

Monitoring the interplay between longevity risk and health dynamics: the case of a Life Care Reverse Mortgage portfolio

Giovanna Apicella¹, Emilia Di Lorenzo², **Giulia Magni³**, Marilena Sibillo⁴

¹University of Udine, Italy

²University of Naples Federico II, Italy

³Sapienza University of Rome, Italy
giulia.magni@uniroma1.it

⁴University of Salerno, Italy

Longevity 21

Rome, September 9th 2026



- ① Background
- ② Methodology
- ③ Numerical Application
- ④ Results
- ⑤ Conclusions
- ⑥ References



1 Background

2 Methodology

3 Numerical Application

4 Results

5 Conclusions

6 References

Context

-  Population ageing and the increase in **life expectancy** are not associated with an increase in **healthy life expectancy**.
-  *Male–female health survival paradox:*
Females **live longer** than males but spend **more years in functional dependence**.
-  These demographic trends directly affect products jointly exposed to **longevity risk** and **long-term care risk**, such as the Life Care Reverse Mortgage (LCRM).

Italian Demographic Evidences: 2024



-0.4

Product explanation

Life Care Reverse Mortgage (LCRM)

A **hybrid insurance product** financed through the **home equity**.

💡 One product, two regimes:

- 🏠 The **RM regime**: The policyholder receives a stream of lifetime income as long as he/she remains **healthy and functionally able**.
- 💓 The **LTC regime**: offering enhanced benefits, activated when **functional dependence** occurs.

📖 Apicella, G., Di Lorenzo, E., Magni, G., Sibillo, M. (2025). Life Care Reverse Mortgages: monitoring the net cashflows of a new hybrid insurance product. *Insurance: Mathematics and Economics*, 103170.

Main Features

- ✓ Unlocks **housing wealth** through a loan
- ✓ Allows heirs to **redeem** the property (question of bequest)
- ✓ Supports **ageing** staying at home, safely
- ✓ Provides financial support to elderly people in case of **functional dependence**
- ✓ Embeds an automatic **Prescriptive Saving Program**.

Q Prescriptive Saving Program

LCRM **redistributes** financial resources from periods of *good health* to periods of *functional dependence*.

The borrower commits to this saving strategy simply by entering the contract.

Actuarial Relevance

- ❗ LCRM portfolios are simultaneously exposed to **longevity**, **functional dependence**, **financial**, and **house price** risks.
- ➡ The **quantitative modelling** of these risks is essential for product valuation, pricing and risk management.

🔍 Research question

How does the interaction between **longevity** and **functional dependence** risks influence the lender's **profit/loss profile** under different **portfolio gender compositions**?

1 Background

2 Methodology

3 Numerical Application

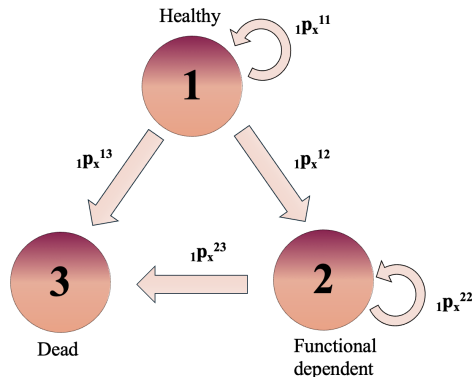
4 Results

5 Conclusions

6 References

Health States

Given the health state space $S = \{1, 2, 3\}$



$$\star \quad {}_1p_x^{11} + {}_1p_x^{12} + {}_1p_x^{13} = 1$$

$$\star \quad {}_1p_x^{22} + {}_1p_x^{23} = 1.$$

${}_1p_x^{ij}$ is the probability that a person aged $x + 1$ is in health state j , conditional on being in state i at age x .

LCRM Cash Flows

At $t=0$, the policyholder pays no premiums.

Inflows (Lender): $\tilde{V}_t$

 **Accrued loan at contract termination** (death of policyholder) through:

-  repayment by heirs, or
-  proceeds from the sale of the property.

