# The Marketability Discount of Controlling Blocks of Shares*

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**Abstract**

An inherent difficulty in estimating the marketability discount on large blocks of shares is the illiquidity of the market for corporate control. To date, there are no estimates of the marketability discount that explicitly take into account this feature of the market. We use the results from our previous work (Albuquerque and Schroth, 2013) to evaluate the relevance of various determinants of the marketability discount and to estimate the marketability discount under different firm, industry and macroeconomic scenarios for both firms that are publicly traded and firms that are privately held.

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# 1. Introduction

To value a controlling block of shares on the equity of a corporation requires measurement of three main components.[1] First, a controlling block holder may add value to the corporation through effective monitoring or management skills. We call these shared security benefits, because they accrue simultaneously to all the shareholders of the corporation. Second, a controlling shareholder may extract private benefits of control. We think of these private benefits primarily as the social status and social network access for individual block holders, or as the economic synergies for corporate block holders that come with owning a large equity stake on a corporation. Shared and private benefits give rise to what is called a control premium. Third, a controlling shareholder faces an illiquidity cost associated with the difficulty of selling a large equity stake. Such large stakes are traded in over-the-counter markets and often at a discount with respect to prices at which dispersed shareholders trade shares of the same company in more liquid, public stock markets. These illiquidity costs give rise to what is called a marketability discount. In this article, we study the magnitude and the determinants of the marketability discount. The main goal is to propose a simple method to estimate the marketability discount to be applied to the valuation of controlling blocks.

To illustrate consider the case of Panasonic's acquisition of 70% of Sanyo's equity, on a fully diluted basis, from Mitsui-Sumitomo Bank, Daiwa SMBC and Goldman Sachs (see Kruse and Suzuki, 2009, for details). On November 7th, 2008, Panasonic announced that it was in talks to acquire a majority stake in Sanyo. On that day, Sanyo's share price jumped from 145 yen to 204 yen (the price had been 114 yen on October 27th, 2008). Later, on November 24th, 2008, Panasonic made an offer of 120 yen per share. The three banks rejected the offer, stating that the price was too low. Goldman Sachs added that the price per share should be at least 200 yen, in order to reflect the control premium. Panasonic raised its bid price on

[1] Our definition of a controlling block of shares is 35% or more of the equity of a corporation.

December 3rd, 2008, to 130 yen per share, and the two Japanese banks accepted the offer. Goldman Sachs still rejected it, and suggested it may purchase the Japanese banks' stakes. But on December 16th, 2008, and following the demise of Lehman Brothers, Goldman Sachs announced its first-ever quarterly loss since going public in 1999. Further, on December 19th, 2008, Goldman Sachs accepted a price of 131 yen, i.e., a discount of almost 10% with respect to the pre-announcement price, while Sanyo's shares in the market dropped to 136 yen.

This example illustrates several important features about illiquidity costs: (i) that finding a buyer that can increase shareholder value following the take over is difficult; (ii) that unexpected events may occur, forcing a block holder to sell immediately, even when the block holder is a large financial institution; and, (iii) that the block may have to be sold at a discount relative to the share price in the public market following such events. To the best of our knowledge, there are no estimates of the marketability discount that consider these illiquidity costs. This glaring lack of estimates is surprising for two reasons. First, the need to price these dimensions of illiquidity is not new. In the famous case of Mandelbaum et al. v. Commissioner of Internal Revenue (1995), the court points to the limited evidence on the proper size of the discount on the value of large blocks relative to the value of exchange traded shares. Second, the predominance of high ownership concentration as a form of corporate governance is by now well established. High ownership concentration is a pervasive phenomenon in public corporations in many countries, including the United States;[2] it is also, by definition, an integral part of privately held corporations.

In this paper, we start by reviewing the estimation results in our previous work, Albuquerque and Schroth (2013, henceforth AS). AS provide a model of the

[2] Holderness (2009) provides evidence that 96% of U.S. public firms have block holders and that these block holders own in aggregate an average of 39% of the common stock. Using a sample of large U.S. corporations, Dlugosz et al. (2006) find that 75% of all firm-year observations have block holders that own at least 10% of the firms' equity. See Morck (2007) for evidence outside the United States.

value of a controlling block of shares and obtain estimates of its various components, shared security benefits, private benefits of control and marketability discount for a cross-section of block trades between 1990 and 2010. In this article, we take these estimates and identify the main cross-sectional and time-series determinants of the marketability discount for publicly traded firms. We show how to apply our results to estimate the adjustment to the value of a controlling block that is due to the marketability discount for both publicly traded firms and privately held firms as in the Mandelbaum valuation case.[3]

## 2. Literature Review

### 2.1 Measures of the value of control

There is a vast literature that tries to measure the value of control. One approach is to use the voting premium, measured directly as the price difference between shares with different voting rights (e.g., Masulis et al., 2009), or indirectly using deviations from put-call parity (Kalay et al., 2011) and equity-loan values (Christoffersen et al., 2007). This approach features two main drawbacks. First, by studying prices *per share*, it measures the *marginal* value of control. Instead, we are interested in the *total* value of control, which is the aim of any valuation analysis of controlling blocks. Second, these measures rely on prices of traded, liquid securities and, therefore, cannot be used to estimate the marketability discount. Moreover, these measures only capture the net value of control, i.e., the difference between the benefits and the costs of control, without isolating each component. The method we describe below uses a direct measure of the liquidity costs arising from the lack of

