# A comparative study of two-population models for the assessment of basis risk in longevity hedges

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# Longevity Swaps

- longevity swaps:
  - longevity risk transfer solution for pension schemes/annuity providers
  - flexible alternative to buy-ins/buy-outs
  - 2014 longevity swaps volume: £25.4bn $\rightsquigarrow$ more than the double the volume written in 2011-13 (Hymans Robertson LLP (2015))
- two types of swaps
  - indemnity based (bespoke): floating leg matches pension payments
  - index based: floating leg depends on mortality index (eg LifeMetrics, Deutsche Börse, ...)

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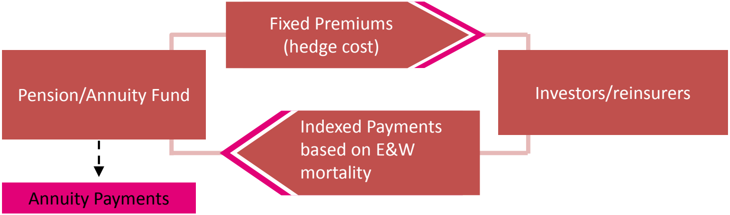
# Index Based Longevity Swap



# Indemnity based vs Index based Swap

- index based swap
  - standardized $\rightsquigarrow$ more efficient, cheaper solution
  - mismatch between pension payments and floating (index) cash flows $\rightsquigarrow$ basis risk
  - almost all swap transactions so far were bespoke
- key issue: understanding basis risk components
  - demographic risk: difference between the two populations — book and index
  - sampling risk: volatility in mortality experience
  - structural risk: due to limited range of hedging instrument

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# Where Does This Research Come From?

- research project sponsored by *Life and Longevity Market Association* and the *Institute of Actuaries*

Life & Longevity Markets Association | Institute and Faculty of Actuaries

- aim: develop a practical tool for analysing basis risk
- joint research group from Cass Business School and Hymans Robertson

Cass Business School CITY UNIVERSITY LONDON | HYMANS ROBERTSON The Spirit of Independence

- Phase I (2013-14) report available at `http://www.actuaries.org.uk/sites/all/files/documents/pdf/ifoa-llma-longevity-basis-risk-report.pdf`

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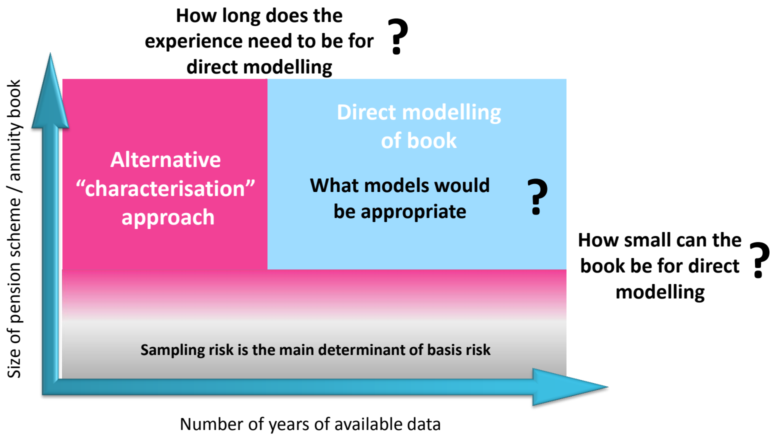
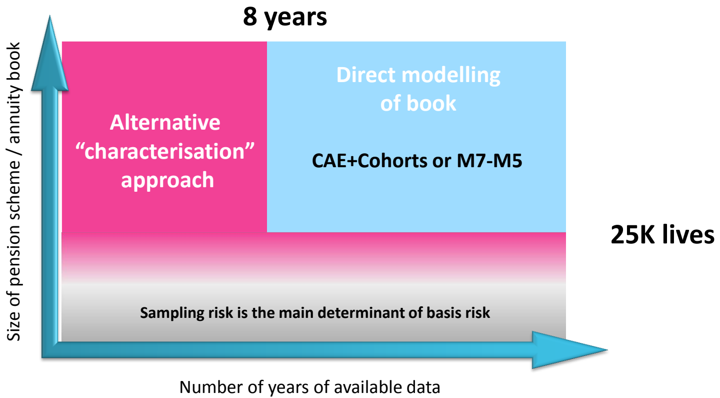
# Multi Population Mortality Models

