Buy-Ins, Buy-Outs, Longevity Bonds, and Value Creation

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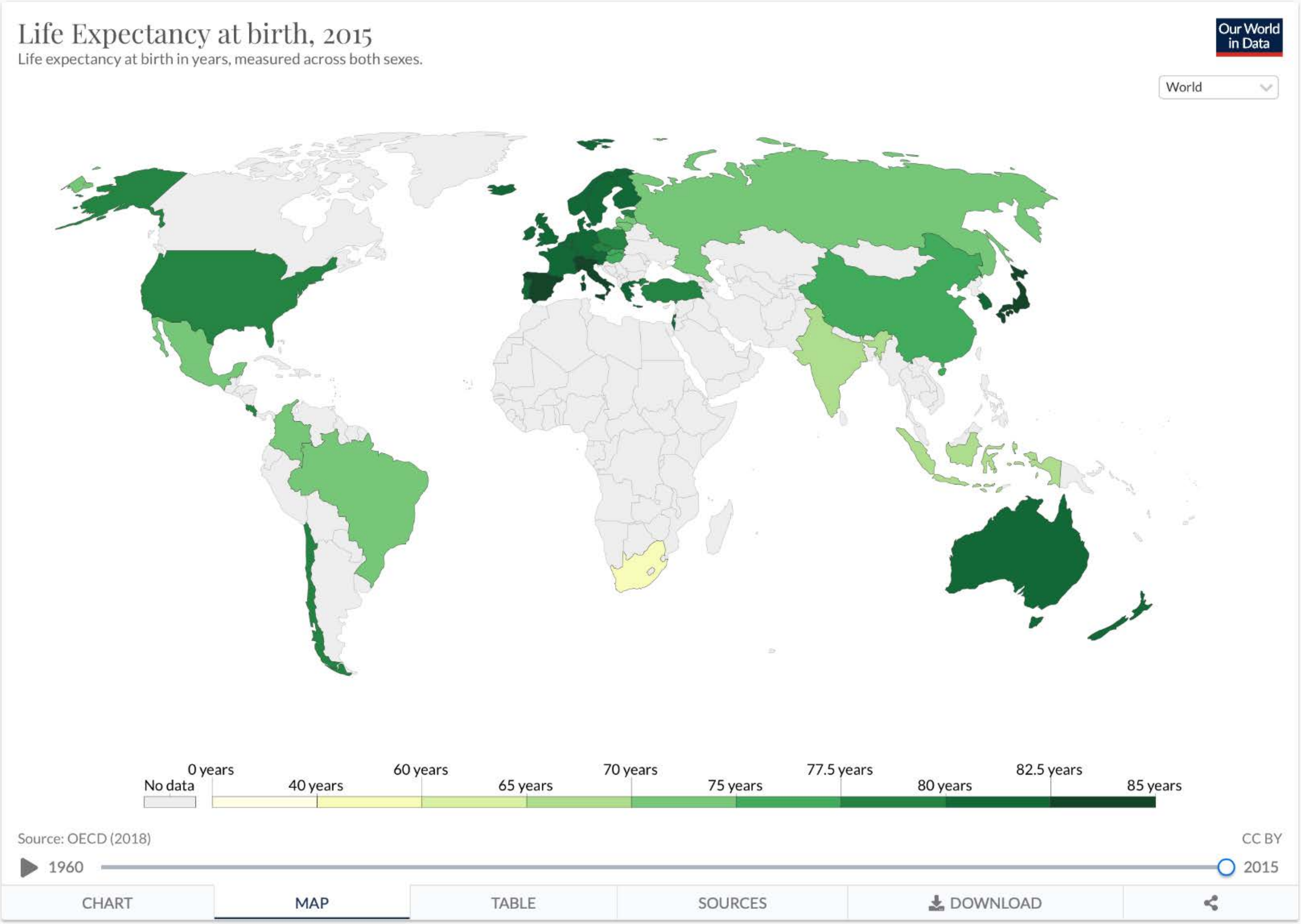
Agenda

- Longevity Risk
- Some of the Literature
- Corporate Objectives
- The Judgment Proof Problem and Under-Investment Problem
- Corporate Risk Management
  - Buy-in, Buy-out
  - Longevity Bonds
- The Creation of Value

