# Bayesian portfolio-specific mortality

Frank van Berkum

joint work with K. Antonio and M. Vellekoop

Actuarial Science and Mathematical Finance
Faculty of Economics and Business
University of Amsterdam
&
Netspar

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UNIVERSITY OF AMSTERDAM

# Outline

# Population vs portfolio-specific mortality

- Life insurers and pension funds value their liabilities using prospective population mortality rates to account for future mortality developments
- To account for selection effects, these population mortality rates are often multiplied with an age-dependent portfolio-specific factor
- Recent research has focussed on modelling of population mortality rates
  - New model structures / factors (Cairns et al. (2006); Plat (2009*a*))
  - New models for modelling of time series (van Berkum et al. (2014))
  - Taking information from other countries into account (Li and Lee (2005); Cairns et al. (2011))
  - Taking parameter uncertainty into account through Bayesian modelling (Czado et al. (2005))

# Definition of variables

- $d_{t,x}$ is the observed deaths during calendar year $t$ aged $x$ at death
- $e_{t,x}$ is the average population aged $x$ during calendar year $t$ (exposure)
- The mortality rate $q_{t,x}$ can be approximated by $1 - \exp[-m_{t,x}]$, with $m_{t,x} = d_{t,x}/e_{t,x}$, the observed death rate
- We distinguish the following groups:
  - The population of a country (superscript $^{\text{pop}}$, e.g. $d^{\text{pf}}_{t,x}$)
  - The portfolio under consideration ($^{\text{pf}}$)
  - The difference between the population and the portfolio referred to as the 'rest' ($^{\text{rest}}$)
    such that: $\text{pf} + \text{rest} = \text{pop}$ )

# Overview of literature (1)

Define the difference in mortality between the population and a portfolio as a portfolio-specific factor (or experience factor)

- Plat (2009*b*) considers observed portfolio-specific factors ($P_{t,x} = \frac{q^{\text{pf}}_{t,x}}{q^{\text{pop}}_{t,x}}$) and models these directly using OLS/WLS
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  - Both $q^{\text{pf}}_{t,x}$ and $q^{\text{pop}}_{t,x}$ are observed
- Olivieri (2011) considers portfolio-specific mortality in a credibility theory setting, e.g.
  - $D_{t,x}|Z_{t,x} \sim \text{Poisson}(e_{t,x} \cdot m_{t,x} \cdot Z_{t,x})$, with $Z_{t,x} \sim \text{Gamma}(\alpha_{t,x}, \beta_{t,x})$
  - The parameters $\alpha_{t,x}$ and $\beta_{t,x}$ are updated yearly using available historical mortality observations
  - The mortality rate $m_{t,x}$ is assumed to be observed

Note: under certain assumptions $q_{t,x} \approx 1 - \exp[-m_{t,x}]$

# Overview of literature (2)

- Gschlössl et al. (2011) and Richards et al. (2013):
  - Estimate a baseline (possibly smoothed) mortality rate, and
  - Estimate risk factors, either given or estimated simultaneously with baseline mortality
  - Limited number of years is observed, so mortality trend is either absent or deterministic

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  - Limited number of years is observed, so mortality trend is either absent or deterministic
- Villegas and Haberman (2014) distinguish a reference population and subpopulations:
  - The reference population is England; subpopulations are different socio-economic classes
  - For the reference population they estimate Lee-Carter with a cohort effect
  - For the subpopulations they estimate Lee-Carter type deviations from this reference population

# Illustration of data

Define $\theta^{\text{pf}}_{t,x} = \frac{m^{\text{pf}}_{t,x}}{m^{\text{pop}}_{t,x}} = \frac{d^{\text{pf}}_{t,x}/e^{\text{pf}}_{t,x}}{d^{\text{pop}}_{t,x}/e^{\text{pop}}_{t,x}}$

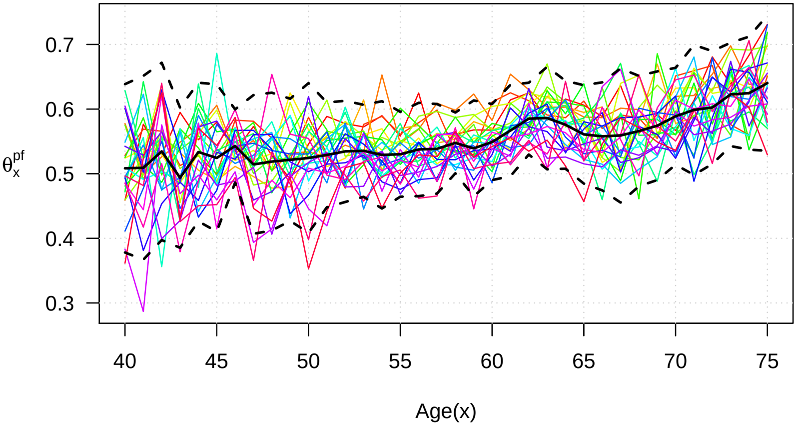


Figure: Colored lines are yearly observations. Black lines represent 2,5th, 50th and 97,5th percentile using estimated mean and variance of (and assuming a normal distribution for) $\theta^{\text{pf}}_{t,x}$.