Outflows (Lender)

 $R^{RM} = \frac{LS_0}{a_{x,0:\overline{n}|T}^{LCRM}};$

 $R^{LTC} = R^{RM}(1 + \beta)$

Outflows: focus on R^{RM}

$$R^{RM} = \frac{LS_0}{a_{x,0:\overline{n}|T}^{LCRM}} \quad \Longleftarrow$$

Valuation basis
Unisex - Male - Female

- i** $a_{x,0:\overline{n}|T}^{LCRM}$: actuarial present value of a life annuity issued to a person aged x , with maximum duration T , accounting for health-state transitions.
- i** $LS_0 = \sum_{t=1}^T \tilde{V}_t (\prod_{s=1}^t (1 + \tilde{r}_s)^{-1})_{t-1/1} \tilde{q}_x$

$$\tilde{V}_t = \tilde{A}_t - \max(\tilde{A}_t - \tilde{H}_t, 0) = \min(\tilde{A}_t, \tilde{H}_t)$$

$$\tilde{A}_t = \alpha H_0 \prod_{s=0}^{t-1} (1 + \tilde{r}_s + \pi), \quad \tilde{H}_t = \alpha H_0 \prod_{s=0}^{t-1} (1 + \tilde{r}_s^{HV})$$

Profit/Loss function

$$P_t = -R^{RM} n_t^{(1)} - R^{LTC} n_t^{(2)} + V_t n_t^{(3)} \quad (1)$$

- ♥ $n_t^{(1)}$ number of active LCRM contracts for **healthy** borrowers;
- ♿ $n_t^{(2)}$ number of active LCRM contracts for **functionally dependent** borrowers;
- ✖ $n_t^{(3)}$ number of **ceased** contracts due to the borrower's death.

N_0 is the initial number of LCRM contracts

$$n_t^{(1)} = N_0 \sum_{h=1}^{m-t} {}_h p_{x+t-1}^{11}$$

$$n_t^{(2)} = N_0 \sum_{h=1}^{m-t} {}_1 p_{x+t-1}^{12} {}_h p_{x+t}^{22}$$

$$n_t^{(3)} = N_0 - n_t^{(1)} - n_t^{(2)}$$

and with transition survival probabilities:

Healthy-to-healthy

$${}_h p_{x+t-1}^{11} = \prod_{s=0}^{h-1} {}_1 p_{x+t-1+s}^{11}$$

Dependent-to-dependent

$${}_h p_{x+t}^{22} = \prod_{s=0}^{h-1} {}_1 p_{x+t+s}^{22}$$

1 Background

2 Methodology

3 Numerical Application

4 Results

5 Conclusions

6 References

Contract Assumptions

LCRM portfolio assumptions

 Parameter	Value
 Initial number of contracts N_0	1,000
 Age at issue	75
 Year of issue	2013
 Maximum contract duration (m)	31 years
 Initial house value H_0	200,000 €
 Lending percentage α	30%
 Enhancement coefficient β	1.5

Data and Calibration

-  Quarterly Italian nominal house price indices (Q1 1970–Q4 2012).
Geometric Brownian Motion (GBM) model.

$$dH_t = \mu H_t dt + \sigma H_t dZ_t \quad (2)$$

-  Monthly 10-year Italian government bond yields (Apr 1991–Dec 2012).
CIR model.

$$dr_t = k(\theta - r_t)dt + s_t\sqrt{r_t}dZ_t \quad (3)$$

Data and Calibration

- ♥ Italian mortality data (1963-2012), Human Mortality Database (HMD).

Lee Carter model.

$$\ln(m_{x,t}) = \alpha_x + b_x k_t \quad (4)$$

- 👤 Italian Transition probabilities among 3 scenarios (high, medium and low) and 3 health states from De Angelis & Di Falco (2016) technical basis (2013–2043).

