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The Effective Number of Risk Factors in UK Property Portfolios

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Abstract

Ang (2012) argues that a meal consisting of different foods with very similar nutrients might not be considered balanced and so a portfolio cannot be considered well-diversified if it is dominated by very few risk factors. In other words, while most investors would consider a portfolio that is spread across a large number of properties to be well diversified it may be undiversified if it is *effectively* driven by only a few risk factors.

Using the diversification ratio of Choueifaty and Coignard (2008) we examine the *effective* number of risk factors in the UK from holding an increasing number of Local Authorities, using the annual returns of 210 'properties', over the period 1981 to 2010. The main findings show that the effective number is very small, as a result of the high correlation between the various property-types and regions. The study therefore raises the question as to how well-diversified are current institutional portfolios in the UK.

Keywords: *Risk Factors, Direct Real Estate, Sector and Regional Portfolios*

The Effective Number of Risk Factors in UK Property Portfolios

1. Introduction

Recent research has highlighted that factor risk portfolios often have superior Sharpe ratios to asset classes portfolios because factors have lower correlations than that between asset classes (see *inter alia*, Jones et al., 2007; Ilmanen, 2011 and Romahi and Santiago, 2012). Ang (2012) makes an analogy between risk factors and nutrients arguing that factor risk is "reflected in different assets just as nutrients are obtained by eating different foods. (...) Assets are bundles of different types of factors just as foods contain different combinations of nutrients." In other words, a meal consisting of different foods with very similar nutrients might not be considered balanced. This suggests that a portfolio that may appear to be well-diversified, due to being spread a wide range of asset, may in fact be undiversified if is concentrated in assets that are essentially exposed to the same risk factor (Ang, 2012). Indeed, Meucci (2009) argues portfolio managers comprehend a portfolio to be well-diversified if it is not heavily exposed to individual factors.

A factor is any characteristic that is important in explaining the return and risk of a group of assets. The oldest and most well-known model of asset returns is the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966), which suggests that there is only one risk factor that arises from exposure to the market and is captured by beta, the sensitivity of an asset's return to the market. Ross (1976) proposed a different theory of what drives asset returns Arbitrage Pricing Theory (APT). This approach suggests that the expected return of assets can be modelled as a function of various macroeconomic factors or theoretical market indexes. Importantly, APT, unlike the CAPM, did not explicitly state what these factors should be. Instead, the number and nature of these factors were likely to change over time and vary across markets. Thus the challenge of building factor models became, and continues to be, essentially empirical in nature, but this has turned out to be its major weakness.

Since the nature and number of the priced factors are unspecified by the APT, two approaches have been used to empirically implement the theory. Some researchers extract the factors from asset returns through statistical techniques such as principal component analysis and find evidence for between 1 and 10 factors in the stock market (see *inter alia*, Roll and Ross, 1980; Chamberlain and Rothschild, 1983; Chen, 1983; Lehman and Modest, 1985; Connor and Korajczyk, 1985 and 1986; Trzinka, 1986; Groenewold and Fraser, 1997 and Jones, 2001). An alternative approach is to work with various macroeconomic variables and/or observable firm characteristics to explain the cross-sectional variations in estimated expected returns and finds evidence of between 3 and 5 factors in equity returns (see *inter alia*, Chen et al., 1986; Fama and French, 1992, 1993, 1996; Carhart, 1997; and Fama and French, 2013). But due to fact that the theory in itself does not identify the number of relevant factors, researchers continue to add new factors, which are too many to enumerate here.

A similar problem exists in the real estate market as studies find that while multi-factor models appear to explain a significant proportion of the variation in real estate returns the number of factors varies from 3 to 12 and often includes different variables and depends on whether direct or indirect real estate returns are examined and the methodology employed (see *inter alia*, Chen et al., 1998; Peterson and Hseih, 1997; Naranjo and Ling, 1997 and 1998; Baroni et al., 2001; Payne, 2003; Sing, 2004; Liow, 2004; Hoskins et al., 2004; Liow,

et al., 2006; Lizieri et al., 2007; Baum and Farrelly, 2009; Fahad, 2010; West and Worthington, 2006; Fuerst and Marcato, 2009; Blundell et al., 2011; and Schulte et al., 2011).

This confusion as to the number of risk factors to use in the real estate market prompted us to take an alternative approach and calculate the extent of diversification in a portfolio, as an indicator of the *effective* number of risk factors in a portfolio. Intuitively, whenever the assets in the portfolio are close to being uncorrelated, the effective number of risk factors will be close to N, the number of assets in the portfolio, indicating a large number of factors are driving the return and risk of the portfolio. On the other hand, when there is a high degree of correlation among different assets the extent of diversification is limited indicating that the assets in the portfolio are effectively driven by a single factor. In other words, the extent of diversification in a portfolio also implies the *effective* number of risk factors in a portfolio.