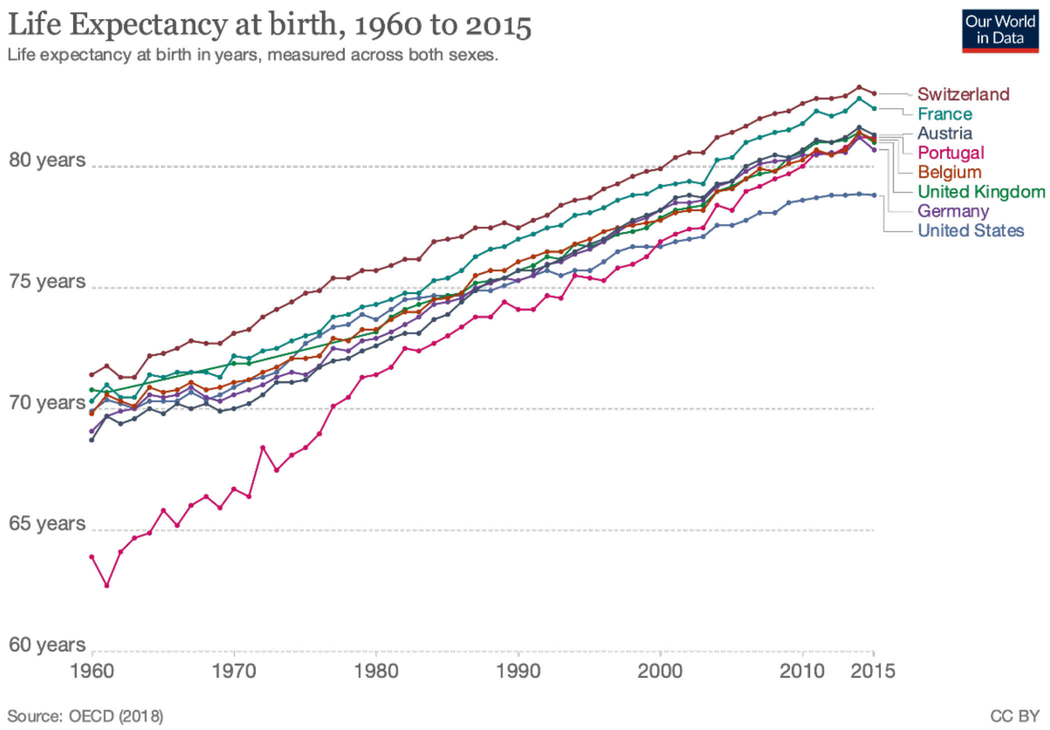
Longevity

- What is it?
- Who does it affect?
  - Individuals, insurers, re-insurers, pension funds, corporations, the markets for longevity risk transfer
- Why is it a risk?