- 1 Background
- 2 Methodology
- 3 Numerical Application
- 4 Results**
- 5 Conclusions
- 6 References

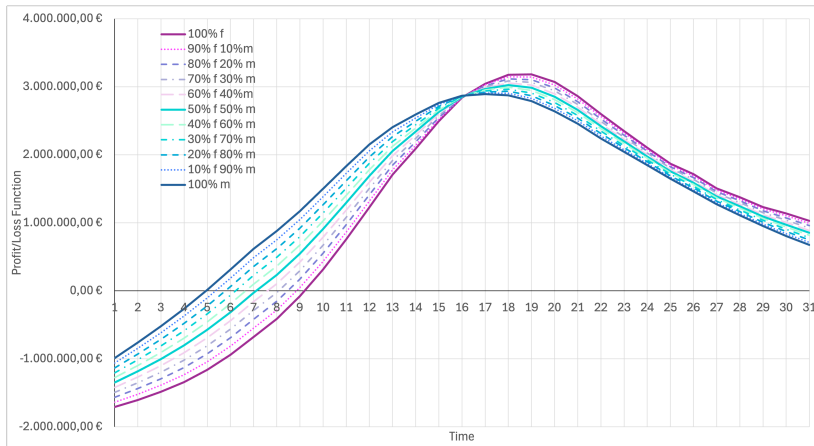
Results

From the calibrated and projected models and the LCRM contract parameters, we obtain:

i Quantity	Female	Unisex	Male
LS_0	43 376.03	44 469.36	46 065.20
NNEG	16 623.97	15 530.64	13 934.80
R (classical RM)	4 458.02	4 894.45	5 653.98
R^{RM} (LCRM)	2 052.29	2 312.58	2 635.54
R^{LTC} (LCRM)	5 130.74	5 781.44	6 588.86

Results

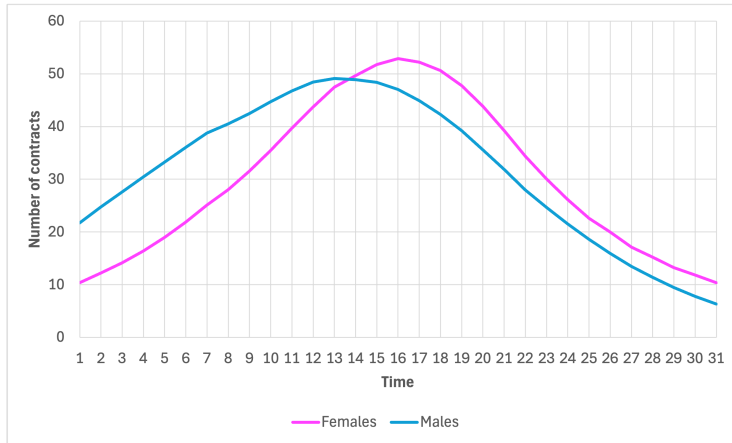
Profit/Loss Function: Portfolio Gender Composition



Results

Number of ceased contracts

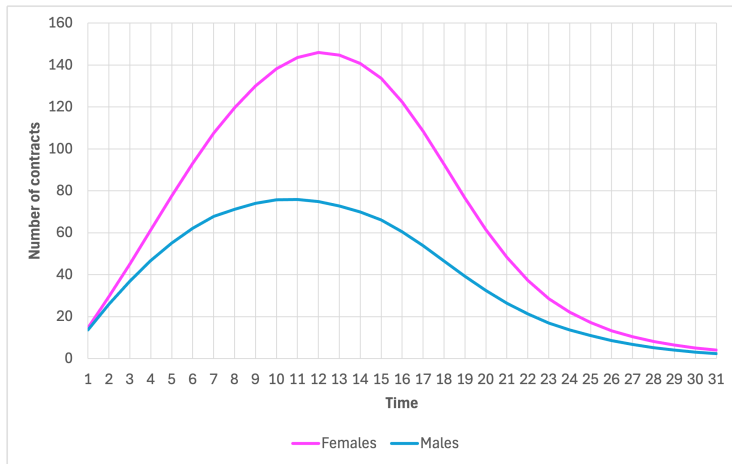
Starting from initial $N_0 = 1,000$ males and females respectively