Because of its simplicity the number of properties in a portfolio still serves as a prominent measure of portfolio diversification for most investors. This implies that the *effective* number of risk factors in a portfolio is equal to the number of assets in a portfolio. But even a portfolio containing a large number of assets may be so heavily concentrated in a few assets that it is effectively undiversified as the portfolio is driven by say only one or two risk factor. In other words, simply looking at the nominal number of constituents in a portfolio is a simplistic indication of how well-diversified a portfolio is, since it does not convey any information about the fractions of wealth invested into its' constituents. Thus, Woerheide and Persson (1993) suggest measures of market concentration, or information theory, that take account of differences in the constituent weights are better measures of diversification than simply counting the number of constituents in the portfolio. But the weight-based measures suggested by Woerheide and Persson (1993) quantifies the extent of diversification in a portfolio independently of the risk characteristics of the constituents of the portfolio, yet the benefits of diversification stem from the differing risk characteristics of the assets in the portfolio. So any good measure of portfolio diversification should take account of the covariance structure of the portfolio, as well as the asset weights in the portfolio.

A number of risk-based measures of diversification have been suggested (see *inter alia*, De Wit, 1997; Goetzmann et al., 2005; Cheng and Roulac, 2007; Choueifaty and Coignard, 2008; and Hight, 2010). The diversification ratio of Choueifaty and Coignard (2008) is particularly attractive as it not only measures the extent of diversification in a portfolio but can be decomposed to show the amount of asset concentration and the correlation structure of the portfolio, the desirable characteristics of any good diversification measure. In addition, Choueifaty et al. (2013) show the square of the diversification ratio can be interpreted as the *effective* number of independent risk factors, or degrees of freedom, represented in the portfolio. Hence, we use the diversification ratio of Choueifaty and Coignard (2008) to infer the *effective* number of risk factors in UK real estate portfolios of portfolio sizes from one to 40 assets, using annual data over the period from 1981 to 2010.

The remainder of this paper is structured as follows. Section 2 reviews previous studies trying to estimate the effective number of risk factors in the real estate market. The following section presents the advantages and disadvantages of the three main measures of diversification that could be used to identify the *effective* number of risk factors in a portfolio. The data and an initial analysis is presented in Section 4. The results of a simulation study are presented in Section 5. The final section concludes the study.

2. Previous Studies of Risk Factors in Real Estate

In the securitised real estate market a large number of studies have examined whether the three factors identified in the US security market by Fama-French (market risk, size and value) or the four factor model of Carhart, which adds momentum to the three factor model of Fama-French, also applied to real estate. For instance, McIntosh et al. (1991) found that small REITs deliver higher returns than large ones, without being subject to higher risk. A size effect was also found by Chen et al. (1998) however no value effect was present in their results. Peterson and Hseih (1997) find risk premiums of Equity Real Estate Investment trusts (EREITs) behave similar to the Fama-French (1993) factors.

Due to institutional changes in the US REIT market in 1990 the significance and number of factors has changed. For instance, Chui et al. (2003) find that in the pre-1990 period a negative size effect and a positive value effect was present in addition to momentum, turnover and analyst-coverage effect, whereas only the momentum effect remained in the post-1990 period. Clayton and MacKinnon (2003) show that while EREITs were primarily driven by the same factors as large cap stocks from 1979 to 1992, this reversed from 1993 to 1998. The size factor becoming even stronger over time (see *inter alia*, Chiang et al., 2004 and 2005; Chiang et al., 2006; Anderson et al., 2005; and Lee and Stevenson, 2007). However, when Schulte et al (2011) examined whether the Fama-French factors explained cross-sectional return differences in European real estate equity returns they find that while a value effect exists no significant small-size effect was evident.

Other studies have examined whether macroeconomic variables could be used to explain securitised real estate returns. Naranjo and Ling (1997) show that four macroeconomic factors (the growth rate in real per capita consumption, the real treasury-bill rate, the term structure of interest rates, and the unexpected inflation rate) explain returns on commercial real estate. In their later work, Ling and Naranjo (1998) included an additional factor: stock market performance. Chaudhry, et al (2004) use a dataset of REITs from 1994 to 2000 and find three factors: leverage, performance measures and earnings variability can predict REIT returns. Payne (2003) investigated the effects that shocks to macroeconomic variables would have on the excess returns of three broad classifications of REITS (equity, mortgage and hybrid) and finds that unexpected changes in the broad stock market index was positively significant to all three types of REITS, unexpected changes to the growth of industrial production was negatively significant for hybrid and mortgage REITS, unexpected changes to inflation and default risk insignificant for all three types, unexpected changes to the term structure negatively related to equity and hybrid REITS and unexpected changes to federal funds rates adversely affect mortgage and hybrid mortgages. Liow, et al. (2006) examined the impact of six macroeconomic factors (GDP growth, INDP growth, unexpected inflation, money supply, interest rate and exchange rate) on property stock returns in Singapore, Hong Kong, Japan and the UK. The results showed some disparities in the significance, as well as direction of impact in the macroeconomic risk factors across the property stock markets. Using Independent Component Analysis Lizieri et al (2007) find 12 factors that accounts for 71% of the total variance in US REIT returns. This supports the work of Fahad (2010) who also used principal component analysis and found that while over the period from 1997 to 2009 there were three distinct factors in the international REIT market, during the period of the Global Financial Crisis (2007-2009) international returns seemed to be driven by only two factors as a result of increased integration between Europe and North American real estate equity returns.

