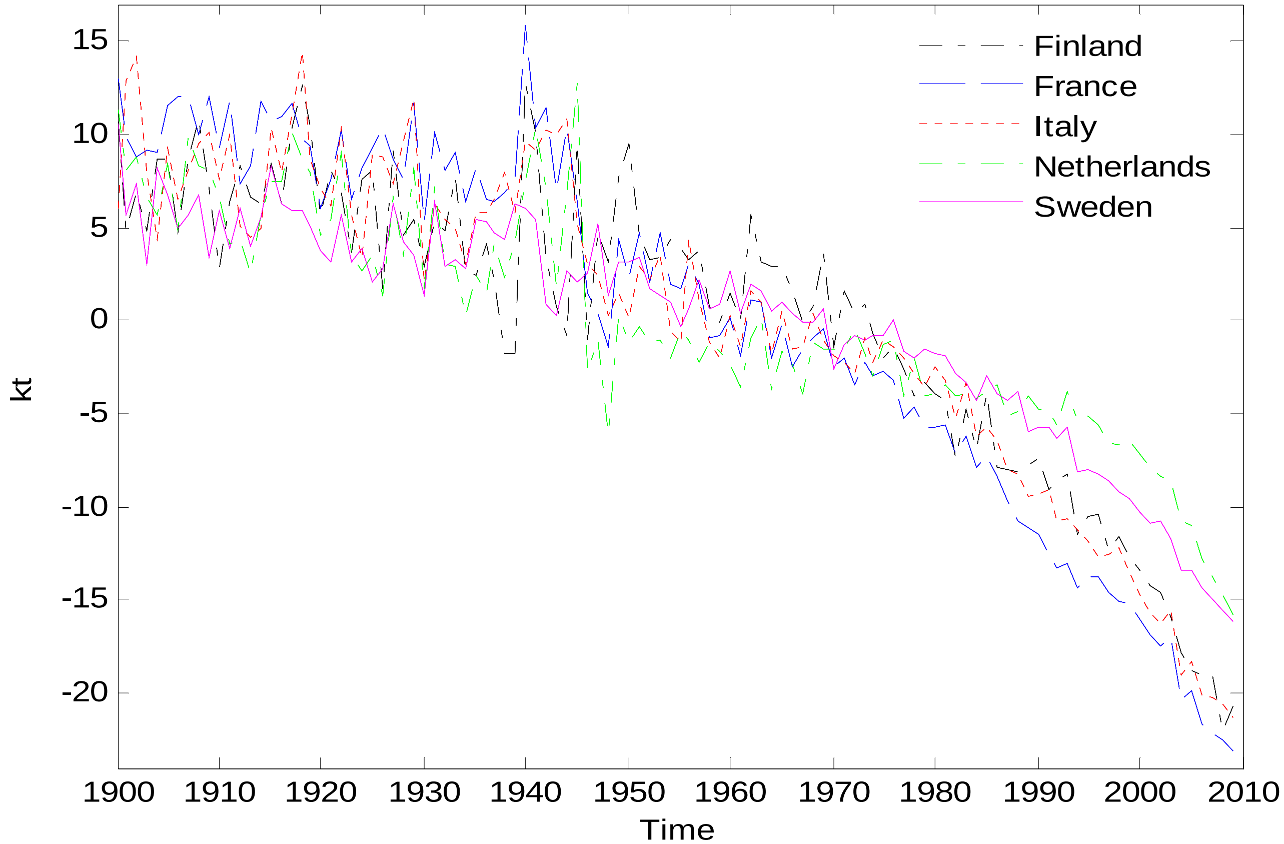
    - Clayton (lower tail dependence):

$$C_C(u_1,u_2,\ldots,u_N;\theta)=\left(\sum_{i=1}^{N}u_i^{-\theta}-n+1\right)^{-1/\theta}$$

Examining and Selecting the Copula Model for Modeling Mortality Dependence across Countries

Mortality Data

- Using the data from HMD
- Demonstrating with five countries, Males.



Goodness of Fits-Kt

Table 1 : Finland_kt

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-284.77	286.77	289.46	8	8	8
T	-276.08	279.08	283.12	7	1	1
JD	-275.30	280.30	287.02	1	6	6
VG	-275.78	279.78	285.17	5	4	4
NIG	-275.34	279.34	284.72	3	2	2
GHST	-275.55	279.55	284.93	4	3	3
HYP	-275.84	279.84	285.22	6	5	5
GH	-275.33	280.33	287.06	2	7	7

Goodness of Fits-Kt

Table 2 : France_kt

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-241.74	243.74	246.43	8	8	2
T	-237.79	240.79	244.83	6	1	1
JD	-238.42	243.42	250.15	7	7	8
VG	-237.73	241.73	247.12	5	5	6
NIG	-237.53	241.53	246.91	2	2	3
GHST	-237.68	241.68	247.06	4	4	5
HYP	-237.60	241.60	246.99	3	3	4
GH	-237.53	242.53	249.26	1	6	7

Goodness of Fits-Kt

Table 3 : Italy_kt

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-256.84	258.84	261.53	8	8	7
T	-252.76	255.76	259.80	7	5	4
JD	-249.74	254.74	261.47	4	4	6
VG	-247.24	251.24	256.63	2	1	1
NIG	-251.79	255.79	261.17	5	6	5
GHST	-252.64	256.64	262.02	6	7	8
HYP	-249.59	253.59	258.97	3	3	3
GH	-247.08	252.08	258.81	1	2	2

