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# MULTI-POPULATION MORTALITY MODELLING:

## A Danish Case Study

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Longevity 11, Lyon, 2015

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## Plan

- Introduction and motivation for multi-population modelling
- Modelling Danish sub-population mortality
- Economic capital

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# 1. Motivation for multi-population modelling

## A: Risk assessment

- Multi-country (e.g. consistent demographic projections)
- Males/Females (e.g. consistent demographic projections)
- Socio-economic subgroups (e.g. blue or white collar)
- Smokers/Non-smokers
- Annuities/Life insurance
- Limited data  $\Rightarrow$  learn from other populations

$\rightarrow$  reserving calculations; diversification benefits

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# Motivation for multi-population modelling

## B: Risk management for pension plans and insurers

- Retain systematic mortality risk; versus:
- ‘Over-the-counter’ deals (e.g. longevity swap)
- Standardised mortality-linked securities
  - linked to national mortality index
  - $< 100\%$  risk reduction: basis risk

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# Multi-Population Challenges

- Data availability
- Data quality and depth
- Model complexity
  - single population models can be complex
  - 2-population versions are more complex
  - multi-pop .....
- Multi-population modelling requires
  - (fairly) simple single-population models
  - simple dependencies between populations

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## 2. A New Case Study and a New Model

- Sub-populations differ from national population
  - socio-economic factors
  - other factors
- Denmark
  - High quality data on ALL residents
  - 1981-2005 available (later data soon)
  - Can subdivide population using covariates on the database

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## Danish Data

- *What can we learn from Danish data that will inform us about other populations?*
- Key covariates
  - Wealth
  - Income
- $Affluence = Wealth + 15 \times Income$

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## Problem

- High income  $\Rightarrow$  “affluent” *and low mortality* BUT
- Low income  $\not\Rightarrow$  not affluent, high mortality
- High wealth  $\Rightarrow$  “affluent” *and low mortality* BUT
- Low wealth  $\not\Rightarrow$  not affluent, high mortality

Empirical solution: use a combination

- Affluence,  $A = \text{wealth} + K \times \text{income}$
- $K = 15$  seems to work well *statistically* as a predictor
- Low affluence,  $A$ , predicts poor mortality

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## Subdividing Data (after much experimentation!)

- Males resident in Denmark for the previous 12 months
- Divide population in year  $t$ 
  - into 10 equal sized Groups (approx)
  - using *affluence*,  $A$
- Individuals can change groups up to age 67
- Group allocations are locked down at age 67

(better than not locking down at age 67)

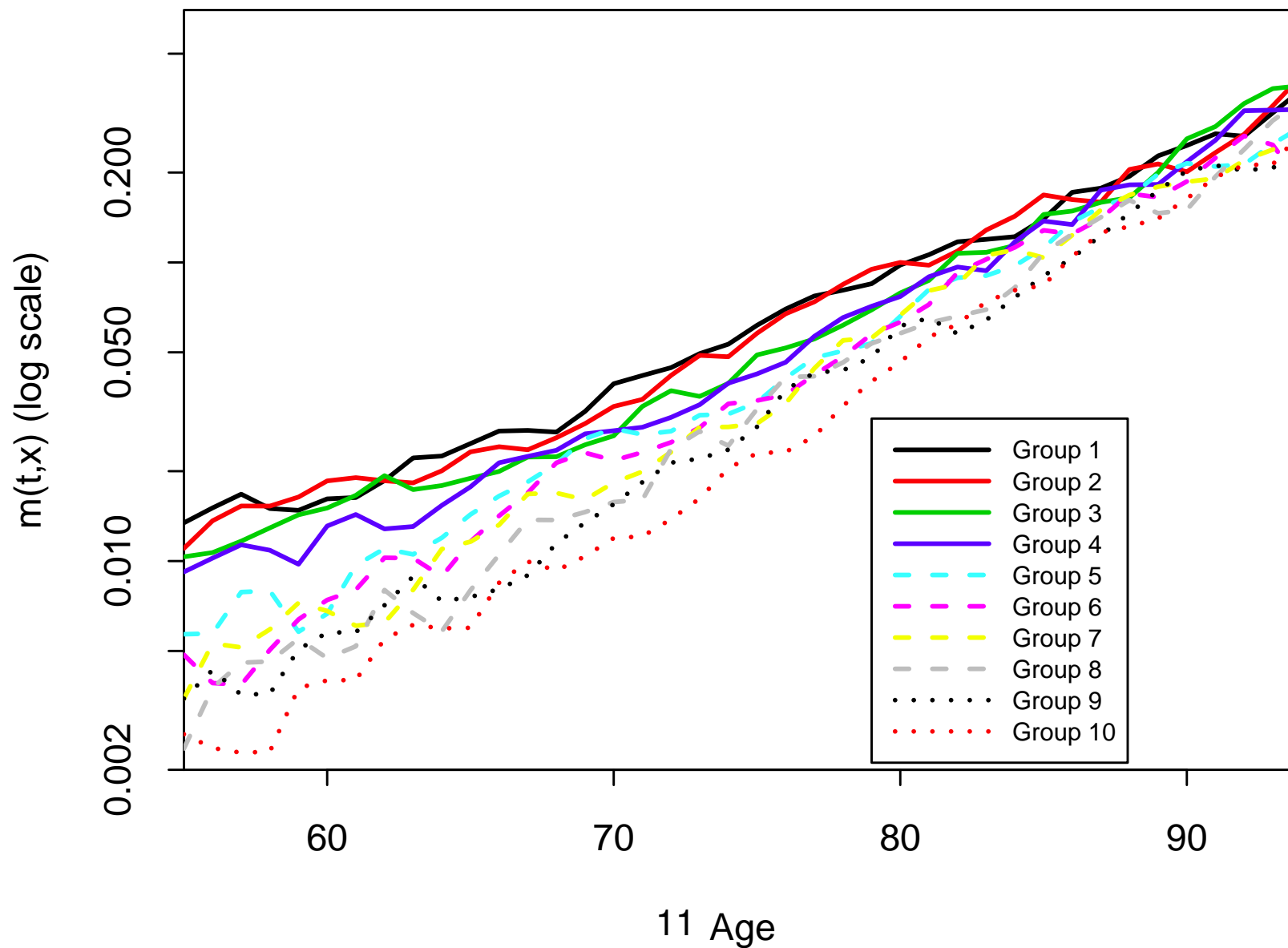
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## Subdivided Data

- Exposures  $E^{(i)}(t, x)$  for groups  $i = 1, \dots, 10$   
range from over 4000 down to 24
- Deaths  $D^{(i)}(t, x)$   
range from 152 down to 6
- Crude death rates  $\hat{m}^{(i)}(t, x) = D^{(i)}(t, x) / E^{(i)}(t, x)$
- Small groups  $\Rightarrow$  Poisson risk is important

# Crude death rates 2012

## Males Crude $m(t,x)$ ; 2012



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## Modelling the death rates, $m_k(t, x)$