Sing (2004) found that macroeconomic risk factors are priced quite distinctly in direct and securitised real estate markets. For instance, Baroni et al. (2001) find that listed real estate volatility is almost entirely captured by two risk factors (macro-economic risk and equity market risk), which explain 87% of total variance. In contrast, Baroni, et al. (2001) direct property in the Paris area over 1973-1998 was driven by four factors, the two driving the listed market (macro-economic risk and equity market risk) plus two property factors (a business cycles risk and physical property risk (average price per square metre), which explained nearly 85% of total variance of the dataset. De Wit and van Dijk (2003) find that direct office property returns are explained by four variables: GDP growth; unemployment rate changes, inflation and market vacancy rates. Liow (2004) identified five macroeconomic factors between office and retail excess returns in Singapore: GDP, industrial production, unexpected inflation, short-term interest rate and market portfolio. Hoskins et al. (2004) compared the relationships of macroeconomic variables on the commercial property markets in Australia, Canada, the UK and the US to find that three factors: GDP, unemployment and inflation as main determinant factors. Pai and Geltner (2007) used US property level data to create property specific factors, analogous to the classic Fama and French (1993) factors. Property specific factors included size, the performance of Tier I and Tier III locations and yield. Fuerst and Marcato (2009) conduct a similar analysis for the UK and included two additional property specific factors: lease length and portfolio concentration. West and Worthington (2006) find that the factors that explain the performance of Australian real estate differ across the property-types. Inflation being an influential factor in office, retail, industrial and listed property trust returns, Industrial production being important in determining retail, industrial and listed property trust returns and employment being significant in office, retail and industrial returns. Interest rates are also a significant risk factor across all types of property portfolios, while the market return is a significant factor in retail, industrial, listed property trusts and property stocks. Schatz and Sebastian (2009) investigated the relationship between real estate returns in Germany and the UK and find comparable results in terms of significance, order, magnitude and sign. Specifically they found a negative relationship between the property indices and unemployment rates, and a positive link with both property markets and the respective consumer price index and government bond yields.

In the unlisted property funds market Baum and Farrelly (2009) identify three key sources of risk: market, stock and financial structure. Market risk emanates from the market segments to which the portfolio is exposed. Stock risk refers to property specific risk and reflects the characteristics of individual properties owned within each market. These include: building quality, development activity, vacancy, lease length and type/credit quality of tenants. Financial structure risk reflects the risks within the fund structure itself. Blundell (2005) suggested a risk assessment methodology intended to capture as broad a selection of potential sources of risk as possible. The work was updated by Blundell et al. (2011) who find six factors had a significant impact upon portfolio performance such as: average asset size, property type and geographic exposure, average lease length, tenant credit strength, and the portfolio vacancy rate. Fuerst and Matysiak (2013) found that significant explanatory power (some 70%) of performance in European unlisted property funds resulted from just four factors: market exposure, leverage and two factors which reflect the underlying direct property characteristics. While, Farrelly and Matysiak (2012) suggest that the variation in total returns can be attributed to five risk factors in the UK: the four identified by Fuerst and Matysiak (2013) plus lagged performance.

3. Measures of Diversification

Ever since the seminal work of Evans and Archer (1968) researchers have used the following equation to examine the effect of the number of assets in a portfolio on its risk (variance):

$$\sigma_p^2 = \frac{1}{N}\bar{\sigma}_i^2 + \frac{(N-1)}{N}\bar{\sigma}_{i,j} \qquad (1)$$

Where: σ_p^2 is the total risk (variance) of the portfolio, $\bar{\sigma}_i^2$ is the average risk of all the assets, $\bar{\sigma}_{i,j}$ is the average covariance among all assets, and N is the number of assets in the portfolio.

Equation 1 shows that as $N \to \infty$, $1/n \to 0$ the first term on the right hand side of the equation tends to zero and the variance of a portfolio eventually falls towards the average of the covariance among all assets, as (N-1)/N tends to 1.

Using equation (1) numerous studies in the financial literature demonstrate that increasing the number of assets in a portfolio results in the reduction of portfolio total risk. The studies showing that the connection between increasing portfolio size and portfolio risk takes the form of a rapidly decreasing asymptotic function. In other words, most of the elimination in risk occurs within the first few assets after which any gain in risk reduction is marginal (see *inter alia*, Fisher and Lorie, 1970; Moa, 1970; Jacob, 1974; Klemkosky and Martin, 1975; Elton and Gruber, 1977; and Surz, 2000). These results have also been confirmed in the direct real estate market (see *inter alia*, Jones Lang Wootton, 1986; Brown, 1988 and 1991; Myer, et al, 1997; De Wit, 1997; Byrne and Lee, 2000; and Cheng and Roulac, 2007). Thus, investors probably consider a portfolio to be well-diversified if it contains a large number of assets with small weights in each asset.

There are at least two weaknesses in using the number of assets in a portfolio as a measure of diversification. First, only in the case where all asset returns arise from a multivariate normal distribution with equal means, equal variances and equal covariance is counting the number of constituents a reasonable measure of diversification. If the data is non-normal and displays heterogeneity the number of assets in a portfolio is a poor measure of diversification[1]. In other words, simply counting the number of assets in a portfolio can only be regarded as a measure of diversification under very restrictive assumptions that are not seen in the real world.