- Basis risk assessment requires a two-population mortality model $\rightsquigarrow$ huge selection to choose from
- Narrowing down the long list of possible models?
  - ▷ Define criteria for 'good' practical model
  - ▷ Review existing models vs criteria
  - ▷ Define what modelling approach is appropriate in which cases

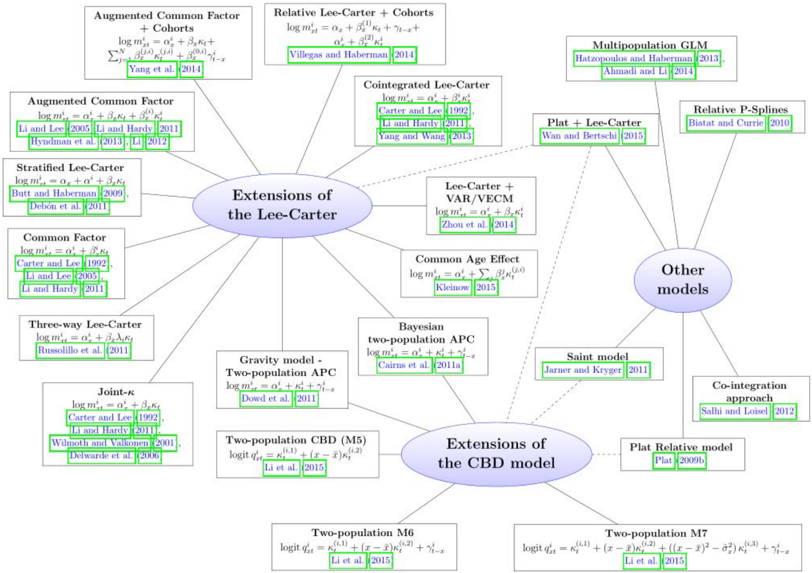
# Key Questions



# Main Findings



# Universe of Multipopulation Models



# Criteria to shortlist models (CMI wp 25, Cairns et al. (2008, 09), Haberman and Renshaw (2011))

- Practical
  - Easy to implement
  - Transparent
  - Parsimonious
  - Compatible with available data
  - Disentangle level and improvement differences
- Central estimates (deterministic)
  - Consistent with historic data
  - Consistent with expected mortality characteristics (e.g. compensation law)
  - Goodness of fit of rates and rate differences
  - Able to incorporate cohort effect

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- Correlations
  - Non-perfect correlations between year on year changes in mortality at different ages
  - Non-perfect correlations between mortality rates in the two populations
- Reasonableness
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  - Handle different book sizes, lengths of back history, portfolio heterogeneity
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# Shortlisting an Appropriate Model(s)

- Some criteria allowed us to narrow down the list of models based on their properties: for instance
  - Non-perfect correlations between mortality rates in the two populations $\rightsquigarrow$ all models with a single period term (eg common factor)
  - Compatible with available data $\rightsquigarrow$ co-integrated models
  - Easy to implement/Transparent $\rightsquigarrow$ Bayesian models, P-splines
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# General Mathematical Formulation

- two populations: reference (R) and book (B)
- $D^i_{xt}$, $i =$D, R: number of deaths at age $x$, calendar year $t$, population $i$; similarly for the matching exposures $E^i_{xt}$
- reference population