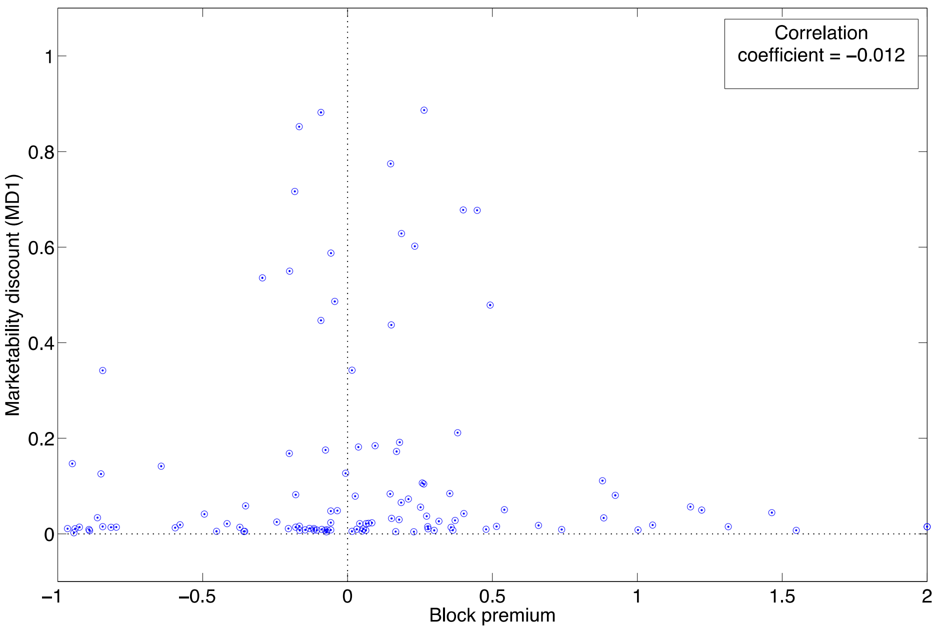
[3] We believe that our work carries important implications for pricing, for tax purposes, for asset sales, etc., but we also acknowledge that it is of limited use for financial reporting. FAS 157 and IFRS 13 prohibit block discounts in financial reporting when individual shares of the company are traded in an active market. This is done for the benefit of comparable reporting and any discount that may occur if and when a block is traded is to be treated as a transaction cost.

marketability of the block, which are of first order importance for companies with controlling blocks.

Another strand of the literature, which started with Barclay and Holderness (1989), measures the value of control via the block price premium, i.e., the difference between the negotiated price per share in the block and the dispersed shares price. As we argue in our prior work (Albuquerque and Schroth, 2010), the block premium summarizes different dimensions of benefits and costs of control. Therefore, while informative, the block premium is neither a direct measure of the benefits nor of the costs of control. Not surprisingly, we find a very low unconditional correlation between the block premium and the marketability discount measure we use here (see Figure 1). Moreover, the economy-wide, and firm- and industry-specific determinants of the marketability discount we uncover here cannot explain variation in the block premium.[4]

[4] The results are omitted here, but available upon request.

**Figure 1: Scatter plot of the block premium against the Marketability Discount measure in Albuquerque and Schroth (2013)**




## 2.2 Other valuation approaches to illiquid assets

There is a related literature that studies the pricing of illiquid assets (see Longstaff, 1995, Abudy and Benninga, 2011, and the surveys by Amihud et al., 2005, and Damadoran, 2005). This literature measures the marketability discount associated with stocks with trading restrictions such as the vesting of stock grants. Our analysis differs from the one in these papers in two critical ways. First, it considers a different set of illiquidity costs, which are more in line with the costs faced by controlling shareholders, as opposed to trading restrictions. Second, we study the pricing of large blocks of shares, which differs from the pricing of a single share because of the control premium, among other things.

## 3. The Albuquerque and Schroth (2013) Model

In this section, we review the AS study so as to provide guidance for how we obtained the estimates of the marketability discount that we then use in the empirical exercise in Section 4.

### 3.1 Illiquidity costs

The block valuation model in AS studies the pricing of blocks by combining the three components of block value mentioned in the introduction. First, by assuming that a controlling block holder of a public corporation affects firm value (e.g., Holderness and Sheehan, 1988; Barclay and Holderness, 1989; and Pérez-González, 2004), the model naturally predicts that a block holder that is not pressed to sell will only do so if a potential buyer arrives that can increase firm value further. Second, AS assume that the controlling block holder is forced to sell if hit by a liquidity shock. In that case, the block holder may sell to a buyer that generates a lower firm value and thus be paid a “fire sale” price. The potential absence of a value-increasing buyer and the likelihood of liquidity shocks leading up to a potential fire sale give rise to a marketability discount on the value of the block.[5] Third, the model allows for pecuniary and non-pecuniary private benefits of control, as long as these are not diverted from the firm’s cash flow.[6] These are assumed to be positively correlated with the asset value generated by the controlling shareholder and do not lead to an additional trading motive.

[5] In real life settings, not all liquidity shocks lead to fire sales since the block holder may have a reservation value that arises from actions taken to avoid a fire sale, for example, using the block as collateral for a loan or finding a white knight. AS show that these considerations result in estimates of the marketability discount that are a lower bound to the true values. Hence, our approach, gives a conservative estimate of these discounts.