Second, the degree of diversification in a portfolio depends not only on the number of assets in a portfolio but also on the fractions of wealth invested into the constituents (Woerheide and Persson, 1993). Morrell (1993) argues that an uneven distribution of property values is an important source of risk in property portfolios and that the potential benefits of diversification from holding a large number of properties can be swamped by the existence of differential weights within the portfolio. In other words, simply holding a large number of properties is not sufficient in itself to guarantee a reduction in risk, as the distribution of properties within the portfolio is an equal, or perhaps a more important determinant of portfolio risk. Indeed, studies show that property portfolios that are unequally- or value-weighted show lower levels of total risk reduction that equally-weighted portfolios as

[1] A number of studies in the real estate market show that property data is non-normal (see, Young et al., 2006, for a review).

portfolio size increases (see *inter alia*, Brown, 1987 and 1991; Morrell, 1993; Schuck and Brown, 1997; Byrne and Lee, 2000; and Cheng and Roulac, 2007).

To account for the weight distribution in a portfolio Woerheide and Persson (1993) introduce measures of market concentration from economics (Herfindahl Index) as well as measures from information theory (Entropy) to portfolio theory in order to assess the diversification of a portfolio.

The Herfindahl index, is a widely-used measure of market concentration in the industrial organization literature (Scherer, 1980) but as also been proposed as a measure of diversification in portfolios (Woerheide and Persson, 1993). The *HI* is calculated by squaring the investment weights of each constituent in a portfolio and then summing the results:

$$HI = \sum_{i=1}^{N} w_i^2 \tag{2}$$

Where, *HI* is the Herfindahl Index and w_i is the weight of asset *i* in the portfolio. If a portfolio's Herfindahl Index *HI* value is close to zero, it means that the portfolio is distributed among a lot of assets and so would be considered as diversified. In contrast, if a portfolio's Herfindahl Index *HI* value is close to one, the portfolio is concentrated in few assets and so would be considered as undiversified. In other words, the lower the Herfindahl Index *HI* the more diversified the portfolio[2].

Another approach towards assessing the degree of diversification in a portfolio stems from information theory (Woerheide and Persson, 1993). Loosely speaking, information theory is concerned with the quantification of the *disorder* of a random variable, with its most prominent measure being entropy (Shannon, 1949). Hence, another way to interpret portfolio weights is to see them as the probability of a "randomly" chosen asset in a portfolio. The corresponding measure is the weight entropy (E_w) measure of diversification:

$$E_w = \sum_{i=1}^{N} w_i \ln(1/w_i) = -\sum_{i=1}^{N} w_i \ln(w_i) \tag{3}$$

Where, E_w *is* the entropy of the portfolio, w_i is the weight of the asset *i* in the portfolio and $\ln(w_i)$ is the natural logarithm of the weight of asset *i* in the portfolio. The entropy of a portfolio has a minimum value of zero if $w_i = 1$ and $w_j = 0$, $i \neq j$ for j = 1, …., n. Entropy takes a maximum of $\ln(w_i)$ if and only if $w_i = 1/n$ for i = 1,…,n., i.e. the portfolio is naïvely diversified. In other words, the higher the portfolios entropy the more diversified the portfolio (see, Bera and Park (2008) for more details).

The advantage of the weight-based measures, over simply counting the number of assets in a portfolio, is that if we take the reciprocal of the portfolio's Herfindahl Index, or the exponent of its Entropy, we can calculate the *effective* number of assets in the portfolio. In particular, if and only if is the portfolio equally-weighted is the *effective* number of assets in the portfolio equal to its actual number, in all other cases the *effective* number will be less. So number based measures of diversification can only be regarded as a measure of naïve diversification. For instance, using either weight-based method to examine the diversification in a 40 asset portfolio, but one in which 90% is concentrated in one asset, would indicate the *effective* number of risk factors in the portfolio is close to one even though

[2] Woerheide and Persson (1993) transform the Herfindahl Index to calculate what they call a Diversification Index (*DI = 1-HI*), such that the minimum value is zero and a maximum of one.

the portfolio contains many assets. Hence, the weight-based measures of diversification (Herfindahl Index and Entropy) give a clearly picture of the *effective* number of risk factors in a portfolio than simply count the number of assets in a portfolio.

The major problem with the weight-based measures of diversification is they do not account for the interdependence between assets and differing risk characteristics of individual assets in the portfolio. Schuck and Brown (1997) argue that while "…an uneven distribution of values can have a significant impact on the specific risk level within property portfolios" the actual effect of value skewness is uncertain as it "will be a function of the number of properties, their individual total risks, and the correlation structure between returns." In other words, two portfolios can have the same size and the same weight structure but a completely different risk structure and so markedly different levels of diversification. It is therefore insufficient to measure diversification, or the *effective* number of risk factors in a portfolio, using the Herfindahl Index or Entropy. In studying the extent of diversification, and so the *effective* number of risk factors, within a portfolio we must also consider the covariance structure of the portfolio as well as the asset weights in the portfolio.