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D^R_{xt} &\sim \text{Bin}(E^R_{xt}, q^R_{x,t}) \\
\operatorname{logit} q^R_{x,t} &= \alpha^R_x + \sum_{i=1}^{N} \beta^{j,R}_x k^{j,R}_t + \gamma^R_{t-x} \\
\boldsymbol{k}^R_t &= \boldsymbol{d} + \boldsymbol{k}^R_{t-1} + \boldsymbol{\xi}^R_t, \ \boldsymbol{\xi}^R \sim \text{WN}(0, \Sigma^R) \\
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D^B_{xt} \sim& \mathsf{Bin}(E^B_{xt}, q^B_{x,t}) \\
\operatorname{logit} q^B_{x,t} - \operatorname{logit} q^R_{x,t} =& \alpha^B_x + \sum_{i=1}^{M} \beta^{j,B}_x k^{j,B}_t + \gamma^B_{t-x} \\
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- model the spread between mortality rates in the two populations
  - ▷ level differences: $(\alpha^B_x)$
  - ▷ improvement differences: $(\boldsymbol{k}^B_t) \rightsquigarrow$ VAR(1) implies not long run divergence
  - ▷ cohort differences: $(\gamma^B_c) \rightsquigarrow$ AR(1) implies not long run divergence

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# Reference population specification

Table 2: Mathematical description of the eight models considered for the reference population.

| Model | Formula |
|---|---|
| LC | $\operatorname{logit} q_{xt}^R = \alpha_x^R + \beta_x^R \kappa_t^R$ |
| LC2 | $\operatorname{logit} q_{xt}^R = \alpha_x^R + \beta_x^{(1,R)} \kappa_t^{(1,R)} + \beta_x^{(2,R)} \kappa_t^{(2,R)}$ |
| LC + Cohorts | $\operatorname{logit} q_{xt}^R = \alpha_x^R + \beta_x^R \kappa_t^R + \gamma_{t-x}^R$ |
| LC2 + Cohorts | $\operatorname{logit} q_{xt}^R = \alpha_x^R + \beta_x^{(1,R)} \kappa_t^{(1,R)} + \beta_x^{(2,R)} \kappa_t^{(2,R)} + \gamma_{t-x}^R$ |
| APC | $\operatorname{logit} q_{xt}^R = \alpha_x^R + \kappa_t^R + \gamma_{t-x}^R$ |
| M5 | $\operatorname{logit} q_{xt}^R = \kappa_t^{(1,R)} + (x - \bar{x}) \kappa_t^{(2,R)}$ |
| M6 | $\operatorname{logit} q_{xt}^R = \kappa_t^{(1,R)} + (x - \bar{x}) \kappa_t^{(2,R)} + \gamma_{t-x}^R$ |
| M7 | $\operatorname{logit} q_{xt}^R = \kappa_t^{(1,R)} + (x - \bar{x}) \kappa_t^{(2,R)} + ((x - \bar{x})^2 - \hat{\sigma}_x^2)\, \kappa_t^{(3,R)} + \gamma_{t-x}^R$ |

# ... Reference Population Specification

- England & Wales (EW), ages 60-89, years 1961-2010
- select Lee Carter+cohorts and M7 consistently with existing literature (Cairns et al. (2009), Haberman and Renshaw (2011))
  - ▷ Lee-Carter+cohorts

$$\text{logit}\, q_{x,t}^R = \alpha_x^R + \beta_x^R\, k_t^R + \gamma_{t-x}^R$$

  - ▷ M7

$$\text{logit}\, q_{x,t}^R = k_t^{1,R} + (x - \overline{x})k_t^{2,R} + ((x - \overline{x})^2 - \sigma_x^2)k_t^{3,R} + \gamma_{t-x}^R$$

# Book Population Specification

- generate synthetic datasets based on Club Vita schemes and IMD (postcode based) national mortality data
  - ▷ allows for changing some characteristics of the books
  - ▷ backtesting
- sample books base case:
  - ▷ period 1981-2010
  - ▷ age range 60-89
  - ▷ scheme size 100 000 lives per year (large)
  - ▷ four different distributions according to IMD typical split
- fit models by ML