[6] Private benefits derived by ‘tunneling’ the firm’s cash flow have distortionary effects on value that are not covered by AS. However, Albuquerque and Schroth (2010) show that, due to the incentive alignment effect of large stakes, this form of private benefits extraction is unlikely to have any effects on the value of blocks consisting of more than 35% of shares.

### 3.2 The marketability discount

We define the marketability discount on the controlling block as the ratio of the price inclusive of the illiquidity costs to the counterfactual and unobserved price that would result in the absence of the illiquidity costs. The well-known difficulty with the estimation of the marketability discount is the need to quantify the price that results in the absence of the illiquidity costs. Because this price is unobservable, the marketability discount cannot be directly measured from observable quantities. To deal with this difficulty, AS perform a "structural" estimation of their model. This method uses their model of block valuation and data on block trades to infer the parameters associated with the illiquidity costs: the probability that a value-increasing buyer arrives; the probability of a liquidity shock; and the fire sale price if a liquidity shock hits the block holder. AS show how to estimate the model's parameters, and hence the marketability discount, from data on the valuations of the two different types of shareholders at the time of a block trade: the block holders' valuation, which is imputed into the negotiated block price in the over-the-counter transaction, and the dispersed shareholders' valuation, which determines the exchange share price.

### 3.3 Discussion

To offer some intuition on the workings of the model, consider what happens with or without a liquidity shock. Following a liquidity shock, AS assume that the block is sold at a fire sale price equal to a fraction of the buyer's valuation. In contrast, the outside shareholders do not care about the fire sale discount but rather about the discounted value of future cash flows under the new block. This price difference allows us to quantify fire sale prices.

In the absence of a liquidity shock, the block changes hands only if a value-increasing buyer arrives. In this case, block and share prices differ partly because liquidity shocks penalize block holders more than outside shareholders who are unaffected by the lower expected fire sale price in a future sale. This price difference allows us to quantify the probability of liquidity shocks.

## 4. Data

Data for the marketability discount is from AS. Data on the determinants of the marketability discount come from three sources: the Board of Governors of the U.S. Federal Reserve for characteristics of the aggregate economy, and CRSP and Compustat for industry and firm-level data.[7]

### 4.1 Data on marketability discounts

AS estimate the marketability discount using data on block trades (114 U.S. disclosed-value trades of blocks consisting of at least 35% but not more than 90% of the stock of a company) from Thomson One Banker Acquisitions. From their analysis, we construct two measures of the marketability discount, *MD1* and *MD2*. *MD2* is the marketability discount in percentage terms that can be applied to the price of the shares traded in an active market. *MD1* is the marketability discount in percentage terms that can be applied to the firm value derived from a standard discounted cash flow analysis, or any similar valuation method, which ignores the illiquidity of the controlling block. *MD1* is the appropriate discount in the case of shares of privately held corporations where a share price is unobservable.[8] *MD1* and *MD2* have both a time stamp and a firm identifier. The time stamp on *MD1* and *MD2*

[7] These data are available upon request.

[8] Recall that the value of a controlling block of shares on the equity of a corporation requires measurement of three main components: shared security benefits and private benefits, and the marketability discount. Note that *MD1* and *MD2* are applied to firm values that already account for the shared security benefits provided there is no change in block holder. Therefore, to determine the final value of a block, that valuation must be increased by the private benefits. We refer the interested reader to AS for this additional step.

is the date of the block trade that was used to produce the estimates. The firm identifier in *MD1* and *MD2* refers to the target firm whose shares were traded.

The top panel of Figure 2 contains a scatter plot of *MD1* and *MD2* over time. From the figure, we note two main patterns. First, there seem to be periods of systematically high values of the marketability discount. These periods may be captured by economy-wide variables that characterize periods where liquidity dries up. In fact, in the bottom panel of Figure 2, we plot the yield curve slope, measured by the difference in the 10-year rate and the 3-month Treasury bills rate, over the same time period. The periods of high marketability discount appear to correlate well with the periods where the term structure of interest rates had a negative or low slope. Below, we show that this is true for other economy-wide variables as well. Second, there appears to be a role for industry and firm-level variables as determinants of liquidity, since even during these periods there is considerable variation in *MD1* and *MD2*.