$m^{(k)}(t, x)$  = pop.  $k$  death rate in year  $t$  at age  $x$

Population  $k$ , year  $t$ , age  $x$

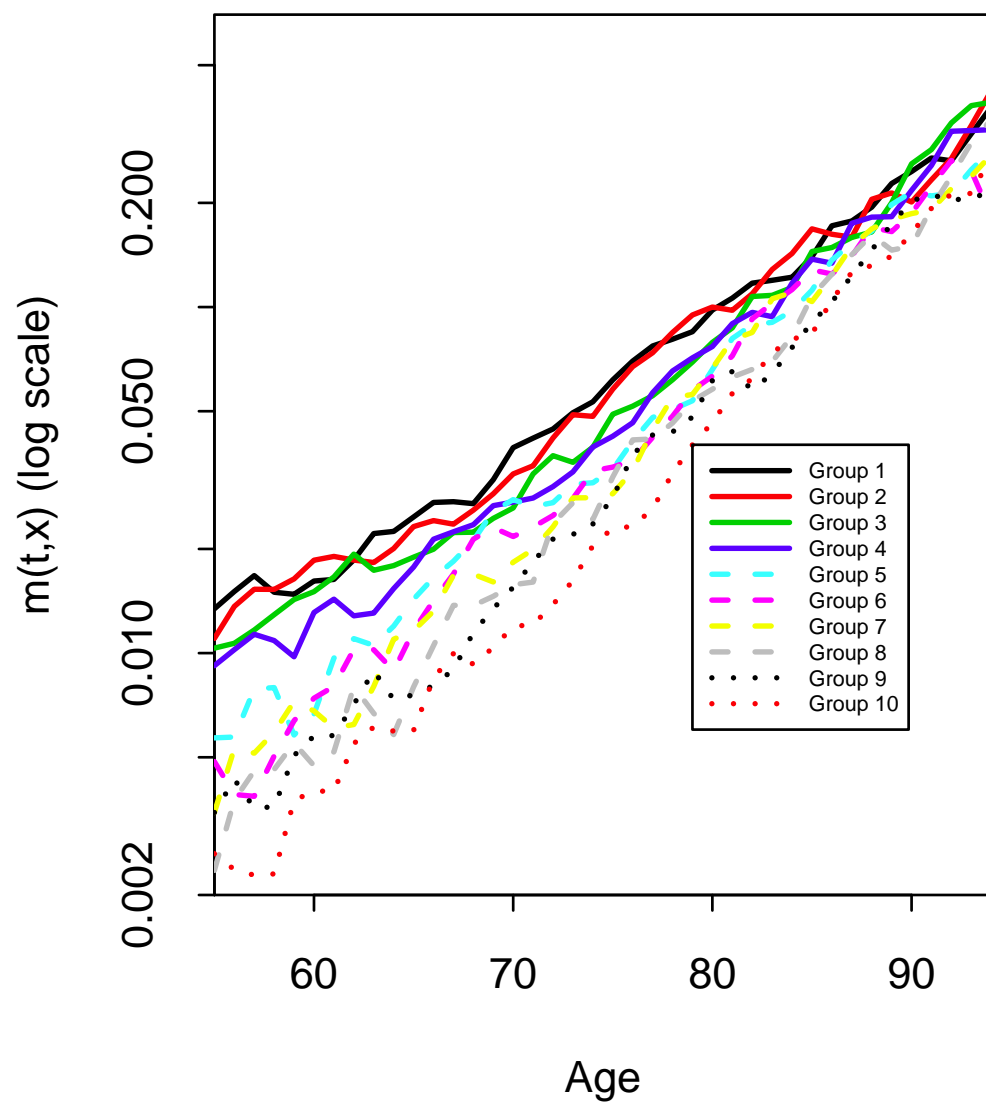
$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

(Extended CBD with a non-parametric base table,  $\beta^{(k)}(x)$ )

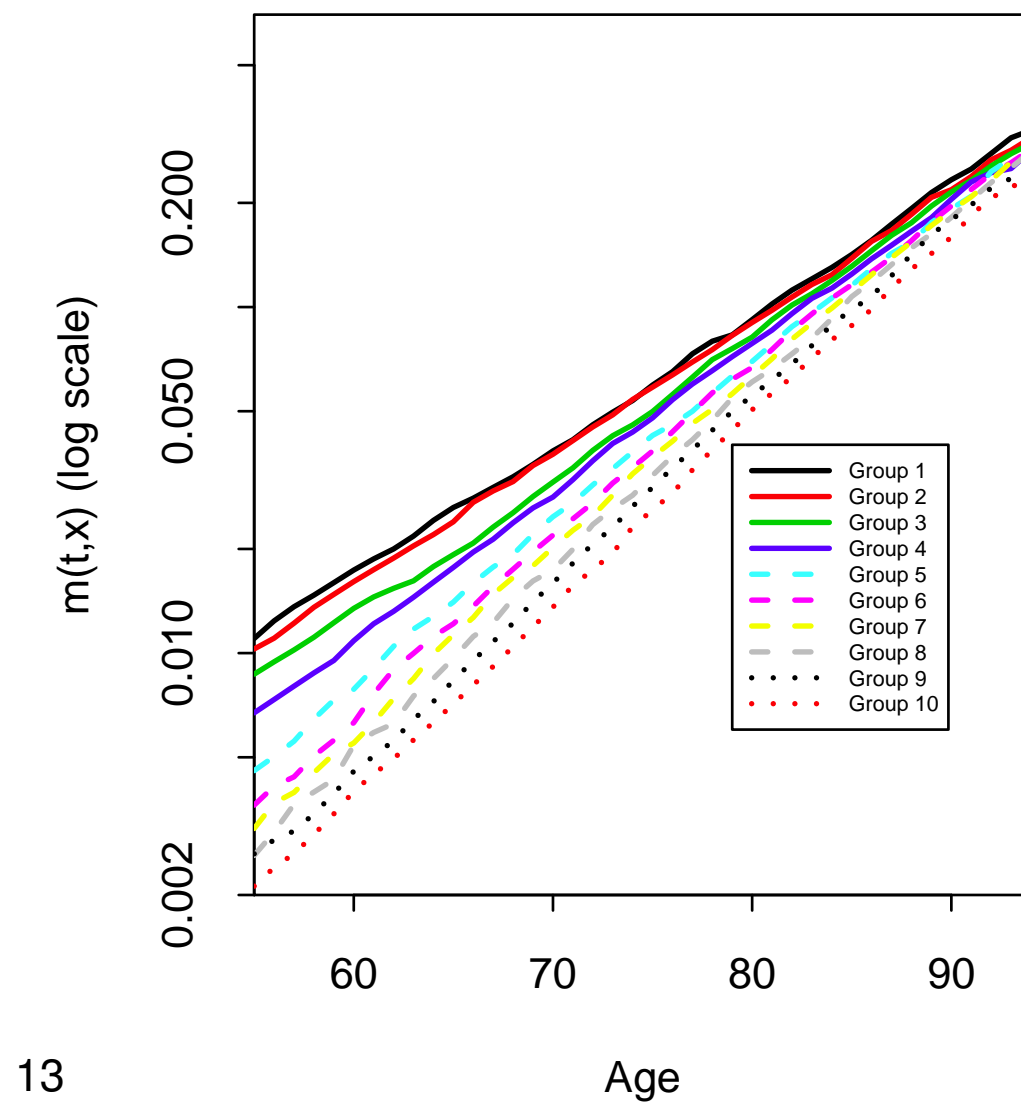
- 10 groups,  $k = 1, \dots, 10$  (low to high affluence)
- 21 years,  $t = 1985, \dots, 2005$
- 40 ages,  $x = 55, \dots, 94$