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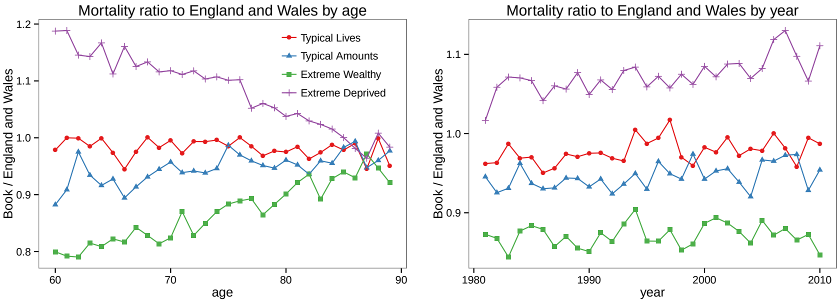
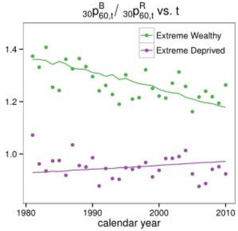


# Book Population Specification

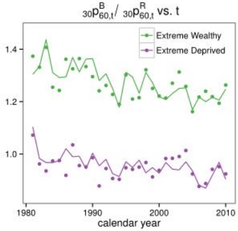
Table 4: Mathematical description of the two-population models considered for goodness-of-fit assessment.

| Original Model | Model tested | Reference Formula $\text{logit}\, q^{R}_{xt}$ | Book Difference Formula $\text{logit}\, q^{B}_{xt} - \text{logit}\, q^{R}_{xt}$ |
|---|---|---|---|
| Common Factor | CF+Cohorts | $\alpha^R_x + \beta^R_x \kappa^R_t + \gamma^R_{t-x}$ | $\alpha^B_x$ |
| Common Age Effect | CAE+Cohorts | $\alpha^R_x + \beta^R_x \kappa^R_t + \gamma^R_{t-x}$ | $\alpha^B_x + \beta^R_x \kappa^B_t$ |
| Relative Lee-Carter with cohorts | RelLC+Cohorts | $\alpha^R_x + \beta^R_x \kappa^R_t + \gamma^R_{t-x}$ | $\alpha^B_x + \beta^B_x \kappa^B_t$ |
| Gravity | Gravity (APC) | $\alpha^R_x + \kappa^R_t + \gamma^R_{t-x}$ | $\alpha^B_x + \kappa^B_t + \gamma^B_{t-x}$ |
| Two-population M5 | M7-M5 | $\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,R)}_t + \gamma^R_{t-x}$ | $\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t$ |
| Two-population M6 | M7-M6 | $\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,R)}_t + \gamma^R_{t-x}$ | $\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + \gamma^B_{t-x}$ |
| Two-population M7 | M7-M7 | $\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,R)}_t + \gamma^R_{t-x}$ | $\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,B)}_t + \gamma^B_{t-x}$ |
| Saint model | M7-Saint | $\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,R)}_t + \gamma^R_{t-x}$ | $\kappa^{(1,B)}_t + (x-\bar{x})\kappa^{(2,B)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,B)}_t$ |
| Plat relative model | M7-Plat | $\kappa^{(1,R)}_t + (x-\bar{x})\kappa^{(2,R)}_t + ((x-\bar{x})^2 - \hat{\sigma}^2_x)\,\kappa^{(3,R)}_t + \gamma^R_{t-x}$ | $\frac{100-x}{100-\bar{x}}\kappa^{(1,B)}_t$ |