**Figure 2: Marketability discounts and Yield spreads over time**

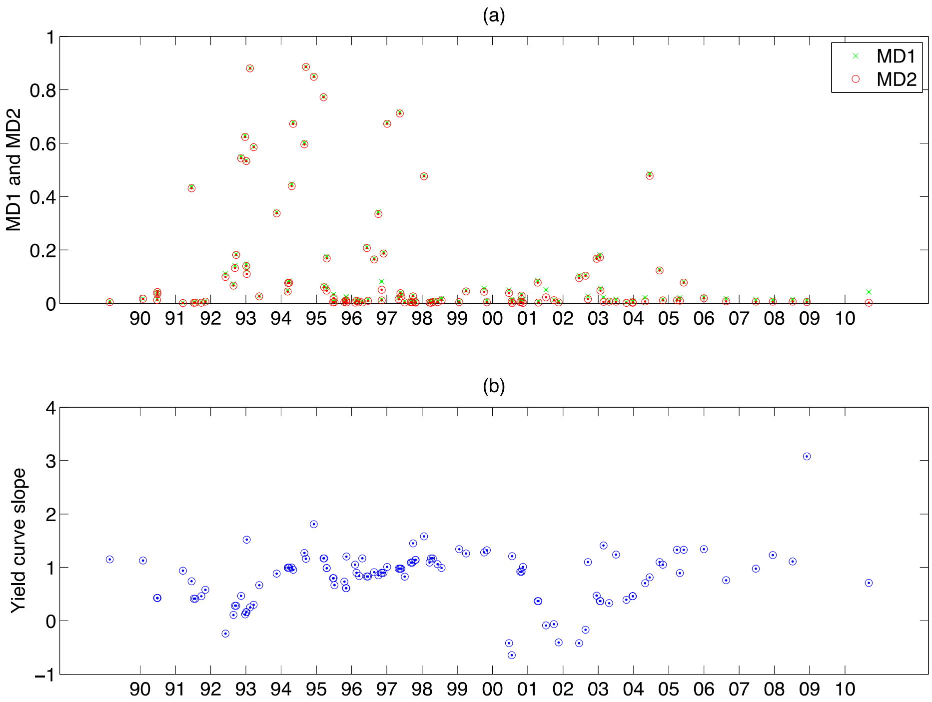
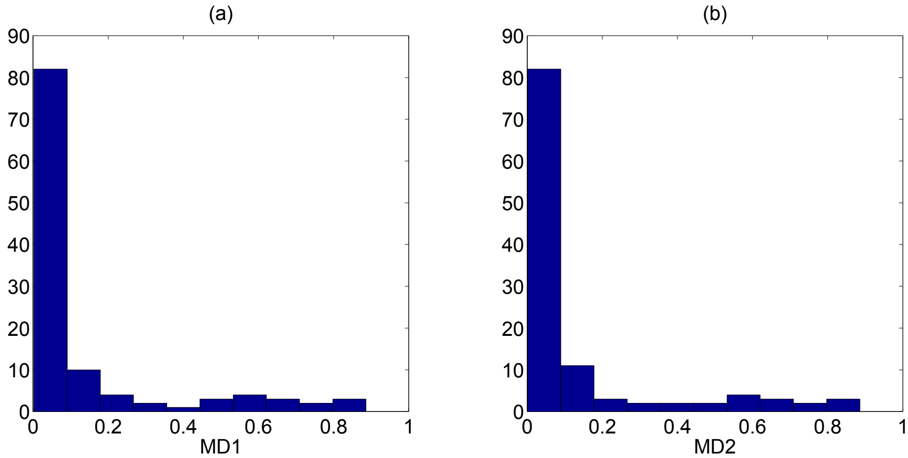



Figure 3 contains histograms of *MD1* and *MD2*. It is interesting to note that marketability discounts are low for a very large fraction of the observations. On average *MD1* is 13.15% and *MD2* is 12.50% and the respective minimum values are 0.2% and 0.1%.

Figure 3: Distribution of Marketability Discounts in Albuquerque and Schroth (2013)




### 4.2 Determinants of the marketability discount

We classify the determinants of the marketability discount into two groups: economy-wide determinants and industry and firm-level determinants. These variables are measured with a lag relative to respective marketability discounts. Table 1 describes these determinants in detail.

**Table 1: Description of determinants of marketability discount**

| Variable | Description |
|---|---|
| **Economy-wide variables** | |
| Market return | Annualized average daily returns on the equally-weighted portfolio of all NYSE, AMEX and NASDAQ stocks for the last month before the deal (%). |
| GDP growth | Average annual growth rate in US GDP per capita in the last quarter prior to the trade (%). |
| Market volatility | Standard deviation of the annualized daily returns on the equally-weighted portfolio of all NYSE, AMEX and NASDAQ stocks for the 12 month-period before the deal (%). |
| Yield curve slope | Difference between the interest rate on the 10 and the 3-month Treasury bill (%). |
| Fontaine-Garcia index | Fontaine and Garcia's (2011) monthly index of the value of funding liquidity: the higher the index, the lower the bond market liquidity. |
| **Industry- and firm-specific variables** | |
| Industry's M&A activity | Total M&A activity during the last quarter before the deal, where the target is in the same 2-digit SIC Code as the deal's target ($ Billions). |
| Target minus industry leverage | Difference between the proportion of total debt to total assets on the last fiscal year before the trade announcement between the target firm and the median if all firms in the same 3-digit SIC code. |
| Target volatility | Standard deviation of the target's annualized average daily return for the 12 month-period ending two months before the trade announcement (%). |
| Industry's asset specificity | Median proportion of machinery and equipment to total assets of all firms in the same 3-digit SIC code as the target firm, as defined by Stromberg (2001). |
| Industry's market-to-book ratio | Median ratio of the market value of the firm (book value of debt + market value of equity) to the book value of total assets of all firms in the same 3-digit SIC code as the target firm. |
| Block-to-Industry Size | Ratio of the total controlling block value to the total market capitalization of all NYSE and AMEX firms in the same 2-digit SIC Code as the target. |

We include characteristics of the aggregate economy that may produce shocks to the block holder's liquidity and thus changes in the marketability discount. We expect these shocks to occur in times of tighter aggregate funding liquidity. Our proxy for funding liquidity is the bond liquidity premium index in Fontaine and Garcia (2012) (*Fontaine-Garcia index*). Fontaine and Garcia (2012) identify a monthly liquidity factor from the yield spread between U.S. Treasury bills with the same cash flows but different ages. They interpret the higher yields on otherwise identical older Treasury bills as a premium on the liquidity of on-the-run bonds. We hypothesize that *Fontaine-Garcia index* is positively associated with the marketability discount.[9]

[9] We also considered as candidate proxies for funding liquidity the TED spread, i.e., the spread between the 3-month dollar LIBOR rate and the 3-month Treasury bill, and the Pástor and Stambaugh (2003) stock market liquidity factor. These variables produce similar results, which are available upon request.