# Model-Inferred Underlying Death Rates 2012

## Males Crude $m(t,x)$ ; 2012



## Males CBD-X Fitted $m(t,x)$ ; 2012 Point Estimates



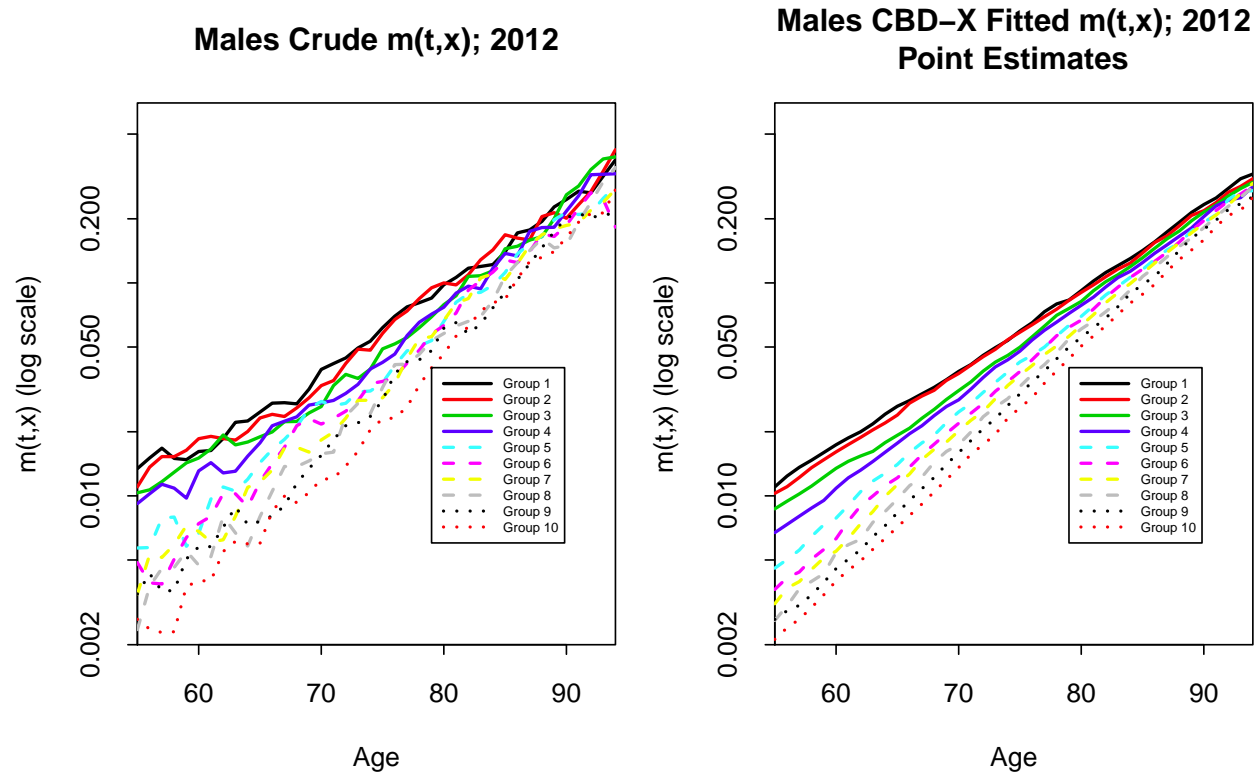
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## Modelling the death rates, $m_k(t, x)$

$$\log m^{(k)}(t, x) = \beta^{(k)}(x) + \kappa_1^{(k)}(t) + \kappa_2^{(k)}(t)(x - \bar{x})$$

- Model fits the 10 groups well without a cohort effect
- Non-parametric  $\beta^{(k)}(x)$  is essential to preserve group rankings
  - Rankings are evident in crude data
  - *“Bio-demographical reasonableness”*:  
more affluent  $\Rightarrow$  healthier

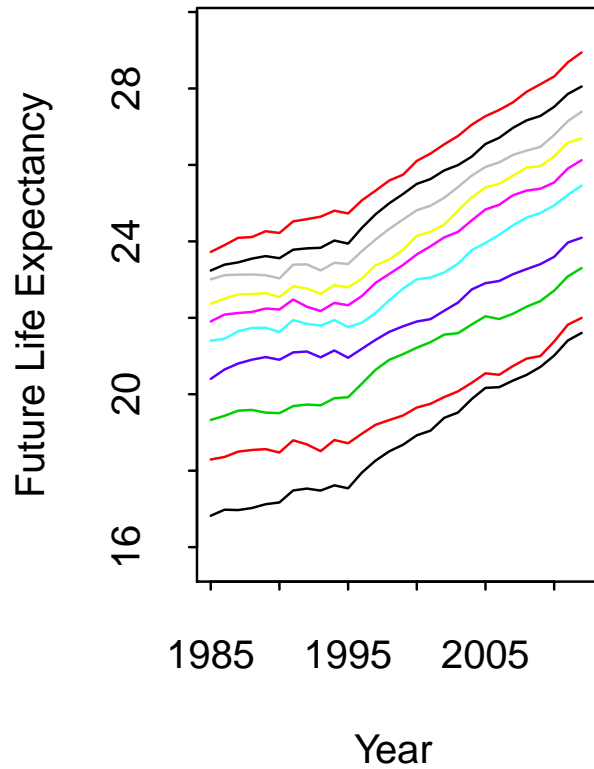
# Model-Inferred Underlying Death Rates 2012



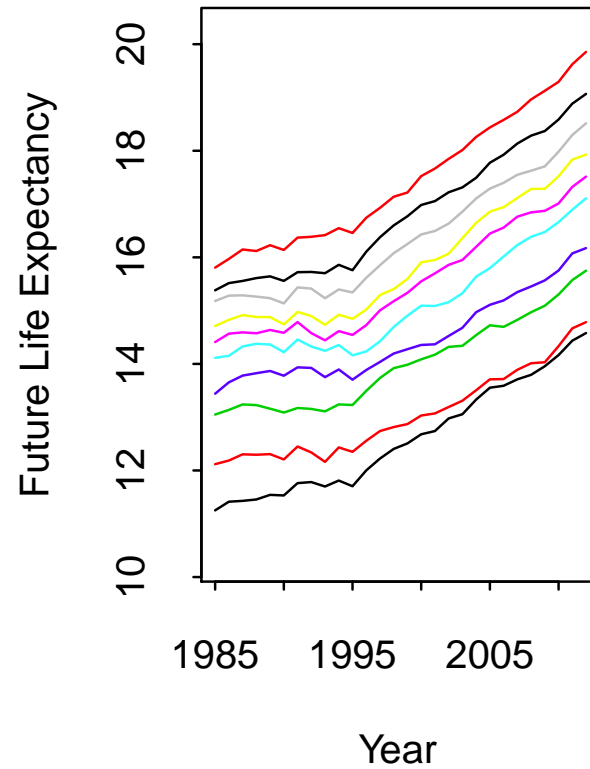
- Gap reduces from over  $5\times$  to  $1.3\times$
- Or **+14 years** difference for Group 1  $\rightarrow$  10, age 55; **+9** at 67.
- Convergence  $\Rightarrow$  way ahead for modelling very high ages???