Choueifaty and Coignard (2008) use the following ratio to measure the diversification of a portfolio:

$$DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sigma_p} \tag{4}$$

Where: *DR* is the diversification ratio, w_i is the portfolio weight in asset *i*, σ_i is the risk of asset *i*, and σ_p is the total risk of the portfolio.

From equation (4) we see that the nominator is the portfolio volatility when every asset is perfectly correlated with the portfolio. The denominator is the actual portfolio risk that will be less than the sum of the individual portfolio risks, as every asset will not be perfectly correlated with the portfolio. Only in the case when the portfolio contains one asset will the diversification ratio be one. So we can classify a diversification ratio close to one as evidence of a poorly diversified portfolio[3]. A higher diversification ratio value therefore is an indication of a more diversified portfolio. The term "higher" refers to a diversification ratio value approaching N (the number of assets in the portfolio).

An attractive feature of the diversification ratio is that it can be decomposed into the two components of any good diversification measure, correlation and weight concentration, as follows (Choueifaty et al., 2013):

$$DR = [\rho_w(1 - CR_w) + CR_w]^{-1/2} \tag{5}$$

where ρ_w is the volatility weighted average correlation of the assets in the portfolio:

$$\rho_w = \frac{\sum_{i \neq j}^{N} (w_i \sigma_i w_j \sigma_j) \rho_{i,j}}{\sum_{i \neq j}^{N} w_i \sigma_i w_j \sigma_j} \tag{6}$$

[3] It is easy to show that the measure suggested by Hight (2010) is a rescaled version of the Diversification Ratio such that its minimum value is zero.

and CR_w is the volatility weighted concentration ratio of the assets in the portfolio:

$$CR_w = \frac{\sum_{i=1}^{N}(w_i\sigma_i)^2}{(\sum_{i=1}^{N} w_i\sigma_i)^2} \quad (7)$$

The intuition behind this decomposition is that the lower (higher) the correlation ratio (ρ_w) and the lower (greater) the concentration ratio (CR_w) the higher (lower) the diversification ratio and the more (less) diversified the portfolio. In the extreme, if the correlation ratio increase to unity, the diversification ratio is equal to one, regardless of the value of the concentration ratio, as portfolios of assets are no more diversified than a single asset. In other words, the higher the correlation ratio (ρ_w) the lower the diversification ratio; even if the portfolio is equally-weighted.

Lastly, Choueifaty et al. (2013) show if there are *F* independent risk factors, and the portfolio allocates risk evenly to these *F* factors, then, the squared diversification ratio is equal to *F*. As such, *F* can be interpreted as the *effective* number of independent risk factors, or degrees of freedom, represented in the portfolio. Hence, the square of the diversification ratio can be interpreted as the *effective* number of independent risk factors in a portfolio.

4. Data and Initial Analysis

In order to investigate the potential number of risk factors in the UK commercial real estate market a very large database of actual real estate data in in various locations across the UK with annual data covering the period from 1981 to 2010 are used.

4.1 Data

The data are derived from the Investment Property Databank (IPD) Local Markets Report (IPD, 2010). The Local Markets Report data are drawn from a comprehensive database consisting of 11,276 properties with a total value of £134,395 million from 283 funds at the end of 2010 (IPD, 2010), and has an almost complete coverage of commercial real estate performance in the UK (Byrne and Lee, 2000a). This provides total annual returns for standard retail, office and industrial properties across the UK between 1981 and 2010[4], giving a sample of 210 'property' possibilities in 133 Local Authorities (LAs), essentially towns and cities, with full time series data.

Similar to previous studies we examine a number of diversification schemes (see *inter alia*, Eichholtz, et al., 1995; Byrne and Lee, 2000; Lee and Stevenson, 2005, and Cheng and Roulac, 2007). First, we examine nine "Single" diversification schemes, i.e. holding a single property-type in a single region. Second, three "Regional" diversification schemes, that is spreading across the LAs within a property-type. Next, we examine three "Property-type" diversification schemes, i.e. diversification across the property-types with a region, as previous studies suggest that this approach to diversification is to be preferred (Fisher and Liang, 2000 and Lee, 2000). The regions used are the 3 "super regions" of the UK; London, the Rest of the South East and the Rest of the UK as suggested by Eichholtz, et al. (1995) and Lee and Stevenson (2005). Lastly, we consider a ''Mixed LAs" scheme, i.e. one in which it

[4] The properties included in the Local Markets Data report are all standing investments in the IPD database, that is, being held in portfolios, not bought or sold or subject to development or substantial improvement expenditure during any given period.

is assumed that investors can invest in any number of LAs across the UK as suggested by Cheng and Roulac (2007). Summary data on the number of Las in the various diversification schemes presented in Table 1.

Table 1: The Number of Data Points

	Sectors			
Regions	**Retail**	**Office**	**Indust**	**Total**
London	14	15	11	**40**
Rest SE	18	15	18	**51**
Rest UK	63	19	37	**119**
Total	**95**	**49**	**66**	**210**

Table 1 shows these data are spread unevenly across both the property-types and regions. Of the three property-types most of the data (45%) are in the Retail property-type, whereas Offices and Industrials only represent 23% and 31% of the data, respectively. Table 1 also shows that 62% of the Office holdings are concentrated in just two regions, London and the South East. In contrast, the Industrial data is less focused on London and more evenly spread across the UK. The spread of Retail is also unevenly spread across the UK with only 15% in London and the majority (66%) in the Rest of the UK. These results reflect a continuous institutional bias towards the South of England for Office property, whereas Retail and Industrial holdings correlate more closely with the urban hierarchy of the UK (see, Byrne and Lee, 2006, 2009, 2010).