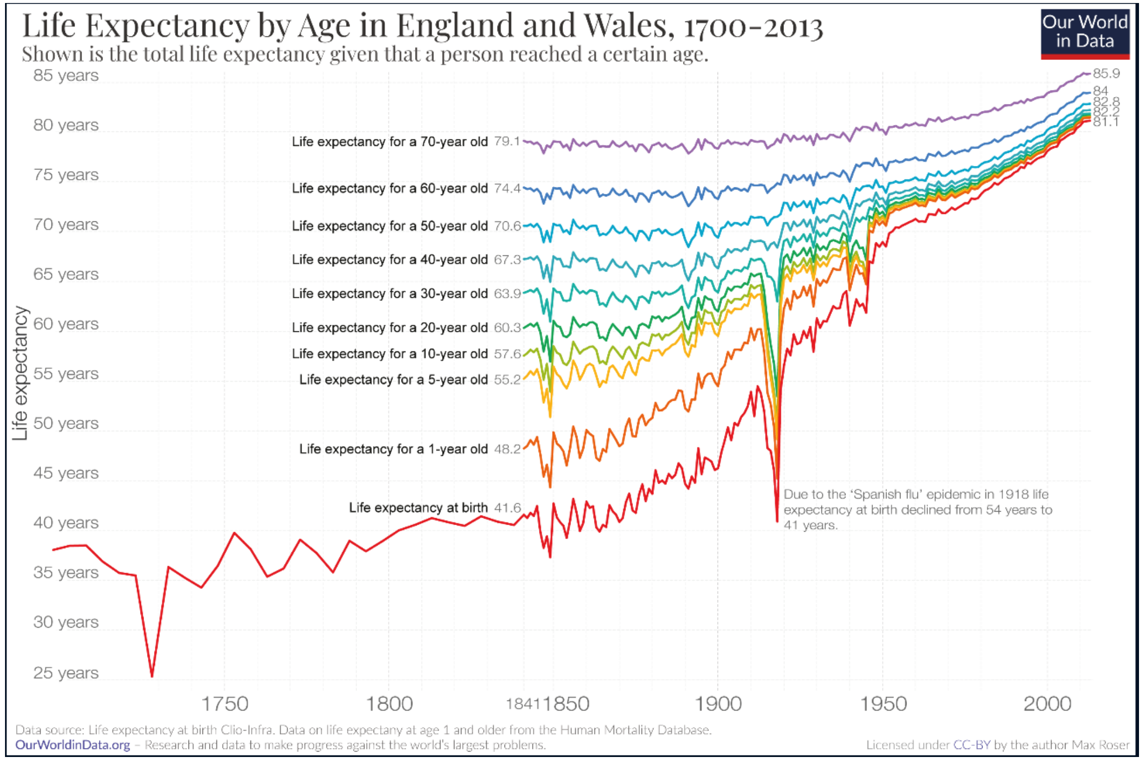


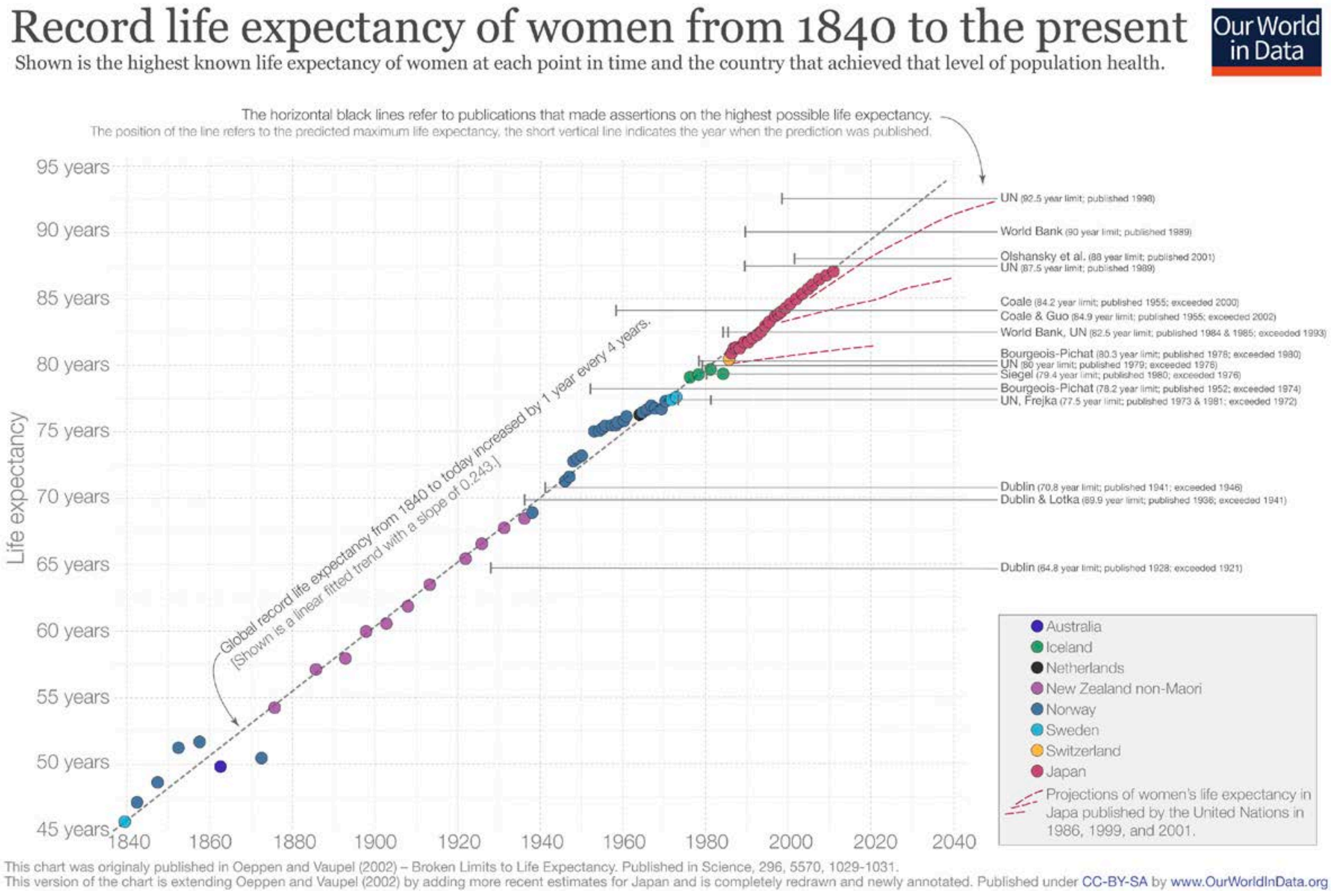
Life Expectation

Life Expectancy at Birth



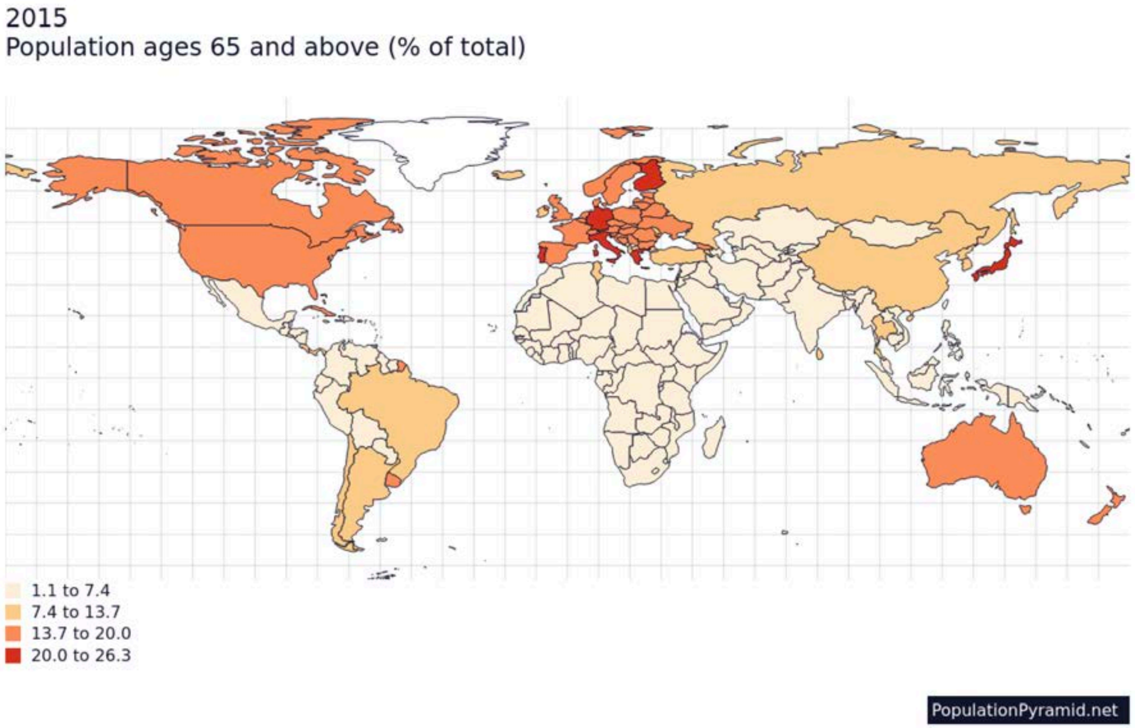
Life Expectancy by Age





Record Life Expectanc y

Aging Populations



- Population Pyramids for United States
- Population Pyramids for China
- Population Pyramids for Japan

literature

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Financial Markets

Notation	Description
ω	State of the economy
$\Omega \equiv [0, \zeta]$	Set of states of the economy
$p(\omega)$	Basis stock price: a promise to pay one dollar in state ω and zero otherwise
$P(\xi)$	Sum of the basis stock prices from zero to ξ; $P(\xi) = \int_0^\xi p(\omega)d\omega$
$\Pi(I, \omega)$	Random investment frontier, or equivalently, the payoff then from an investment of I dollars *now*. $D_1\Pi > 0$, $D_2\Pi > 0$
$A(\omega)$	Asset portfolio payoff for the pension fund; DA > 0
$L(\omega)$	Liability portfolio for the pension fund; DL > 0
$B(b)$	Safe zero-coupon bond value for a promised payment then of b dollars
$D(b)$	Risky zero-coupon bond value for a promised payment then of b dollars
P	Put option value
S	Stock value
V	Corporate value