# Life Expectancy for Groups 1 to 10

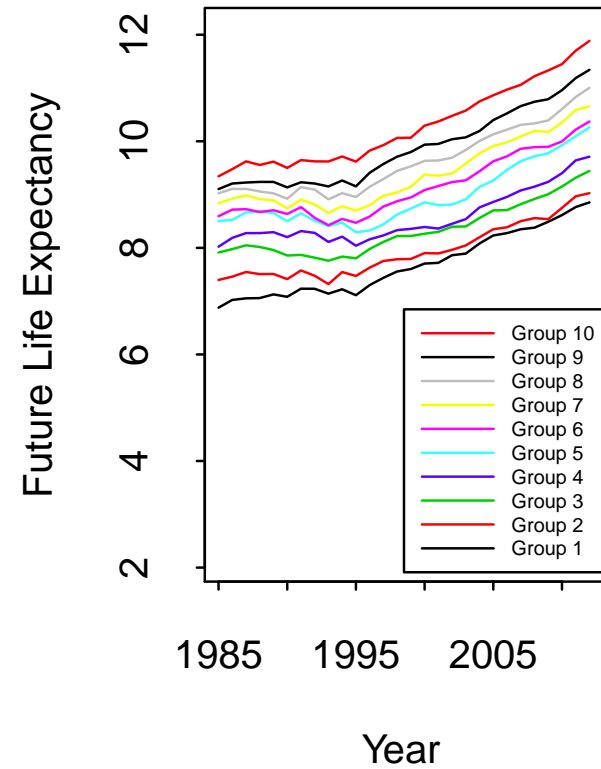
**Males Period EL:  
Age 55**



**Males Period EL:  
Age 65**



**Males Period EL:  
Age 75**

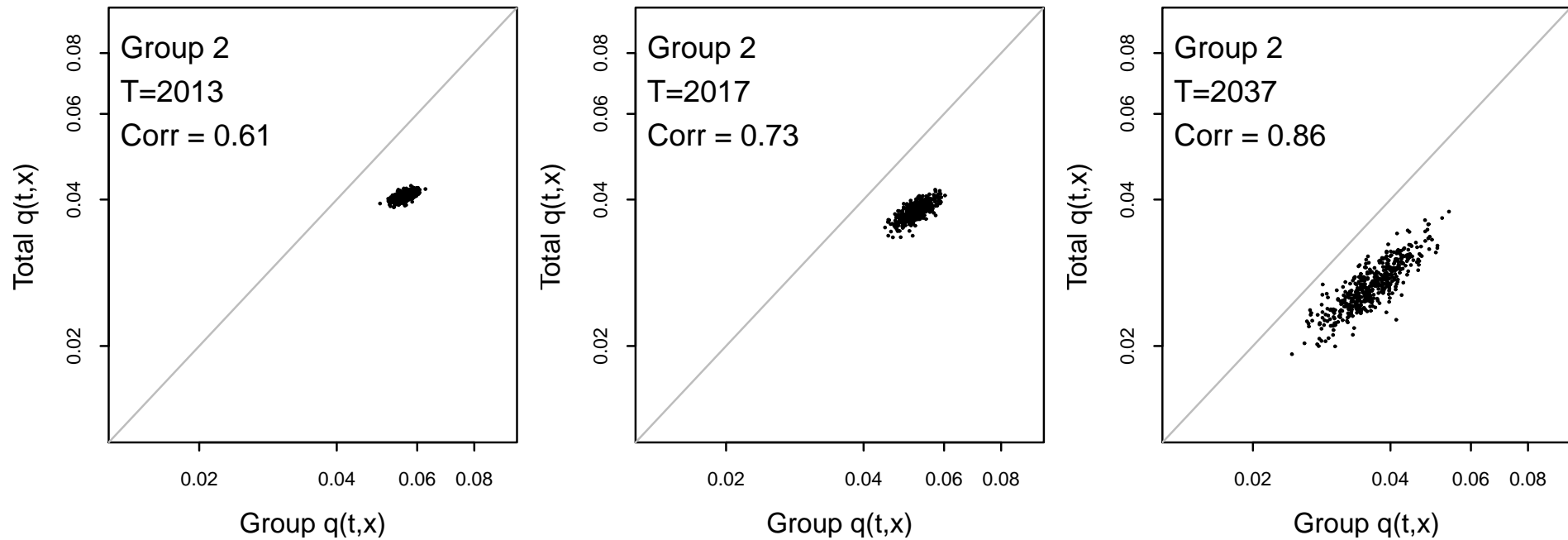


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## Time series modelling

- $t \rightarrow t + 1$ : Allow for correlation
  - between  $\kappa_1^{(k)}(t + 1)$  and  $\kappa_2^{(k)}(t + 1)$
  - between groups  $k = 1, \dots, 10$
- Medium/long term:
  - group specific period effects gravitate towards the national trend*
  - ⇒ Bio-demographical reasonableness:
    - groups should not diverge

# Simulated Group versus Population Mortality, $q(t, x)$



As  $T$  increases: +1 year; +5 years; +25 years

- Scatterplots become more dispersed
- Shift down and to the left
- Correlation increases

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## Forecast Correlations

Deciles are quite narrow subgroups

More diversified e.g.