4.2 Descriptive Statistics

Table 2 shows the descriptive statistics for the property-type and regional data over the period 1981 to 2010. Panel A of Table 2 shows that the best performing property-type on average was Industrials, while Offices performed the worst. However, Panel B of Table 2 shows that on average high returns are not necessarily associated with higher levels of risk (standard deviation). Although the highest risk was associated with the property-type showing highest return (Industrials) the lowest level of property-type risk was in Retail, which offered the second best returns. The regional data confirm this observation since the region with the lowest average returns (Rest of South East) showed the second highest level of risk, because of the dominance, by value, of Offices within the region. The Rest of the UK region, which showed the second best average returns, had the lowest risk level. The worst performing single property-type/region as measured by the ratio of return to risk was Offices in London. The best performing single property-type/region was Industrials in the Rest of the South East.

Table 2: Descriptive Statistics

Panel A: Mean	**Retail**	**Office**	**Indust**	**Total**
London	10.7	8.7	11.7	**10.2**
Rest of South East	9.6	7.8	11.0	**9.5**
Rest of UK	9.5	9.3	10.9	**9.9**
Total	**9.7**	**8.7**	**11.0**	**9.9**
Panel B: SD	**Retail**	**Office**	**Indust**	**Total**
London	11.2	13.0	11.6	**12.0**
Rest of South East	10.9	10.8	11.9	**11.2**
Rest of UK	10.3	11.4	11.6	**10.9**
Total	**10.6**	**11.7**	**11.7**	**11.2**
Panel C: Correlation	**Retail**	**Office**	**Indust**	**Total**
London	0.81	0.81	0.89	**0.76**
Rest of South East	0.84	0.87	0.89	**0.78**
Rest of UK	0.81	0.80	0.84	**0.72**
Total	**0.81**	**0.78**	**0.84**	**0.73**

Lastly, Panel C of Table 2 shows the overall average correlation between the 210 'properties' is 0.73, which is considerably higher than that reported in Byrne and Lee (2000) at 0.61. Of the property-types the highest correlation was in Industrials, while the lowest was in Offices. The regional portfolio with the lowest correlation was in the Rest of the UK and the highest in the Rest of the South East. This result matches that of Eichholtz et al. (1995), who found that the further away from London, the lower the correlation. Such a high level of correlation within property-types and regions suggests that the effective number of risk factors in the UK property market is likely to be small.

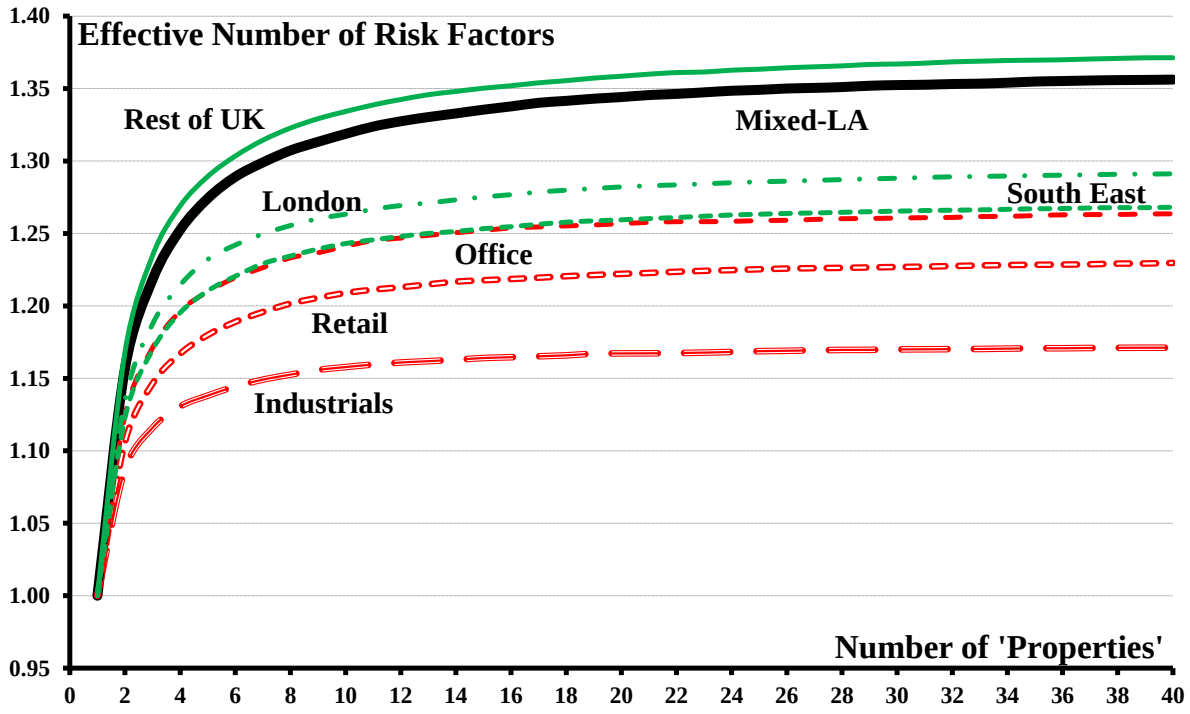
4.3 The Potential Number of Risk Factors