# Challenges to overcome

## Extending existing literature

- Yearly observations of portfolio-specific factors can be very volatile
  - Instead of modelling realised portfolio-specific factors, we model death counts that are assumed to be Poisson distributed, as is common in population mortality modelling

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  - Instead of modelling realised portfolio-specific factors, we model death counts that are assumed to be Poisson distributed, as is common in population mortality modelling
- Population and portfolio-specific mortality are ideally estimated simultaneously
  - Using a Bayesian set-up, we can achieve this in a statistically justifiable manner
- It is unclear to what extent the volatility in yearly observations is caused by Poisson randomness or uncertain portfolio-specific factors
  - Using a Bayesian set-up, we obtain the posterior distribution of portfolio-specific factors which allows illustration of the uncertainty in portfolio-specific factors

# Outline

Introduction and motivation

Model setup

Results

Conclusion

# Illustration of available observations

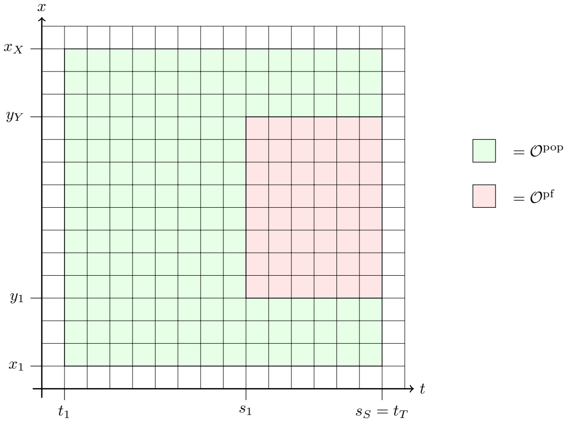


Figure: For the portfolio and the rest (red cells) only limited observations are available. The dataset is extended with observations of the corresponding population (green cells) to estimate a more stable mortality trend.

# Model setup

- Recall the Lee-Carter model:
  - $\ln \mu_{t,x} = \alpha_x + \beta_x \cdot \kappa_t$
- If only observations on the population are available:
  - $D^{\text{pop}}_{t,x} | \mu_{t,x} \sim \text{Poisson}(e^{\text{pop}}_{t,x} \cdot \mu_{t,x})$ for $(t,x) \in \mathcal{O}^{\text{pop}}$
- If observations on the portfolio and the rest are available:
  - $D^{i}_{t,x} | \mu_{t,x}, \Theta^{i}_{x} \sim \text{Poisson}(e^{i}_{t,x} \cdot \mu_{t,x} \cdot \Theta^{i}_{x})$ for $i \in \{\text{pf}, \text{rest}\}$ and $(t,x) \in \mathcal{O}^{\text{pf}}$
  - $\Theta^{i}_{x}$ is an age-dependent portfolio-specific factor
  - We do not want to assume whether mortality in the portfolio or in the rest is higher or lower than the baseline mortality
    - We impose (*a priori*) $E[\Theta^{i}_{x}] = 1$ for $i \in \{\text{pf}, \text{rest}\}$ and $\forall x$

# Prior distributions

## Age and period effects

For the age effects (Czado et al. (2005)):

- $e_x = \exp(\alpha_x) \sim \text{Gamma}(a_x, b_x)$
- $\beta_x \overset{\text{iid}}{\sim} \text{N}(\mu_\beta, \sigma_\beta^2)$
  - For the hyperparameters $\mu_\beta$ and $\sigma_\beta^2$ we use conventional priors[1]

[1] See our working paper for more technical details.

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For the period effect we assume a random walk with drift:

- $\kappa_t = \kappa_{t-1} + \delta + \varepsilon_t, \quad$ with $\kappa_1 = 0$ and $\varepsilon_t \overset{\text{iid}}{\sim} \text{N}(0, \sigma_\varepsilon^2)$
  - For the hyperparameters $\delta$ and $\sigma_\varepsilon^2$ we use conventional priors

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1. Gamma (independent over ages and groups)
   $\Theta_x^i \sim \text{Gamma}(c_x^i, c_x^i) \quad$ for $i \in \{\text{pf}, \text{rest}\}$

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1. Gamma (independent over ages and groups)
   $\Theta_x^i \sim \text{Gamma}(c_x^i, c_x^i) \quad \text{for } i \in \{\text{pf}, \text{rest}\}$
2. Lognormal (dependence over ages, independent over groups)
   $\ln \Theta_x^i = \rho_\Theta \ln \Theta_{x-1}^i + \varepsilon_x^i$ with $\varepsilon_x^i \sim \text{N}(-\frac{1}{2}\sigma_{\theta^i}^2(1-\rho_{\theta^i}), \sigma_{\theta^i}^2(1-\rho_{\theta^i}^2))$
   - $\ln \Theta_{x_1}^i \sim \text{N}(-\frac{1}{2}\sigma_{\theta^i}^2, \sigma_{\theta^i}^2)$
   - For remaining parameters $(\rho_{\theta^i}, \sigma_{\theta^i}^2)$ we use conventional priors