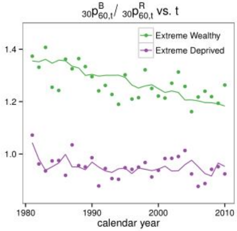
# Fitting (Rates and) Rates Differences



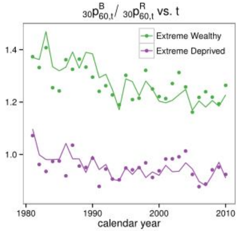
(a) CF+Cohorts



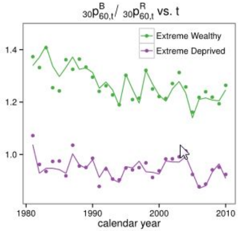
(b) CAE+Cohorts



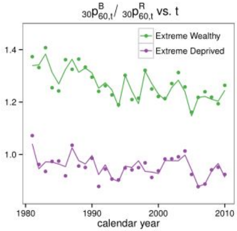
(c) RelLC+Cohorts



(d) Gravity



(e) M7-M5



(f) M7-M6

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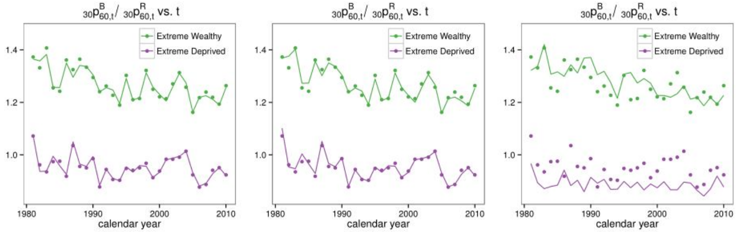


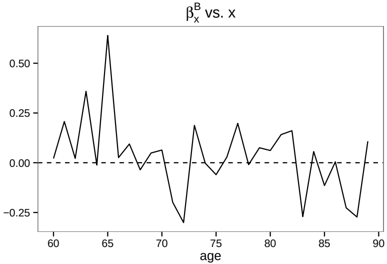

(g) M7-M7 (h) M7-Saint (i) M7-Plat

Figure 7: Fitted vs. observed ratio of 30 year period survival probabilities at age 60 for the “Extreme Wealthy” and the “Extreme Deprived” sample schemes.

- Some models are too simple to fit observed differences

# Main Message

- avoid models with non parametric book specific age response term $\beta_x^B$
  - not enough book data to estimate $\beta_x^B$
  - may produce over-smoothed aggregate demographics metrics $\rightsquigarrow$ model behaves as if it implied perfect correlation
  - example: RelLC+Cohort



# Goodness of Fit vs Parsimony

Table 5: Effective number of parameters and AIC for the book part of different two-population models fitted to the four test books.

| Model | Number of reference parameters | Number of book parameters | Typical Lives | Typical Amounts | Extreme Wealthy | Extreme Deprived |
|---|---|---|---|---|---|---|
| CF+Cohorts | 185 | 30 | 7008 (1) | 7001 (1) | 6921 (1) | 7146 (4) |
| CAE+Cohorts | 185 | 59 | 7036 (2) | 7026 (2) | 6950 (2) | 7130 (3) |
| Gravity | 156 | 116 | 7090 (5) | 7077 (5) | 7010 (5) | 7182 (6) |
| M7-M5 | 226 | 60 | 7043 (3) | 7049 (3) | 6971 (3) | 7102 (1) |
| M7-M6 | 226 | 117 | 7106 (6) | 7099 (6) | 7033 (6) | 7166 (5) |
| M7-M7 | 226 | 146 | 7123 (7) | 7128 (7) | 7052 (7) | 7188 (7) |
| M7-Saint | 226 | 90 | 7069 (4) | 7074 (4) | 6991 (4) | 7117 (2) |

- CAE+Cohort and M7-M5 have best compromise
- models with book specific cohort have the worst trade-off
- enough to capture level and and slope differences

# CAE+cohorts & M7-M5

- CAE+cohorts:

$$\begin{aligned} \operatorname{logit} q_{x,t}^R =& \alpha_x^R + \beta_x^R k_t^R + \gamma_{t-x}^R \\ \operatorname{logit} q_{x,t}^B - \operatorname{logit} q_{x,t}^R =& \alpha_x^B + \beta_x^R k_t^B \end{aligned}$$

- M7-M5:

$$\begin{aligned} \operatorname{logit} q_{x,t}^R =& k_t^{1,R} + (x - \overline{x}) k_t^{2,R} \\ & + ((x - \overline{x})^2 - \widehat{\sigma}_x^2)) k_t^{3,R} + \gamma_{t-x}^R \\ \operatorname{logit} q_{x,t}^B - \operatorname{logit} q_{x,t}^R =& k_t^{1,B} + (x - \overline{x}) k_t^{2,B} \end{aligned}$$