We include also the growth of U.S. GDP per capita (*GDP growth*) and the average daily return on the equally-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks (*Market Return*). The inclusion of these business cycle proxies is meant to capture two opposing effects: during expansions, (i) investors have stronger balance sheets and are less likely to face liquidity shocks, and, (ii) better alternative investment opportunities may generate a demand for cash. We try to separate these opposing effects by interacting the business cycle variables with variables that describe aggregate funding costs. We argue that having a better alternative investment opportunity would only force the block holder to sell if at the same time the cost of borrowing is high. The proxy for the cost of funding used is the slope of the yield curve (*Yield curve slope*). We expect high *GDP growth* and high *Market return* to have a negative direct effect on the marketability discount, but a positive effect via their interaction with *Yield curve slope*.

We also include in the determinants of the marketability discount the standard deviation of the market return (*Market Volatility*). Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) show that liquidity providers face tighter funding constraints when market returns are low and volatility is high and thereby diminish their role as liquidity providers (see also Chordia et al., 2002). We therefore predict the marketability discount to decrease with *Market Return* and to increase with *Market Volatility*.

We consider several firm and industry determinants of the marketability discount. Williamson (1988) argues that asset liquidation values should be closely related to the asset's redeployability. Shleifer and Vishny (1992) add that, because distressed assets tend to be put to the best use by liquidating them within the same industry, redeployability is a function of the industry's capacity to absorb them. We adopt these ideas about the liquidity of physical assets to the financial asset under consideration. As a proxy for the "financial redeployability" of the block, we use the ratio of the block value to the total market capitalization of all firms in the same 2-

digit SIC group (*Block-to-Industry Size*) (see Gavazza, 2010). Based on this interpretation, we expect the marketability discount to increase with the relative size of the block. However, if block holders have a preference for relatively larger blocks in order to, say, exert control over industry policies, then the marketability discount should vary negatively with *Block-to-Industry Size*.

Another proxy for the redeployability of the block is obtained from the industry's asset specificity. We follow Stromberg (2001) and measure *Industry's asset specificity* with the median proportion of machinery and equipment to total assets of all firms in the industry (non-industry specific assets include land, commercial real estate and cash). As with *Block-to-Industry Size*, *Industry's asset specificity* measures the redeployability of the physical assets rather than of the controlling block. We view *Industry's asset specificity* as a proxy for the amount of industry-specific knowledge required by the controlling block holder and expect more potential buyers of controlling stakes in firms that use generic productive assets. Hence, we predict a positive association between *Industry's asset specificity* and the marketability discount.

We include the total dollar volume of M&A activity involving targets in the same 2-digit SIC group (*Industry's M&A activity*). High *Industry's M&A Activity* could be the result of an increased supply of industry-specific assets, which would increase the marketability discount. High *Industry's M&A Activity* could also reflect high liquidity for industry-specific assets as in Schlingemann et al. (2002) and Ortiz-Molina and Phillips (2011) and, therefore, decrease the marketability discount.

We let the marketability discount vary with the target's leverage relative to its industry's median leverage. We define *Target minus Industry Leverage* as the difference between the target's proportion of long-term debt to assets and the median proportion of long-term debt to assets of all firms in the same 3-digit SIC code. We expect that block holders' price in a larger marketability discount for firms with more long term debt as they are more constrained to borrow to fund any restructuring activities. To control for the time-series variation in investment opportunities in the same industry, we include the median ratio of the market-to-book value of assets of all firms in the same 3-digit SIC code. Finally, we control for the possibility that fire sale prices are affected by the target firm's return volatility.

# 5. A Simple Method to Value the Marketability Discount

## 5.1 Estimating elasticities

We regress the marketability discount on its determinants. We estimate two different econometric specifications, one that uses the logarithm, and another that uses the level, of the each of the discount measures, *MD1* and *MD2*. For statistical reasons, the logarithm specification is preferred since the marketability discount is truncated at zero and highly skewed.