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# Posterior distribution

## Population mortality parameters

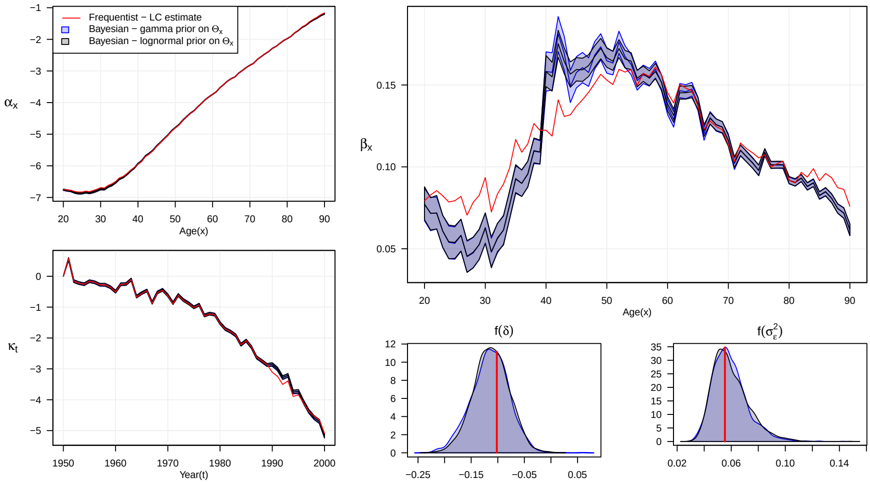


Figure: Population data used for the ages 20-90 and the years 1950-2000, and portfolio data used for the ages 40-75 and the years 1975-2000. The red lines represent frequentist estimates of LC on population data, coloured areas show the 95% credible interval (equal-tailed) for the proposed model.

# Posterior distribution

## Portfolio-specific factors

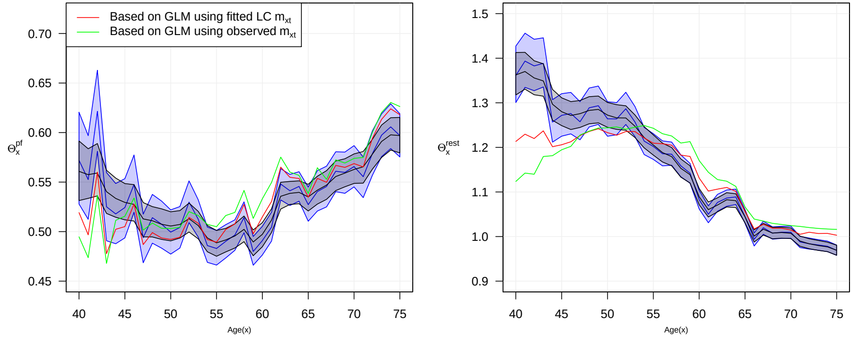


Figure: Population data used for the ages 20-90 and the years 1950-2000, and portfolio data used for the ages 40-75 and the years 1975-2000. The red and green lines represent portfolio-specific factors estimated using a Poisson GLM explaining portfolio deaths with portfolio exposure and population mortality as offset. Coloured areas show the 95% credible interval (equal-tailed) for the proposed model.

# Fitted and projected mortality rates

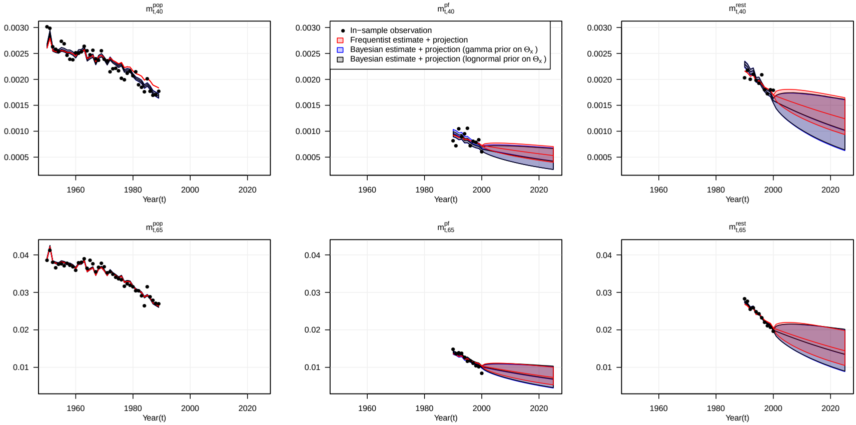


Figure: Fitted and projected mortality rates for $x = 40$ and $x = 65$.

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- We introduce a Bayesian model that accounts for all sources of randomness that affect portfolio-specific mortality
- The posterior distribution of the portfolio-specific factors depend on the prior distribution: assuming a correlation structure between ages leads to smoother portfolio-specific factors
- Case study suggests our model is better able to predict mortality for a small portfolio through the simultaneous estimation of population and portfolio-specific mortality than using a two-step procedure

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