# Reasonable Forecast Levels of Uncertainty (Cairns et al. (2011))

- sources of uncertainty:
  - ▷ process risk (PR) $\rightsquigarrow$ future trajectories of $\boldsymbol{k}^B$ and $\gamma^B$
  - ▷ parameter uncertainty (PU) $\rightsquigarrow$ estimation of parameters of the model, computed via bootstrapping (Koissi et al. (2006), Renshaw and Haberman (2008))
  - ▷ sampling risk (SR) $\rightsquigarrow$ volatility of actual mortality experience ($D_{xt}^B$)
- focus on key metrics
  - ▷ period truncated life expectancy $\rightsquigarrow$ value based hedge
  - ▷ cohort truncated life expectancy $\rightsquigarrow$ cash-flow based hedge
- analyse robustness wrt book size and book length

# Reasonable Forecast Levels of Uncertainty (Cairns et al. (2011))

- sources of uncertainty:
  - ▷ process risk (PR) $\rightsquigarrow$ future trajectories of $\boldsymbol{k}^B$ and $\gamma^B$
  - ▷ parameter uncertainty (PU) $\rightsquigarrow$ estimation of parameters of the model, computed via bootstrapping (Koissi et al. (2006), Renshaw and Haberman (2008))
  - ▷ sampling risk (SR) $\rightsquigarrow$ volatility of actual mortality experience ($D_{xt}^B$)
- focus on key metrics
  - ▷ period truncated life expectancy $\rightsquigarrow$ value based hedge
  - ▷ cohort truncated life expectancy $\rightsquigarrow$ cash-flow based hedge
- analyse robustness wrt book size and book length

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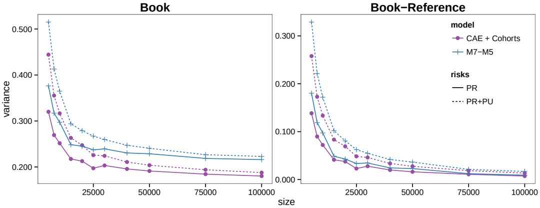
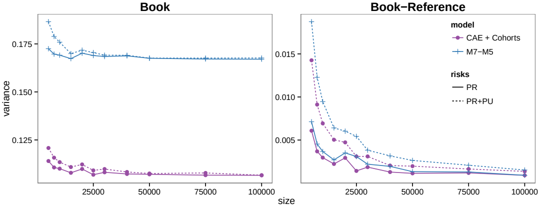
# Robustness wrt Book Size

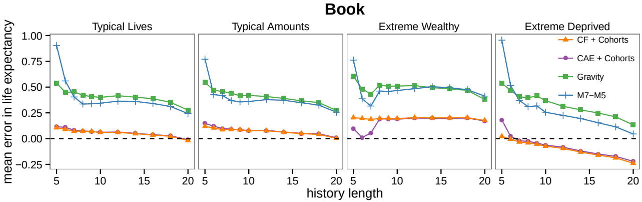



30 year curtailed period life expectancy at age 60 in 2020

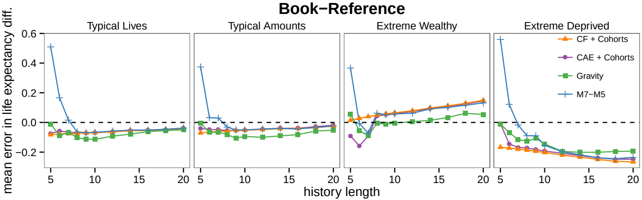



25 year curtailed cohort life expectancy at age 65 in 2011

# Robustness wrt Book History

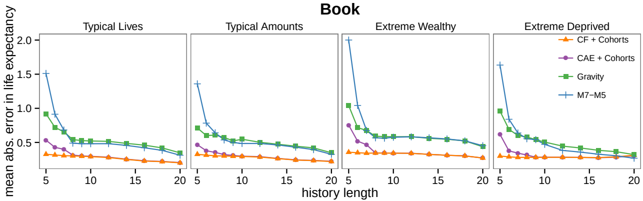



Forecast mean error in the forecast of 30 year period curtailed life expectancy at age 60