For MD1, and for the logarithm specification, we obtain,

$$\ln \widehat{MD}1 = -\underset{(0.62)}{1.98} + \underset{(0.12)}{0.08} \times \mathit{Yield\ curve\ slope} + \underset{(0.19)}{0.51} \times \mathit{Fontaine\text{-}Garcia\ index} - \underset{(1.48)}{7.39} \times \mathit{Market\ return} - \underset{(2.21)}{7.16} \times \mathit{Market\ volatility} - \underset{(8.06)}{17.93} \times \mathit{GDP\ growth} + \underset{(3.68)}{6.27} \times \mathit{Yield\ curve\ slope} \times \mathit{GDP\ growth} + \underset{(0.60)}{0.69} \times \mathit{Yield\ curve\ slope} \times \mathit{Market\ return} + \underset{(0.31)}{1.05} \times \mathit{Target\ minus\ industry\ leverage} + \underset{(0.47)}{0.94} \times \mathit{Industry's\ asset\ specificity} - \underset{(0.23)}{0.33} \times \mathit{Industry's\ market\text{-}to\text{-}book\ ratio} - \underset{(2.70)}{0.73} \times \mathit{Block\text{-}to\text{-}Industry\ size} - \underset{(0.03)}{0.12} \times \mathit{Industry's\ M\&A\ activity} - \underset{(0.21)}{0.46} \times \mathit{Target\ volatility}\ ; R^2 = 0.74.$$

For MD2, and again for the logarithm specification, we obtain very similar estimates, both qualitatively and quantitatively:

$$\ln \widehat{MD}2 = -\underset{(0.72)}{1.95} + \underset{(0.14)}{0.16} \times Yield\ curve\ slope + \underset{(0.23)}{0.62} \times Fontaine\text{-}Garcia\ index - \underset{(1.73)}{6.89} \times Market\ return - \underset{(2.58)}{8.59} \times Market\ volatility - \underset{(9.42)}{19.95} \times GDP\ growth + \underset{(4.30)}{8.10} \times Yield\ curve\ slope \times GDP\ growth + \underset{(0.70)}{0.28} \times Yield\ curve\ slope \times Market\ return + \underset{(0.36)}{1.21} \times Target\ minus\ industry\ leverage + \underset{(0.55)}{1.20} \times Industry's\ asset\ specificity - \underset{(0.27)}{0.60} \times Industry's\ market\text{-}to\text{-}book\ ratio - \underset{(3.16)}{1.25} \times Block\text{-}to\text{-}Industry\ size - \underset{(0.03)}{0.18} \times Industry's\ M\&A\ activity - \underset{(0.25)}{0.63} \times Target\ volatility\ ; R^2 = 0.76.$$

Associated with each variable, there is the estimated coefficient and underneath it, in parenthesis, the standard deviation of the estimate. The high $R^2$ in both regressions indicate that the specifications explain well the marketability discount. The $R^2$ in the regressions that use the level of the discounts as the left hand side variable are in the range of 60%.

In Table 2, we convert the estimated coefficients from these regressions into elasticities that give the percentage change in the marketability discount induced by one percentage change in each of the liquidity determinants, keeping all others constant. By and large, the two specifications produce very similar results, with the logarithmic specification occasionally yielding slightly more statistically significant estimates.

**Table 2: The elasticities of the marketability discount with respect to its determinants**

| Econometric Specification | Log on levels | | Levels on levels | |
|---|---|---|---|---|
| | Marketability discount 1 | Marketability discount 2 | Marketability discount 1 | Marketability discount 2 |
| **Economy-wide variables** | | | | |
| Market return | -0.792***<br>(0.101) | -0.817***<br>(0.118) | -0.563***<br>(0.136) | -0.578***<br>(0.144) |
| GDP growth | -0.921***<br>(0.115) | -0.989***<br>(0.135) | -0.811***<br>(0.156) | -0.842***<br>(0.165) |
| Market volatility | -1.004***<br>(0.309) | -1.205***<br>(0.362) | -0.692<br>(0.418) | -0.425<br>(0.262) |
| Yield curve slope | 0.621***<br>(0.161) | 0.768***<br>(0.188) | 0.799***<br>(0.217) | 0.847***<br>(0.229) |
| Fontaine-Garcia index | 0.412***<br>(0.156) | 0.499***<br>(0.182) | 0.665***<br>(0.211) | 0.422***<br>(0.132) |
| **Industry- and firm-specific variables** | | | | |
| Industry's M&A activity | -0.477***<br>(0.106) | -0.702***<br>(0.124) | -0.445***<br>(0.143) | -0.478***<br>(0.152) |
| Target minus industry leverage | 0.047***<br>(0.014) | 0.054***<br>(0.016) | 0.039**<br>(0.019) | 0.040**<br>(0.020) |
| Target volatility | -0.182**<br>(0.084) | -0.253**<br>(0.098) | -0.282**<br>(0.114) | -0.303**<br>(0.120) |
| Industry asset specificity | 0.254**<br>(0.126) | 0.327**<br>(0.148) | 0.273***<br>(0.087) | 0.291***<br>(0.097) |
| Industry market-to-book ratio | -0.413<br>(0.290) | -0.738**<br>(0.339) | -0.45<br>(0.392) | -0.494<br>(0.414) |
| Block-to-Industry Size | -0.006<br>(0.021) | -0.01<br>(0.025) | 0.025<br>(0.029) | 0.027<br>(0.031) |

This table reports the elasticities of the marketability and control discounts of majority controlling blocks with respect to their determinants. Their standard errors are shown underneath, in brackets. Estimates followed by ***, **, and * are statistically significant to the 0.01, 0.05, and 0.1 respectively. The variables are defined in Table 1. The elasticities are derived from the regression of the logarithms or the levels of the marketability discounts predicted by the search model in Albuquerque and Schroth (2013) on the determinants shown above, using a sample of 114 acquisitions of controlling blocks between 35% and 90% of the common stock, which occurred between January of 1990 and December of 2010.