Goodness of Fits-Kt

Table 4 : Netherlands_kt

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-266.22	268.22	270.92	8	7	7
T	-253.92	256.92	260.96	6	3	1
JD	-266.06	271.06	277.79	7	8	8
VG	-252.34	256.34	261.72	2	1	2
NIG	-252.75	256.75	262.14	3	2	3
GHST	-253.47	257.47	262.85	5	6	5
HYP	-252.94	256.94	262.33	4	4	4
GH	-252.34	257.34	264.07	1	5	6

Goodness of Fits-Kt

Table 5 : Sweden_kt

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-214.02	216.02	218.71	8	8	7
T	-210.17	213.17	217.21	7	6	5
JD	-204.27	209.27	216.00	1	2	3
VG	-204.95	208.95	214.33	3	1	1
NIG	-208.87	212.87	218.25	5	5	6
GHST	-210.06	214.06	219.45	6	7	8
HYP	-206.34	210.34	215.72	4	4	2
GH	-204.95	209.95	216.67	2	3	4

Goodness of Fits-Copula Model

Table 7 : Standard Residual (Levy) fit Copula Model

Model	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Clayton Copula	53.802	-52.802	-51.456	6	5	5
Time-varying Clayton Copula	55.672	-52.672	-48.635	5	6	6
Gumbel copula	45.690	-44.690	-43.344	8	7	7
Time-varying Gumbel copula	46.776	-43.776	-39.739	7	8	8
Gaussian Copula	76.489	-66.489	-53.033	4	4	3
Time-varying Gaussian Copula (beta=0)	78.785	-67.785	-52.982	3	3	4
T Copula	100.159	-89.159	-74.357	2	2	2
Time-varying T Copula (beta=0)	104.301	-92.301	-76.152	1	1	1

Valuation Framework for Survivor Swaps

Valuation Framework for Survivor Swaps

- Survivor swaps
  - Agreement the periodic exchange of a series of preset payment for mortality-dependent payments
  - Value of notional principal multiplied by a fixed rate receives the value of the notional principal multiplied by the unexpected shock in survival probability (floating-ratepayer)
  - Transfer the unexpected shock in mortality improvement

Valuation Framework for Survivor Swaps

- Risk neutral of survivor probability
  - N-year survival probability

$$ {}_n p_{x_0}^j = p_{x_0}^j(t_0, n) = \prod_{j=0}^{n-1} p_{x_0}^j(t_0 + j) = \exp\left(-A_n^j\right) \quad j = 1, \ldots, 2N $$

  - The ${}_t p_{x_0}^j$ under P measure

$$ F_t^j(x) = Porb_P({}_t p_{x_0}^j \le x) $$

Valuation Framework for Survivor Swaps

- Risk neutral of survivor probability
  - ${}_{t}p_{x_0}^{j}$ P measure to Wang measure Q

$$\tilde{F}_t^{\,j}(x) = \Phi\left(\Phi^{-1}\left(F_t^{\,j}(x)\right) + \lambda_W\right)$$

  - Denuit et al. (2007) shows

$$E_Q\left[{}_{t}p_{x_0}^{j}\right] = \int_0^1 \left(1 - \Phi\left(\Phi^{-1}\left(F_t^{\,j}(y)\right) + \lambda_W\right)\right) dy$$

Valuation Framework for Survivor Swaps

- Fair value of Basis Survivor Swap

$$LS(t_0)=\sum_{t=1}^{T}\left(B(0,t)\sum_{j=1}^{2N}Q_j(0)L_jE_Q\left[{}_tp_{x_0}^{j}\right]\right)-(1+\pi)\sum_{t=1}^{T}\left(B(0,t)\sum_{j=1}^{2N}Q_j(0)L_jH_t^{j}\right)$$

- Fair swap premium

$$\pi=\frac{\sum_{t=1}^{T}\left(B(0,t)\sum_{j=1}^{2N}Q_j(0)L_jE_Q\left[{}_tp_{x_0}^{j}\right]\right)}{\sum_{t=1}^{T}\left(B(0,t)\sum_{j=1}^{2N}Q_j(0)L_jH_t^{j}\right)}-1$$

Valuation Framework for Survivor Swaps

- From the standpoint of pay-fixed, the unexpected loss

$$L(t) = \left((1+\pi)H(t) - S_{x_0}(0,t) \right)$$

- The present value of total unexpected loss

$$PVL = \sum_{t=1}^{T} B(0,t)L(t)$$

Analysis of Fair Charges for Survivor Swaps

Fair Swap Rate(b.p.)

Table 8

Yield Rates	Model	λ = -0.1	λ = -0.15	λ = -0.2
Original yield curve	Dcc T Levy	81.27	105.21	129.00
	No dependence+Normal	60.79	84.61	108.31
Parallel shift up of 2%	Dcc T Levy	54.29	74.70	94.97
	No dependence+Normal	37.26	57.57	77.78
Parallel shift up of 4%	Dcc T Levy	33.15	50.54	67.82
	No dependence+Normal	19.02	36.33	53.56

Analysis of Longevity Risk

Table 10

Time to Maturity	Model	VaR95	VaR99	CTE95	CTE99
25	Dcc T Levy	0.7003	1.0701	0.9339	1.3267
	No dependence+Normal	0.4585	0.6375	0.5682	0.7260

Contribution

- Employs time-varying copula introduce mortality dependence
  - Mortality model: Lee-Carter with JD & GH

- Demonstrate symmetric dependence

- Valuation survivor swap

- Value at Risk (VaR) & Conditional Tail Expectation (CTE) of basis swaps

Thank You