- **Blue collar pension plan**

⇒ equal proportions of groups 2, 3, 4

- White collar pension plan

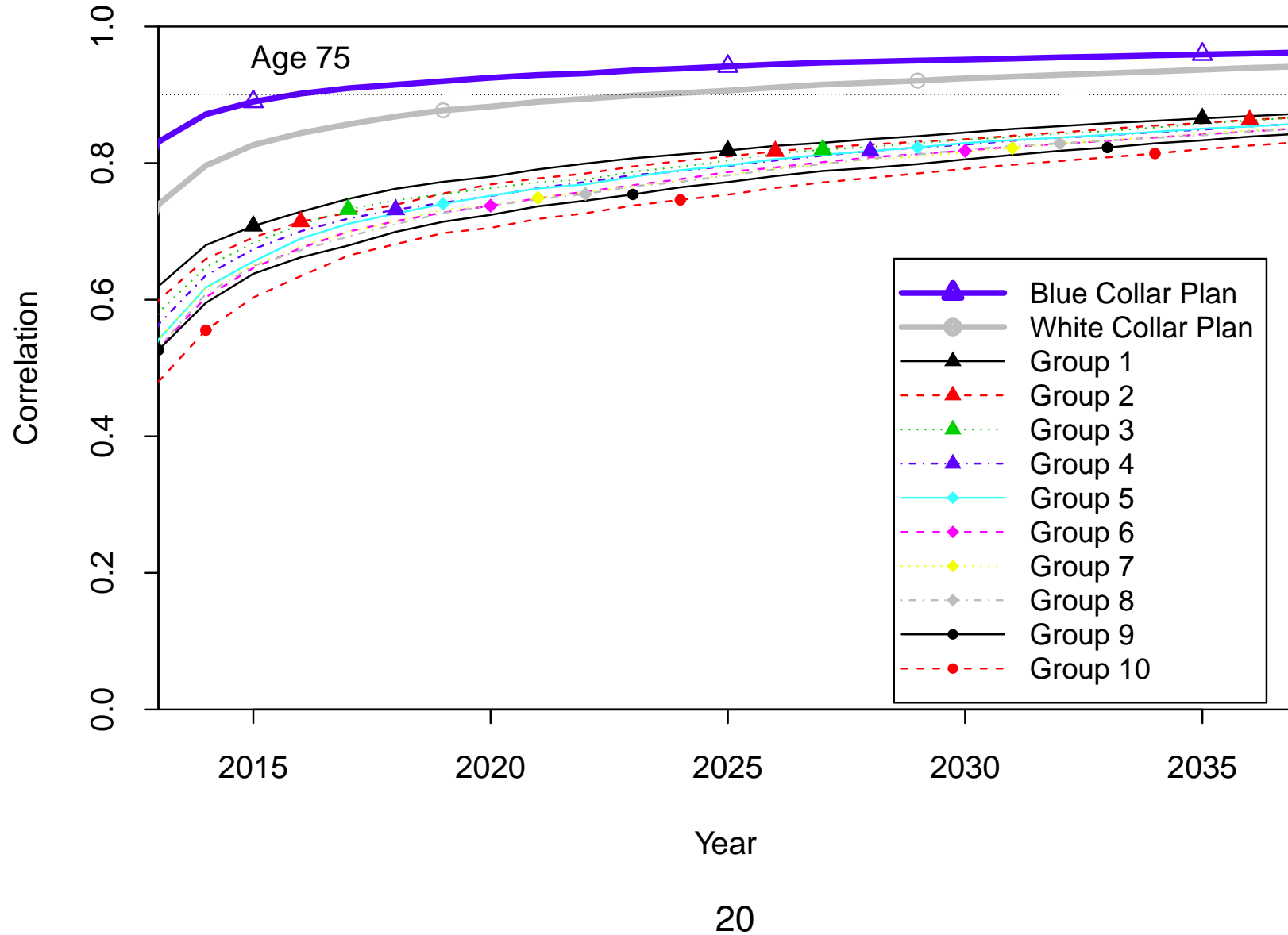
⇒ equal proportions of groups 8, 9, 10

- **Mixed plan**

⇒ proportions  $(0, 0, 1, 2, \dots, 7, 8)/36$  (e.g. amounts)

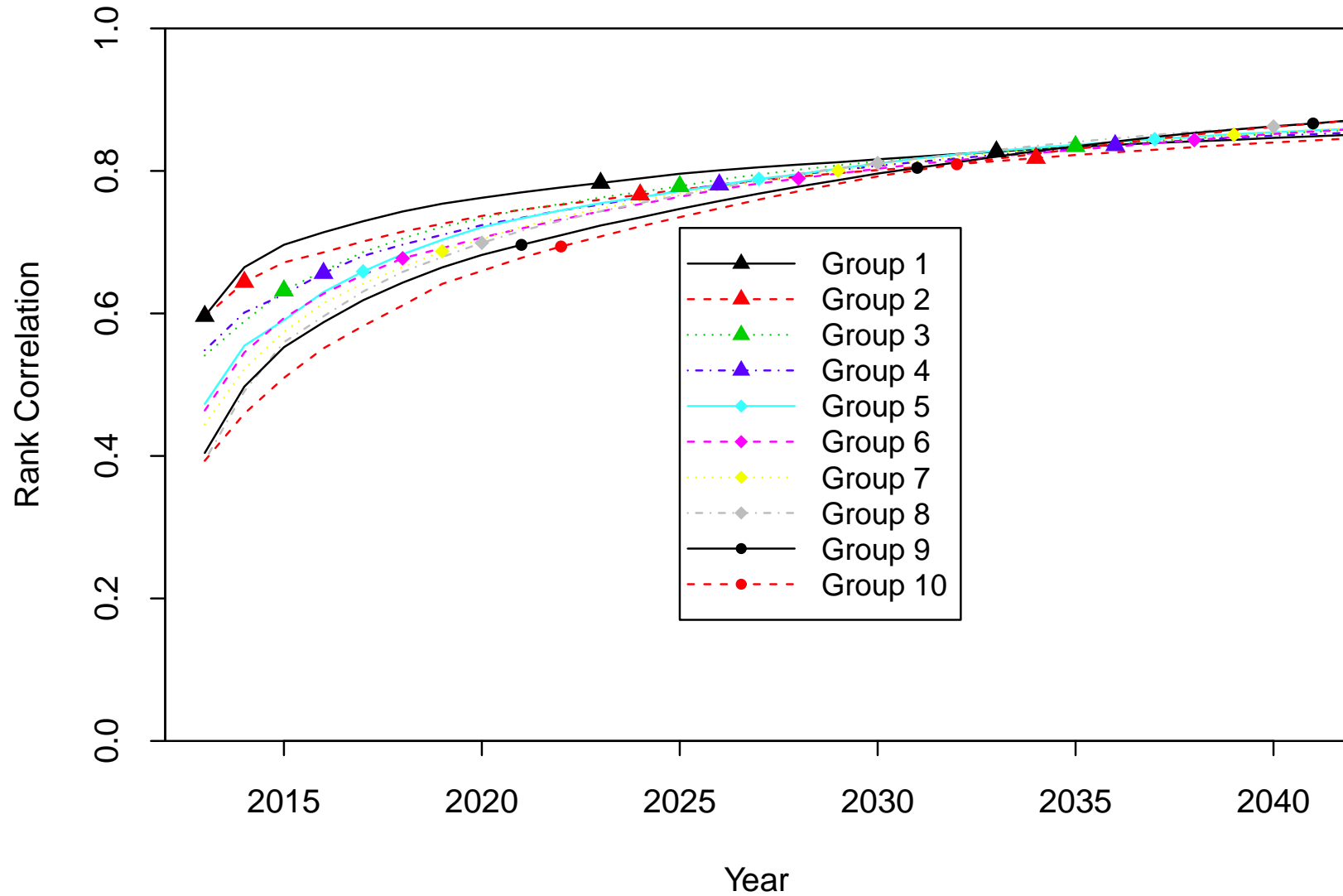
# Forecast Correlations: Mortality Rates at Age 75

## Correlation Between Group $q(t,x)$ and Total $q(t,x)$



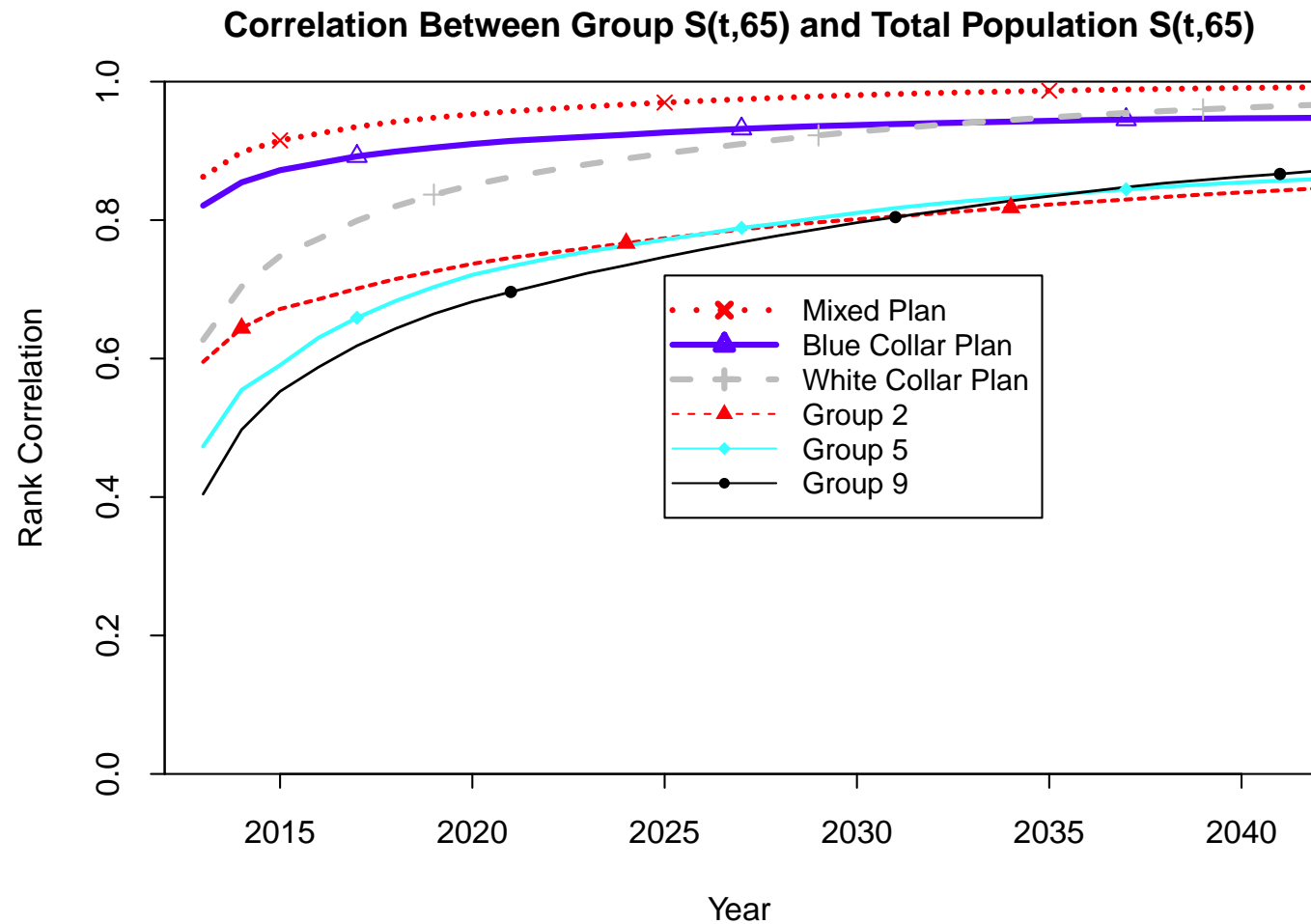
# Forecast Correlations: Cohort Survivorship from 65