To rank each determinant of liquidity by its explanatory power, we compute the partial correlation coefficient between the marketability discount and said determinant. The partial correlation, say between *MD1* and *GDP growth*, is calculated as the correlation between the residual of regressing *MD1* on all its determinants except for *GDP growth* and the residual of regressing *GDP growth* on all the liquidity determinants except itself. Table 3 presents the correlation coefficients between *MD1* and its determinants, as well as their squared values, which give the proportion of the residual variation in the marketability discount that is explained by each determinant. The determinants are ordered by their relevance. The results for *MD2* are almost identical and therefore omitted from the table.

Table 3 presents evidence consistent with the marketability discount being most highly correlated with economy-wide or business cycle determinants of liquidity, *Market Return* and *GDP growth*. Periods of high *Market Return* or *GDP growth* are periods of increased liquidity and lower marketability discounts, as predicted. Quantitatively, we find that in an economic expansion, equivalent to a one standard deviation increase in *GDP growth* relative to its mean, the average *MD1* is 1.5%, whereas in an economic recession, equivalent to a one standard deviation decrease in *GDP growth* relative to its mean, the average *MD1* is 24.8%. The numbers for the average *MD2* are 0.6% and 24.4%, respectively.

Of the remaining economy-wide determinants of liquidity, the proxies for funding costs and funding liquidity, *Yield curve slope* and *Fontaine-Garcia index*, respectively, also appear quantitatively important, but their effect, as measured by the squared partial correlation, is less than one fourth of the effect of the business cycle variables.

Of the industry and firm-level determinants of liquidity, the most significant one is *Industry's M&A Activity*. Periods with high *Industry's M&A Activity* appear to be associated with increased demand for industry assets and low marketability discounts. The effect of *Target minus Industry Leverage* is positive as expected, though smaller with a squared partial correlation of 10.4%. All other industry and firm-level determinants have squared partial correlations that are at or below 5%. Surprisingly, our results suggest that the variables that proxy for the block's redeployability, *Industry's asset specificity* and *Block-to-Industry Size*, are marginally significant at best. This is not conclusive evidence that asset specificity is not relevant to determine liquidity in block trades partly because of the difficulty of finding good proxies for financial redeployability.

**Table 3: Explanatory power of the marketability discounts determinants**

| Determinants | Partial correlation coefficient | Squared partial correlation | Significance (p-value) | Discounts interval following a partial one sample standard devitation decrease/increase in each determinant | |
|---|---|---|---|---|---|
| | | | | Marketability discount 1 | Marketability discount 2 |
| **Economy-wide variables** | | | | | |
| Market return | -0.652 | 0.425 | 0.000 | [ 0.002 , 0.260 ] | [ 0.000 , 0.252 ] |
| GDP growth | -0.639 | 0.409 | 0.000 | [ 0.015 , 0.248 ] | [ 0.006 , 0.244 ] |
| Market volatility | -0.381 | 0.145 | 0.000 | [ 0.082 , 0.181 ] | [ 0.082 , 0.181 ] |
| Yield curve slope | 0.304 | 0.092 | 0.002 | [ 0.075 , 0.188 ] | [ 0.059 , 0.191 ] |
| Fontaine-Garcia index | 0.215 | 0.046 | 0.028 | [ 0.098 , 0.165 ] | [ 0.086 , 0.164 ] |
| **Industry- and firm-specific variables** | | | | | |
| Industry's M&A activity | -0.456 | 0.208 | 0.000 | [ 0.072 , 0.191 ] | [ 0.042 , 0.208 ] |
| Target minus industry leverage | 0.322 | 0.104 | 0.001 | [ 0.093 , 0.170 ] | [ 0.083 , 0.167 ] |
| Target volatility | -0.211 | 0.045 | 0.031 | [ 0.107 , 0.156 ] | [ 0.093 , 0.157 ] |
| Industry asset specificity | 0.204 | 0.041 | 0.038 | [ 0.108 , 0.155 ] | [ 0.097 , 0.154 ] |
| Industry market-to-book ratio | -0.073 | 0.005 | 0.462 | [ 0.111 , 0.152 ] | [ 0.090 , 0.160 ] |
| Block-to-Industry Size | -0.045 | 0.002 | 0.653 | [ 0.128 , 0.135 ] | [ 0.120 , 0.130 ] |

This table reports the correlation coefficient of the marketability discount of majority controlling blocks with each of its determinants after removing the effects of all other determinants. The variables are ranked according to the square of this partial correlation. The significance level (p-value) is for the test that each determinant's partial correlation with the marketability discount is zero. The table also reports the predicted marketability discounts on the controlling block following a decrease or an increase by one sample standard deviation in each determinant, keeping the others constant.

## 5.2 Predicting the marketability discount for privately held corporations: An application to the case of Mandelbaum et al v. Commissioner of Internal Revenue

Next, we use the elasticities in Table 2 to compute the change in the marketability discount given changes in its determinants. To illustrate, we use the actual values in Mandelbaum et al. v. Commissioner of Internal Revenue (1995).