In order to calculate the potential number of risk factors in the UK property market we have substituted the parameters in Table 2 into equation 1 to calculate the standard deviation of portfolios of various sizes from one to a portfolio of infinite size. Next we used equation 4 to calculate the diversification ratio of each portfolio size by dividing the risk of portfolio size N by that of an undiversified portfolio, i.e. the risk of a portfolio of size one. Finally, we squared the diversification ratio to calculate the effective number of risk factors in each portfolio size, for each diversification scheme. The summary data presented in Table 3 and plotted in Figure1.

Table 3: Potential Number of Risk Factors: Infinite Holding

	Sectors			
Regions	**Retail**	**Office**	**Indust**	**Total**
London	1.23	1.23	1.12	**1.19**
Rest of South East	1.19	1.15	1.12	**1.28**
Rest of UK	1.23	1.25	1.19	**1.39**
Total	**1.23**	**1.28**	**1.19**	**1.37**

Figure 1: Number of Risk Factors: Different Diversification Schemes



Given the use of the equal-weighted diversification strategy when using equation 1, the effective number of risk factors will be determined by the volatility weighted average correlation; especially given the high correlation across the dataset. This implies that the effective number of risk factors is likely to be very small, as confirmed in Table 3. The results in Table 3 and Figure 1 show that with the addition of the first few 'properties' the *effective* number of risk factors increases rapidly but then after about 12 LAs, which is equivalent to a holding of about 50 properties, there are only minor increases. The highest number of risk factors is in the Rest of the UK, due to its relatively low correlation and the lowest number of risk factors is in Industrials, due to its high correlation. Nonetheless, even spreading across all property-types and regions with an infinite holding the effective number of risk factors is only 1.37. This implies that investors can be misled in thinking a portfolio is well-diversified, due to the large number of properties in the portfolio, when in fact they are really undiversified as property portfolios in the UK are *effectively* driven by only one risk factor.

5. Simulations

The previous analysis may be deficient because an individual investor owns only one portfolio and results based on averages are not really relevant to his/her particular case, which may be substantially different from the average (Newbould and Poon, 1993), i.e. the average analysis disguises the variability around the average, which could be large. In other words, investors who base their diversification strategies on average results could be leaving themselves open to unpleasant results. Therefore investors, who wish to avoid such adverse surprises from an unfortunate selection, would be better off looking at the bounds around the average rather than the average itself. The descriptive statistics in Table A1 in the Appendix suggest that this could be a serious problem here, because they indicate a wide variation in correlation coefficients within and between sectors and regions. The lowest correlation between any pair of 'properties' is 0.043; which indicates that it would be possible for a portfolio of two 'properties' could show a large number of risk factors. By the same token a portfolio could face one risk factor if an investor were to choose to invest in the two 'properties' that show the highest correlation of 0.972. In other words, since the sampling variation in the correlation coefficients is quite large, the results in Table 3 do not fully convey the actual impact that increasing the portfolio size has on the effective number of risk factors in UK property portfolios.

To calculate the effective number of risk factors in a portfolio of size N we employ a simulation process. Various combinations of the 210 'properties' from the 133 LAs in the dataset were first created at random, using Monte Carlo simulation techniques. Similar to Devaney and Lee (2007) an initial asset was randomly selected from the data set by randomly drawing a 'property' from a uniform distribution, with replacement[5], and its standard deviation calculated. A second 'property' was then randomly selected and added to the first and the standard deviation of the equal-weighted naïve portfolio derived. This process continued until a 40 'property' portfolio was achieved[6]. The choice of this cut-off portfolio size was determined by a number of criteria. First, the limitation on the number of data points within a region/property-type, i.e. there is only 40 'properties' in London. Second, as each LA is required to have at least of four properties, to protect confidentiality, a portfolio of 40 locations across the UK must actually contain, at a minimum, 160 properties. These are

[5] Byrne and Lee (2000) show that simulations with or without replacement as no effect of the results.
[6] Since the simulations used 40 'properties' the analysis was concentrated on the overall sectors and regions.

effective portfolio sizes far in excess of the average for funds in the UK of just over 40 properties (IPD, 2010). Increasing the simulated portfolio size beyond 40 LAs would serve little purpose in the practical sense. It is of course necessary to sample a sufficient number of times to obtain statistically acceptable output distributions and statistics, so we use a sample size of 1,000. The results for the overall data, while the sector and regional results are presented in Figure 1 in the Appendix.