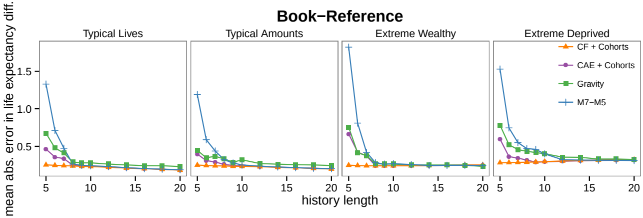



Forecast mean error in the difference of 30 year period curtailed life expectancy at age 60 between the book and the reference

# . . . Robustness wrt Book History




Forecast mean absolute error in the forecast of 30 year period curtailed life expectancy at age 60




Forecast mean absolute error in the difference of 30 year period curtailed life expectancy at age 60 between the book and the reference

# Conclusion

- robustness wrt book size
  - for book sizes $< 15\,000$ lives, PR is unrealistically high distorting the basis risk assessment
  - for book sizes $< 25\,000$ lives, PU is significant distorting the basis risk assessment
- robustness wrt book length
  - History length shorter than 8 years have poor forecasting performance
  - CAE+Cohort has the best out-of-sample performance both in terms of bias and accuracy
  - for history lengths longer than 8 years no model outperform the others in terms of differences

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# Hedge Effectiveness (Cairns et al. (2014))

- measured by

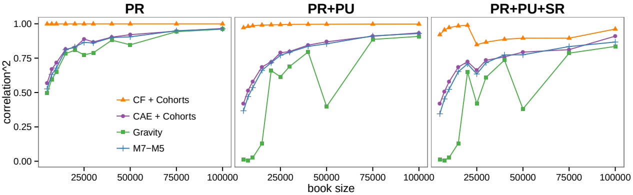
$$R^2 = 1 - \frac{\mathsf{VAR}(L - h^* H)}{\mathsf{VAR}(L)} = \rho^2$$

where

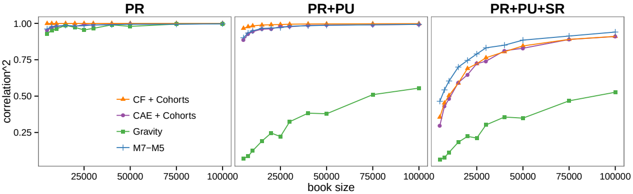
- ▷ $L$ liability, either book period (value hedge) or cohort (cash-flow hedge) life expectancy
- ▷ $H$ hedging instrument $\rightsquigarrow$ corresponding quantity in the reference population
- ▷ $h^*$ optimal hedge ratio
- ▷ $\rho$ correlation coefficient between $L$ and $H$

- focus on analysis wrt book size and length

# . . . Hedge Effectiveness

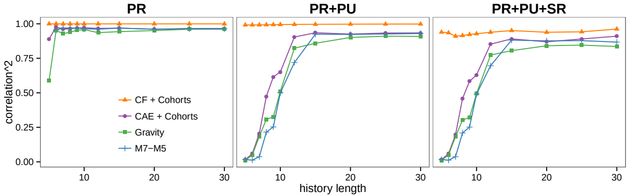



Value hedge example: 30 year curtailed period life expectancy at age 60 in 2020




Cash flow hedge example: 25 year curtailed cohort life expectancy at age 65 in 2011

# . . . Hedge Effectiveness

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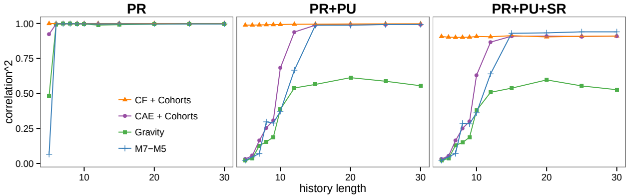


Cash flow hedge example: 25 year curtailed cohort life expectancy at age 65 in 2011

# Main Conclusions

- Our analysis suggest that the most appropriate models are the M7-M5 (more flexible) and the Common Age Effect plus Cohorts
- For the estimation exercise to be reliable
  - 25 000 lives
  - 8 years of available history

  are the minimum requirement