Correlation Between Group  $S(t,65)$  and Total Population  $S(t,65)$



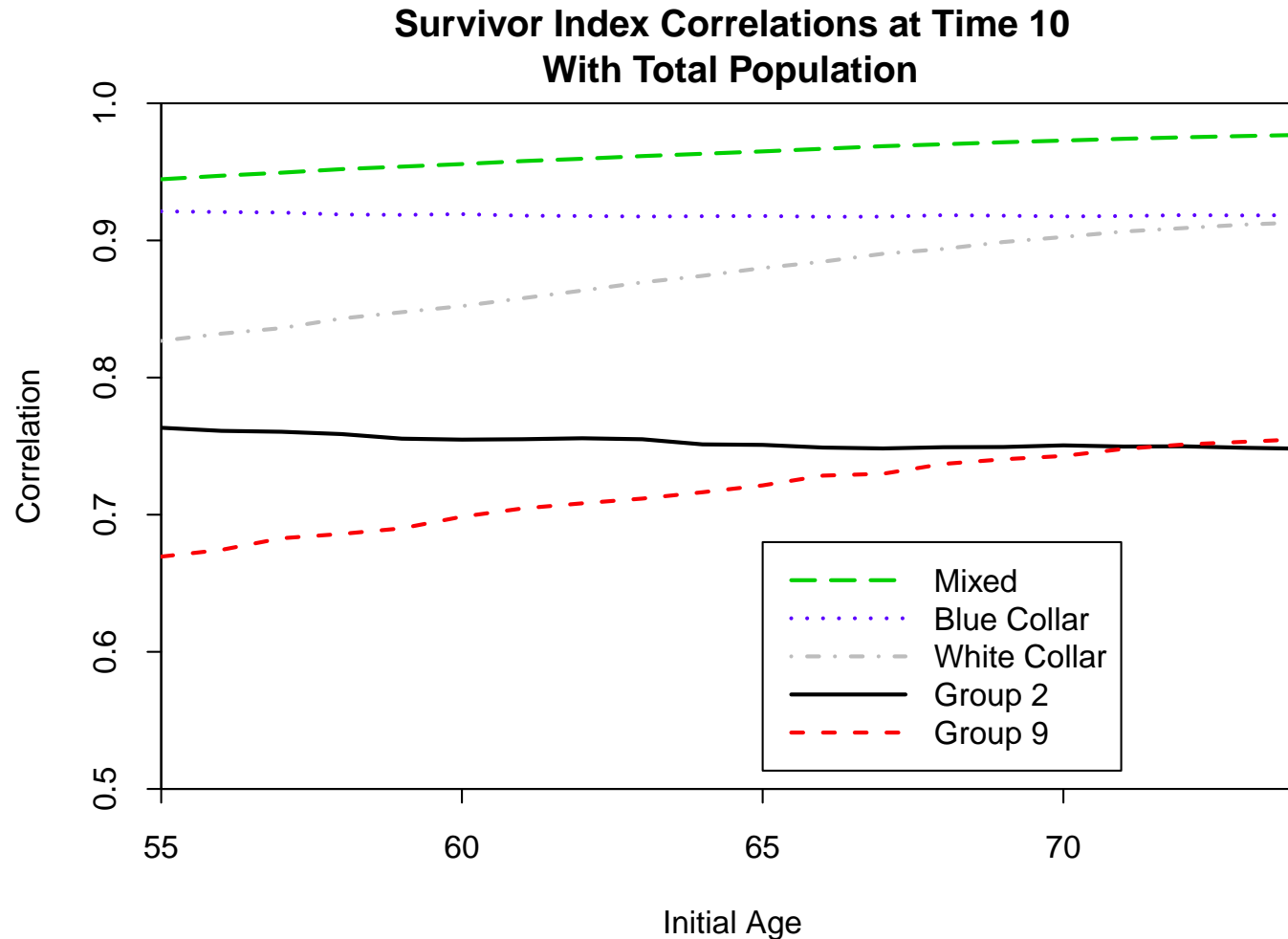
# Forecast Correlations: Cohort Survivorship from 65

How does correlation change with maturity?