In the Court's decision regarding Mandelbaum et al. v. Commissioner of Internal Revenue, the Judge applied a 30% discount with respect to the standard valuation, ruling against the plaintiff's expert claim to apply a 75% discount on the 'virtually illiquid' block of shares of Big M, Inc., a chain of department stores specialized in sporting goods located in northern New Jersey. The Judge argued that he had no other choice but to apply the average observed discount on stocks with trading restrictions, but suggested that this discount was not ideal because the illiquidity of blocks is not the same as trading restrictions on small amounts of shares, and because discounts should vary with deal and macroeconomic effects.

Table 4 shows how to use the information above to get an estimate for the marketability discount to apply in the case of Big M, Inc. stock. To conduct this exercise, we focus on the top three economy-wide determinants and the top three industry- and firm-level determinants of *MD1*. The top part of Table 4 documents the mean and standard deviation of these determinants in the AS sample, under the heading "Sample statistics," and the actual data from Big M, Inc., its industry (3-digit SIC Code 531, Department Stores) and the economy in December of 1990, under the heading "Big M, Inc." The last column in the top part of Table 4, shows the percentage difference between the values for Big M, Inc. and the sample means in AS.

The resulting marketability discount is shown at the bottom part of Table 4. It is calculated by adding to the mean discount (bottom-left) the sum of the changes induced by each variable, which are each estimated by multiplying the percentage change (column 4, Table 4) times their elasticity (columns 1 and 2, Table 2). Interestingly, the Judge's estimate of 30% is remarkably close to the average of the relevant discount (*MD1*) in the AS sample, i.e., 37.13%. In our results, this high discount is explained by the facts that the U.S. economy was growing at a pace below mean and the stock market had experienced low returns, coupled with low M&A activity in the industry. The ameliorating factor was that Big M, Inc. was significantly less levered than its industry peers as compared with the average target in the AS sample.

**Table 4: Estimating the block discount for Big M, Inc. in December 1990**

| Data | Sample statistics | | Big M, Inc. | |
|---|---|---|---|---|
| | Mean | Standard deviation | Values | % difference |
| **Economy-wide variables** | | | | |
| Market return | 12.74% | 15.78% | -1.98% | -115.52% |
| GDP growth | 3.23% | 3.11% | 0.56% | -82.80% |
| Market volatility | 14.03% | 5.25% | 15.96% | 13.73% |
| **Industry- and firm-specific variables** | | | | |
| Industry's M&A activity | 3.879 | 3.682 | 0.569 | -85.34% |
| Target minus industry leverage | 0.045 | 0.280 | -0.139 | -409.41% |
| Target volatility | 39.83% | 40.13% | 24.47% | -38.56% |
| Discounts | Sample | | Predicted | |
| | Mean | Standard deviation | Mean | Standard error |
| Marketability discount 1 | 13.15% | 22.19% | 37.13% | 21.22% |
| Marketability discount 2 | 12.50% | 22.27% | 38.42% | 20.11% |

This table illustrates the estimation of the marketability and control discounts for the controlling block of Big M, Inc., a chain of sports goods retail shops located in New Jersey, using the elasticities reported in Table 2, and the determinants of discounts with an effect significant to the 0.01 level (see Table3).

This exercise underscores the need to condition the measurement of the marketability discount on firm, industry and economy-wide characteristics. One of the main results from our exercise is that the marketability discount varies dramatically across firms, industries and time. Therefore, the application of unconditional averages can lead to biased results. While the Judge for Mandelbaum et al. v. Commissioner of Internal Revenue (1995) raised this concern, at the time there was no way to address it. Of course, the quantitative analysis that we propose is not to be seen as a rule of thumb to be applicable blindly. Instead, judgment is required to consider for example, any special circumstances, facts and variables ignored in the analysis, and the bargaining power of the buyer and seller in each transaction.[10] In addition, one reason for exercising judgement that transpires from our exercise is the high standard error of the estimated marketability discount. In the Mandelbaum case, the estimated mean value of *MD1* has a standard error of 21%.

## 6. Summary and Conclusions

The precise valuation of controlling blocks of shares must recognize the complex effects of the illiquidity of the market for corporate control. Evidence suggests that shares in these blocks trade in over the counter markets at prices that differ substantially from the share price that the same companies trade at in active markets. While it is possible and necessary to solve and estimate a model that spells out this complexity, it is also necessary to devise a way to use the model's results in a straightforward way.

We have analyzed the marketability discount and a laundry list of possible determinants of liquidity. We provided both a ranking of these determinants based on economic and statistical significance and a way of combining this information to estimate the marketability discount on blocks of shares.

Besides its simplicity, two important strengths of our results are that (i) they allow for firm, industry and macroeconomic determinants of liquidity to affect the marketability discount, and (ii) it can be quite accurate relative to the more exact approach in AS with remarkably little data. Economy-wide variables appear to capture well periods of drying up liquidity.

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[10] Knowing for example the prospects for a future IPO or the capacity of the firm to pay dividends may further contribute to improve these estimates.

The simplicity of our approach should not hide the complexity of the issue. The quantitative estimates that can be derived from using the elasticities we offer have to be judged based on the statistical significance of the estimates as well as based on information that has not been quantified in the analysis. For example, in any valuation analysis consideration may be given to behavioral factors such as the bargaining power of the buyer and seller, or non-behavioral factors that we omitted in the analysis. Also, consideration may be given to alternative ways of disposing the asset including the piecemeal sale of the block.

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