Results

Number of functionally dependent contracts

Starting from initial $N_0 = 1,000$ males and females respectively



Summary

From $t = 1$ to $t = 16$:

⇒ **Females:** Higher losses & Lower profits:

↓ Mortality

↑ Probability to become and remain functionally dependent

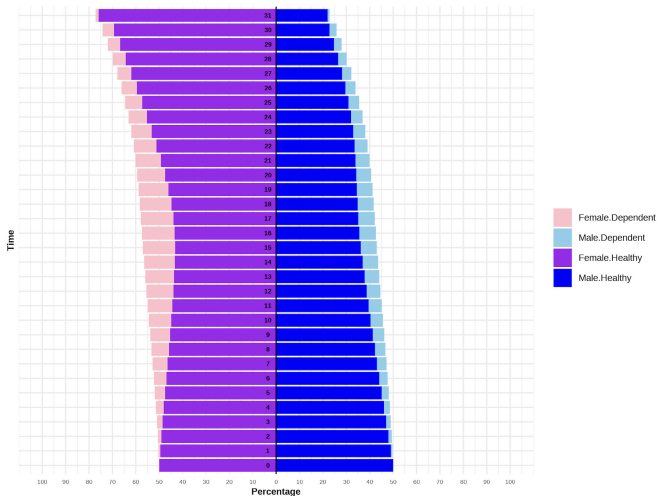
From $t = 16$ onwards:

⇒ **Females:** Higher profits:

↑ Mortality

Results

Portfolio Gender Composition over Time (Initial 50% M, 50% F)



1 Background

2 Methodology

3 Numerical Application

4 Results

5 Conclusions

6 References

Conclusions

Key Findings

- The profitability of a LCRM portfolio is strongly influenced by its gender composition.
- The interaction between longevity risk and disability risk is dynamic and depends on the evolving portfolio composition.
- Ignoring these dynamics may lead to a misleading assessment of the lender's profitability.

1 Background

2 Methodology

3 Numerical Application

4 Results

5 Conclusions

6 References

Main References

- ☰ HMD - *Human Mortality Database*. Max Planck Institute for Demographic Research (Germany), University of California, Berkeley (USA), and French Institute for Demographic Studies (France). Available at: www.mortality.org.
- ☰ Di Falco, L., De Angelis, P. (2016). *Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche*. Società editrice il Mulino spa.
- ☰ Banca d'Italia (2025). *Infostat — Statistiche e Dati Economici*. Available at: <https://infostat.bancaditalia.it/inquiry/home?spyglass/taxo:CUBESET=&ITEMSELEZ=&OPEN=false/&ep:LC=IT&COMM=BANKITALIA&ENV=LIVE&CTX=DIFF&IDX=1&/view:CUBEIDS=BMK0100>
- ☰ OECD, Housing prices (indicator). doi: 10.1787/63008438-en.

Main References

- 📖 Apicella, G., Di Lorenzo, E., Magni, G., Sibillo, M. (2025). Life Care Reverse Mortgages: monitoring the net cashflows of a new hybrid insurance product. Insurance: Mathematics and Economics, 103170.
- 📖 Levantesi, S., Menzietti, M. (2012). Managing longevity and disability risks in life annuities with long term care. Insurance: Mathematics and Economics, 50(3), 391-401.
- 📖 Shao, A. W., Chen, H., Sherris, M. (2019). To borrow or insure? long term care costs and the impact of housing. Insurance: Mathematics and Economics, 85, 15-34.
- 📖 Bohnert, A., Born, P., Gatzert, N. (2014). Dynamic hybrid products in life insurance: assessing the policyholders' viewpoint. Insurance: Mathematics and Economics, 59, 87-99.