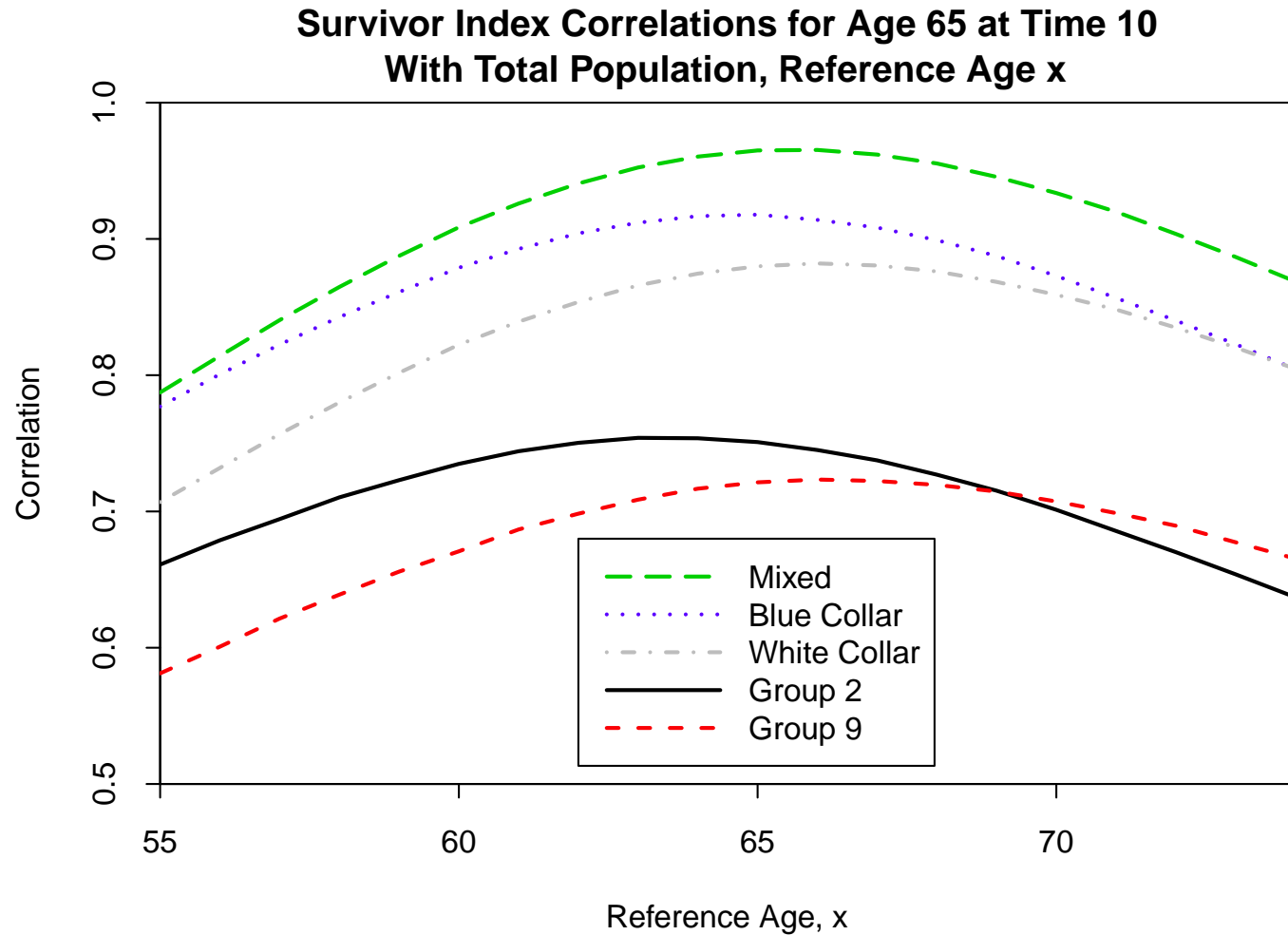
# Forecast Correlations: Cohort Survivorship

Correlations at different ages:  $cor(S_X(10, x), S_{TOT}(10, x))$



# Forecast Correlations: Cohort Survivorship

Different reference ages:  $cor(S_X(10, 65), S_{TOT}(10, x))$



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## Conclusions

- Development of a new multi-population dataset for Denmark  
*strong bio-demographically reasonable group rankings based on a new measure of affluence*
- Unlike multi-country data  
*a priori* ranking of affluence-related groups
- Proposal for a simple new multi-population model
- Mortality rates converge at high ages
- Strong correlations over medium to long term
- Correlations depend strongly on diversity of sub-population

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# Bonus Slides

## A specific model

$$\begin{aligned} \kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) && \text{(random walk)} \\ &\quad - \psi \left( \kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right) && \text{(gravity between groups)} \end{aligned}$$

$$\begin{aligned} \kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) \\ &\quad - \psi \left( \kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right) \end{aligned}$$

where

$$\bar{\kappa}_1(t) = \frac{1}{n} \sum_{i=1}^n \kappa_1^{(i)}(t) \quad \text{and} \quad \bar{\kappa}_2(t) = \frac{1}{n} \sum_{i=1}^n \kappa_2^{(i)}(t)$$

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## A specific model

$$\kappa_1^{(i)}(t) = \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi \left( \kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right)$$

$$\kappa_2^{(i)}(t) = \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi \left( \kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right)$$

Model structure  $\Rightarrow$

- $(\bar{\kappa}_1(t), \bar{\kappa}_2(t)) \sim$  bivariate random walk
- Each  $\kappa_1^{(i)}(t) - \bar{\kappa}_1(t) \sim AR(1)$  reverting to 0
- Each  $\kappa_2^{(i)}(t) - \bar{\kappa}_2(t) \sim AR(1)$  reverting to 0
- $\beta^{(i)}(x)$  vs  $\beta^{(j)}(x) \Rightarrow$  intrinsic group differences

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## Non-trivial correlation structure: between different ages and groups

$$\begin{aligned}\kappa_1^{(i)}(t) &= \kappa_1^{(i)}(t-1) + \mu_1 + Z_{1i}(t) - \psi \left( \kappa_1^{(i)}(t-1) - \bar{\kappa}_1(t-1) \right) \\ \kappa_2^{(i)}(t) &= \kappa_2^{(i)}(t-1) + \mu_2 + Z_{2i}(t) - \psi \left( \kappa_2^{(i)}(t-1) - \bar{\kappa}_2(t-1) \right)\end{aligned}$$

The  $Z_{ki}$  are multivariate normal, mean 0 and

$$\text{Cov}(Z_{ki}, Z_{lj}) = \begin{cases} v_{kl} & \text{for } i = j \\ \rho v_{kl} & \text{for } i \neq j \end{cases}$$

$\rho$  = cond. correlation between  $\kappa_1^{(i)}(t)$  and  $\kappa_1^{(j)}(t)$  etc.

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## Comments

- Model is very simple
  - One gravity parameter,  $0 < \psi < 1$
  - One between-group correlation parameter,  
 $0 < \rho < 1$
- Many generalisations are possible
- But more parameters + more complex computing
- This simple model seems to fit quite well.
- Nevertheless  $\Rightarrow$  work in progress

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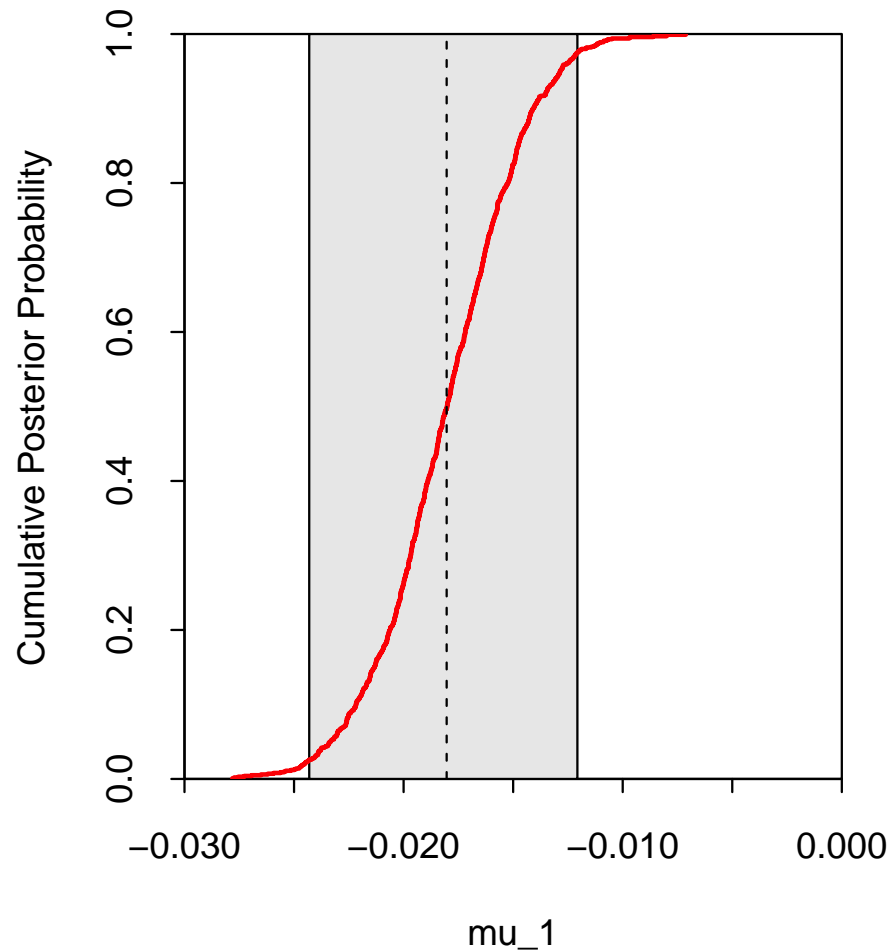
## Prior distributions