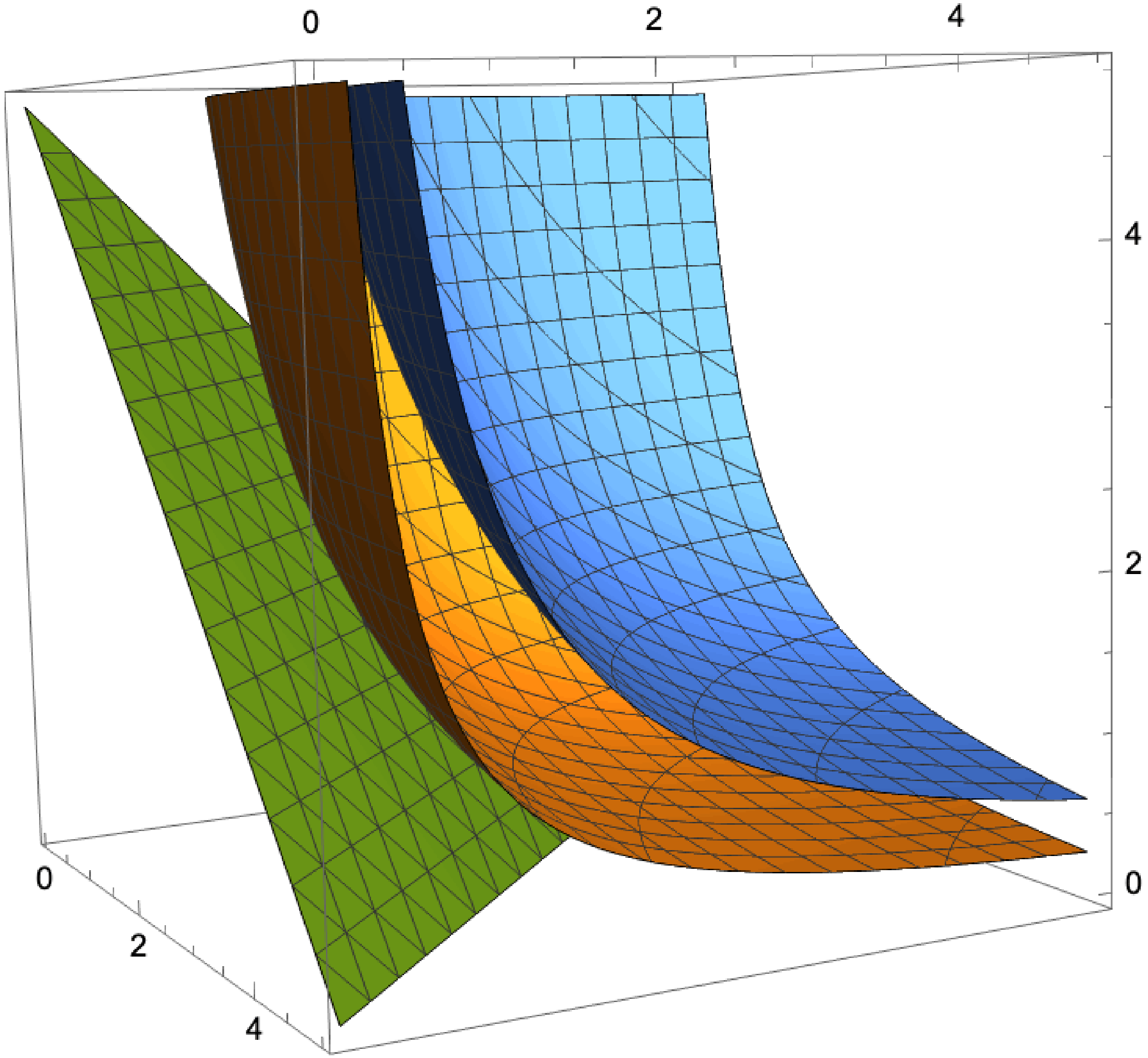
Fisher Model

Constrained Optimization Problem

Expected Utility

Constraints:

Budget Plane and Financing constraint



Financial Markets

Suppose that financial markets are complete, and that Ω represents the set of states of nature.