Figure 2: Maximum, Minimum and Average Risk Factors

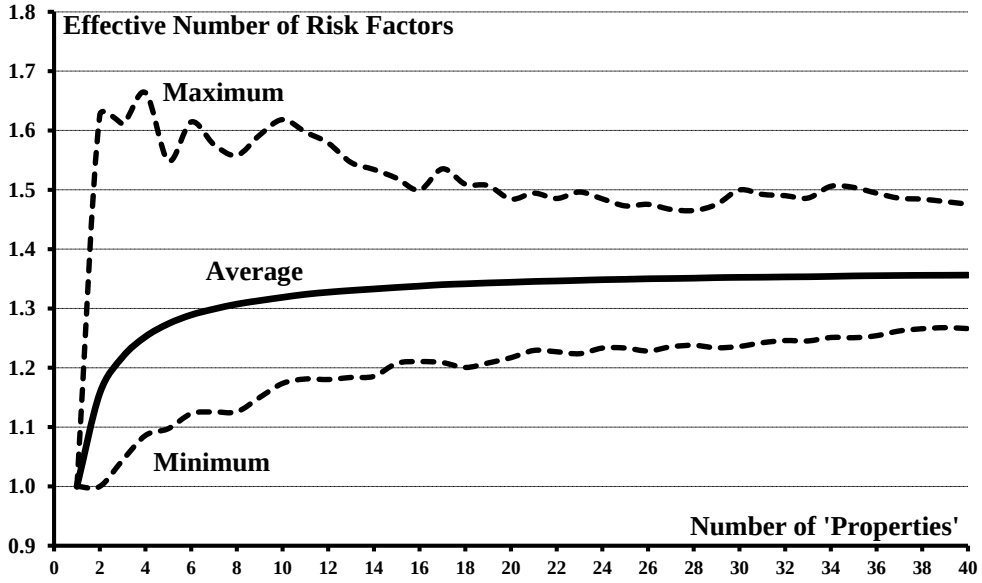


Figure 2 shows that even though the sampling variation within the data set is quite large the maximum effective number of risk factors is low and always less than two. More importantly the minimum number of risk factors, even from holding 40 LAs (equivalent from holding at least 160 properties), is only 1.7. So unless an investor is extremely fortunate and chooses a pair of 'properties' that show a very low level of correlation their portfolio is likely to be effectively driven by one risk factor.

Figure 1 in the Appendix supports the previous analysis and shows that the property-type with the highest number of risk factors in the Office sector, peaking at 1.66. But this is out performed by the Rest of the UK, which peaked at 1.75. The property-type with the least factors is the Industrial sector, with a minimum of only 1.2 risk factors even with a holding of 40 Las. In contrast, the Rest of the UK region achieved 1.1 risk factors with only 6 LAs and 1.3 risk factors with 40 LAs.

6. Conclusion

Using the diversification ratio of Choueifaty and Coignard (2008) and annual returns from 210 'propertes' in 133 LAs in the UK over the 30-year period from 1981 to 2010 we have investigated the *effective* number of risk factors in property portfolio by investing across an increasing number of LAs as opposed to investing in a single LA in the UK. From the results we can draw a number of conclusions.

First, the effective number of risk factors is always very low even with an infinite number of 'properties' in the portfolio, irrespective of the property-type or region. In other words,

investors who focus on trying to diversify their portfolios by spreading across the regions and properties in the UK are unlikely to have a well-diversified portfolio as such portfolios are effectively driven by one risk factor. The study therefore raises the question as to how well-diversified are current institutional portfolios in the UK.

Second, the largest number of risk factors is in the regional portfolios, from spreading across the property-types within a region, and the lowest within the sector portfolios, from spreading across the regional within a property-type. These results confirm the previous studies on sector and regional diversification that indicate the diversification across the property-types within a regional, especially the further away from London, always shows greater risk reduction due to the lower correlation within regional portfolios (see *inter alia*, Jones Lang Wootton, 1986; Brown, 1988 and 1991; Myer, et al, 1997; DeWit, 1997; Byrne and Lee, 2000).

Finally, like all research the analysis is subject to a couple of caveats. First, the private real estate data used here is Local Authority (town level) data, to avoid confidentiality issues; as such the property portfolios are to some extent already diversified. Future analysis therefore needs to examine the effective number of risk factors in individual property data. Second, the low number of risk factors found over the sample period results from the high correlation between the various property-types and regions. However, correlation coefficients are an average of a large number of concurrent asset movements in many economic and financial environments. This implies that in calculating the number of risk factors over such a long time we are essentially assuming that the risk and return characteristics of the various property-types and regions is the same even in periods of boom and bust. But, a number of studies show that risk factors are time varying (see *inter alia*, Liow, 2000 and 2004; Liow, et al., 2006; West and Worthington 2006; and Fahad, 2010). Further studies therefore should examine whether the effective number of risk factors is also time varying. These extensions are left for future research.

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Appendix

Table A1: Summary Correlation Statistics

Sector/Region	Average	Max	Min	SD
Retail	0.808	0.971	0.390	0.077
Office	0.783	0.972	0.086	0.122
Industrial	0.844	0.971	0.356	0.085
London	0.762	0.972	0.478	0.101
Rest SE	0.780	0.964	0.403	0.105
Rest UK	0.719	0.971	0.043	0.138

Figure 1: Sector and Regional Maximum, Minimum and Average Risk Factors