- As uninformative as possible
- $\mu_1, \mu_2 \sim$  improper uniform prior
- $\{v_{ij}\} \sim$  Inverse Wishart
- $\rho \sim \text{Beta}(2, 2)$
- $\psi \sim \text{Beta}(2, 2)$

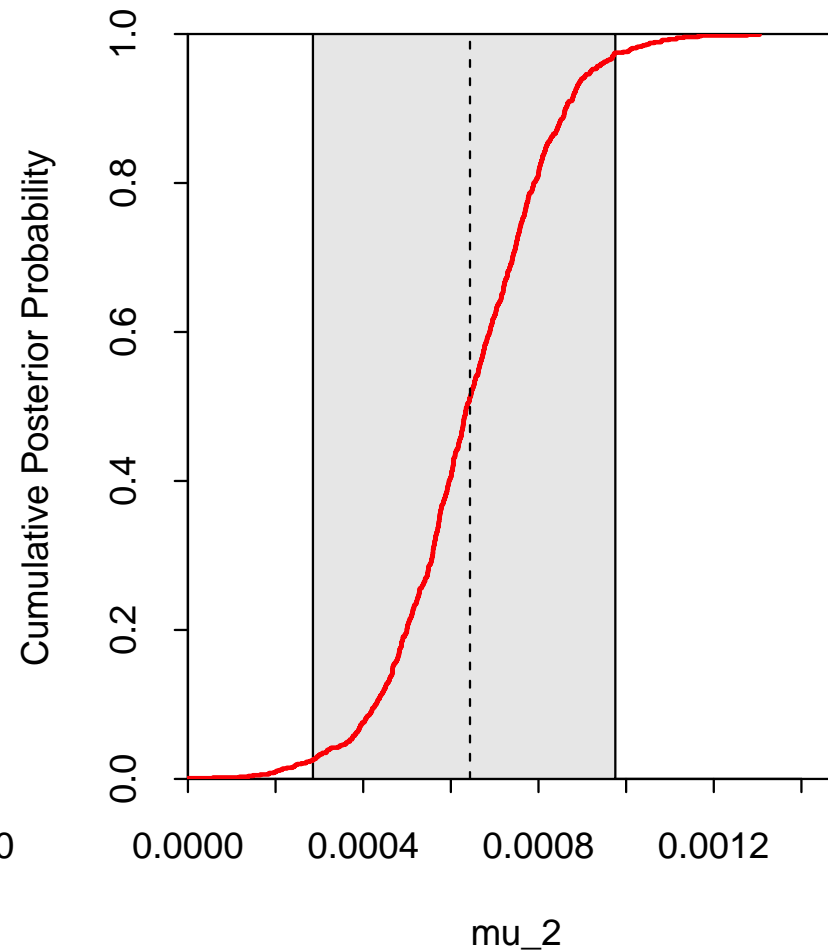
State variables and process parameters estimated using  
MCMC (Gibbs + Metropolis-Hastings)

# Posterior Distributions and 95% Credibility Intervals

Kappa\_1 Drift,  $\mu_1$



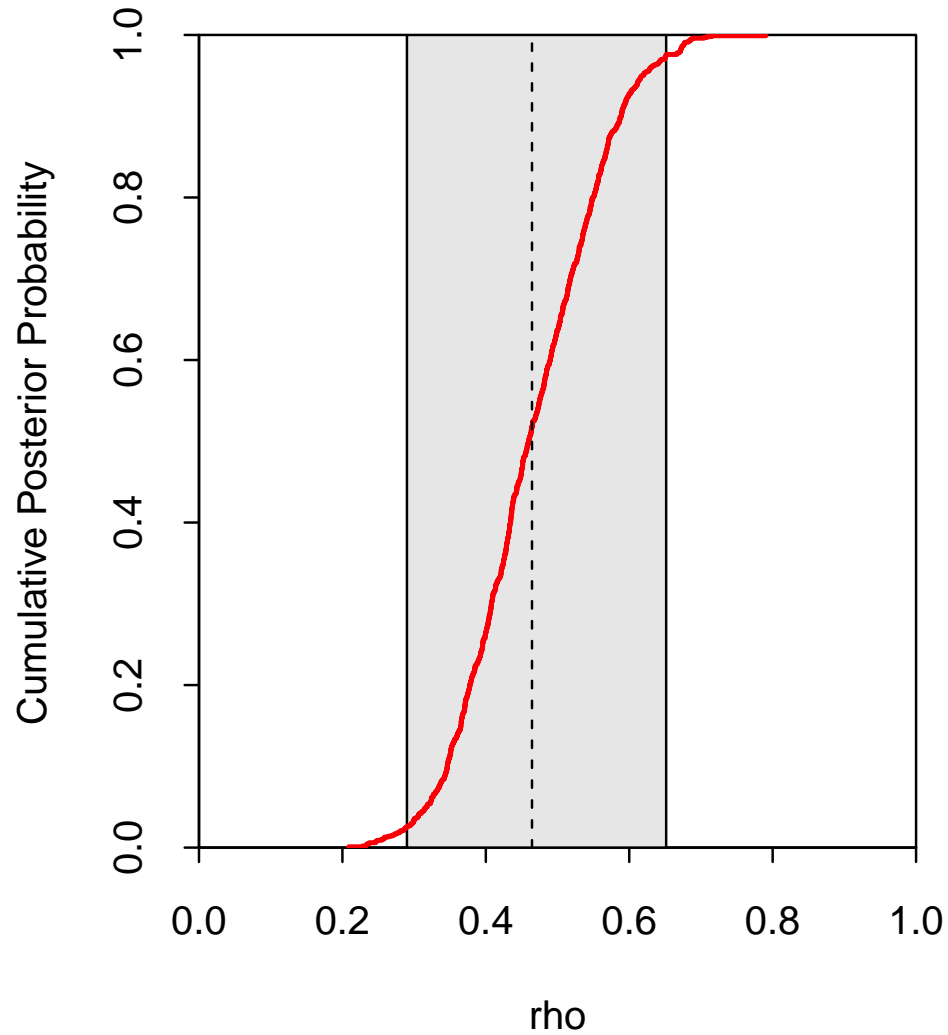
Kappa\_2 Drift,  $\mu_2$



Note:  $-\mu_1 =$  global improvement rate

# Posterior Distributions and 95% Credibility Intervals

Between Group Correlation, rho



Gravity Parameter, psi