Recall that one basis stock exists for each state of nature and that stock pays one dollar in a particular state ω and zero otherwise. Hence all other assets can be described as a portfolio of the basis stock.

Let p(ω) denote the price of the basis stock.

Letting b be the promised repayment on a zero-coupon bond, $\Pi(I, \omega)$ be the corporate payoff and P(ξ) be the sum of the basis stock prices from zero to ξ, A be the asset portfolio payoff, L be the pension liability and S be the stock value. Then the stock value may be represented as

$$S^u = \int_0^{\zeta} max\{0, \Pi + A - L\}\, dP$$

The Corporate Objective Function

Suppose the corporate manager makes a portfolio decision on personal account and investment, risk management, and financing decisions on corporate account. If the manager is compensated with a salary and stock, then that manager makes decisions on corporate account to maximize the stock value of the current shareholders.

Suppose the manager does not make an investment decision. Then corporate payoff may be represented as $max\{0, \Pi + A - L\} = \Pi + A - L + max\{0, L - (\Pi + A)\}$

And so, the stock market value of the unhedged firm is

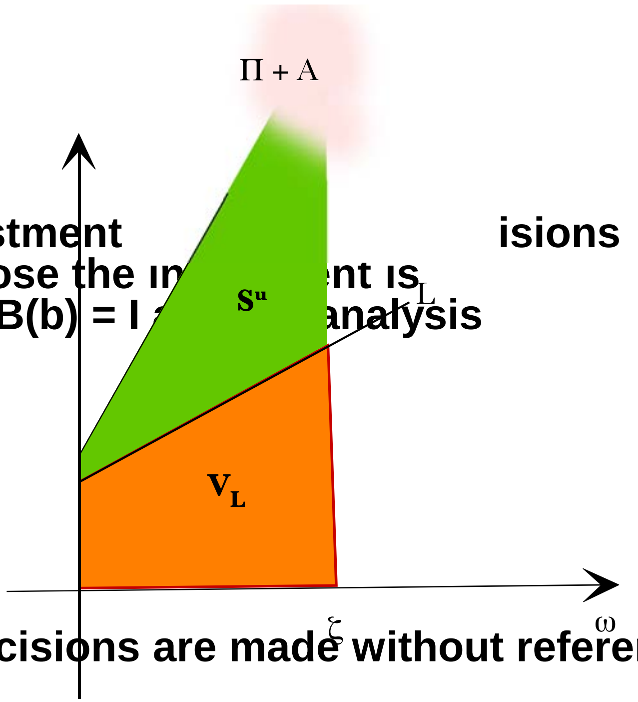
$$S^u = \int_0^{\zeta} (\Pi + A - L) dP + \int_0^{\zeta} max\{0, L - (\Pi + A)\} dP$$
$$= V + V_A - V_L + P^u$$
$$= V + \Delta + P^u$$

No Insolvency Risk

Consider the base case in which the pension plan is fully funded and the operation of the firm has a non-negative payoff.

$$S^u = \int_0^{\zeta} \Pi \, \mathrm{d}P + \int_0^{\zeta} (A - L)\, \mathrm{d}P = V + \Delta$$

Suppose the corporate manager (CEO) makes the investment isions on corporate account. Without loss of generality suppose the in ent is financed with a bond issue. Then b is selected so that B(b) = I analysis easily shows that the CEO makes decisions to



$$\textit{maximize } S^u(I)$$

$$\textit{subject to } B(b) = I$$

Note that this is a Fisher Separation result since the decisions are made without reference to the CEO's risk aversion.

Base Case: No Insolvency Risk

The Lagrange Function for this case is $\boldsymbol{L(I, b, \lambda) = S^u(I) + \lambda\,(B(b) - I)}$

The first order conditions are:

$$\boldsymbol{D_1 L = \int_0^{\zeta} D_1 \Pi \, dP - \lambda = 0}$$

$$\boldsymbol{D_2 L = -\int_0^{\zeta} dP + \lambda \int_0^{\zeta} dP = 0}$$

$$\boldsymbol{D_3 L = B(b) - I = 0}$$

It follow that the condition for an optimal investment is that for a socially efficient investment and satisfies the following condition:

$$\boldsymbol{\int_0^{\zeta} D_1 \Pi(I^e, \omega)\, dP(\omega) - 1 = 0}$$

It should also be noted that this is the condition for a maximum risk adjusted net present value since

$$npv = -I + \int_0^{\varsigma} \Pi \mathrm{d}P$$

Insolvency Risk

Next suppose the corporation is subject to insolvency risk due to an under funded pension plan or a corporate payoff insufficient to cover its operations and investments and its pension and debt liabilities.

The insolvency introduces the judgment proof problem

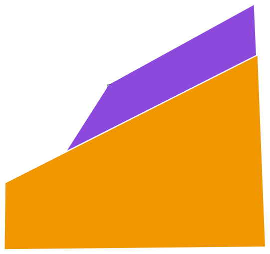
The corporate payoff may be rewritten as $max\{0, \Pi + A - L - b\} = \Pi + A - L - b + max\{0, L + b - (\Pi + A)\}$

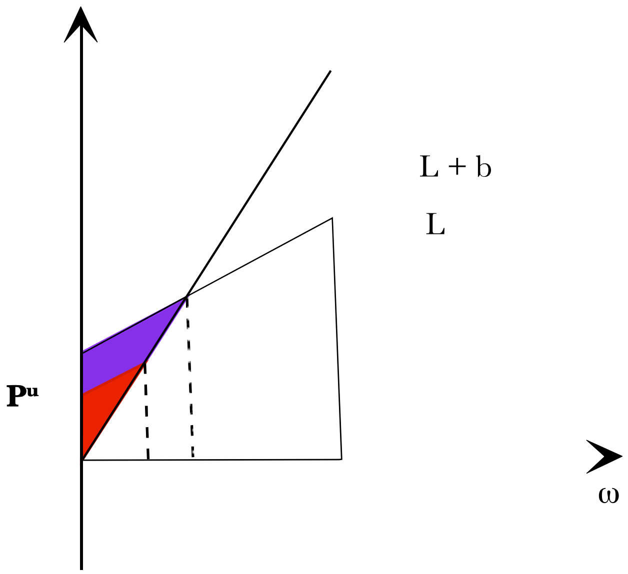
The unhedged stock value is

$$S^u = \int_0^\zeta (\Pi + A - L - b) dP + \int_0^\zeta max\{0, L + b - (\Pi + A)\} dP$$

$$= V + V_A - V_L - B(b) + \left(P^b + P^u\right)$$

The Unhedged Firm





Insolvency Risk and the Unhedged Firm

Now consider the investment decision made by the corporate manager. Given appropriately aligned incentives, the manager will select the investment to maximize the stock value subject to a financing constraint.

Without loss of generality, suppose the firm issues debt to cover the investment expenditure; let $D(b)$ denote the value of the debt, and b represent the promised payment *then*. The manager's constrained maximization problem is

$$\text{maximize } S^u$$
$$\text{subject to } D(b) = I$$

where

$$S^u = \int_{\gamma}^{\zeta} (\Pi + A - b - L)\, dP$$

and

$$D(b) = \int_{\delta}^{\gamma} (\Pi + A - L)\, dP + \int_{\gamma}^{\zeta} b\ dP$$

Case 1: Insolvency Risk and No Hedge

Let $L(I, b, \lambda) = S^u + \lambda(D(b) - I)$ be the Lagrange function for the constrained maximization problem. Then the first order conditions are as follows:

$$D_1 L = \int_\gamma^\zeta D_1 \Pi \, dP + \lambda \left(\int_\delta^\gamma D_1 \Pi \, dP - 1 \right) = 0$$

$$D_2 L = -\int_\gamma^\zeta dP + \lambda \int_\gamma^\zeta dP = 0$$

$$D_3 L = D(b) - I = 0$$

Let I^u denote the optimal investment decision for the manager. Note that the first-order conditions yield the following condition for an optimal investment

$$D_1 L = \int_\delta^\zeta D_1 \Pi \, dP - 1 = 0$$

Comparing first order conditions, it follows by the second-order condition that the manager selects $I^u < I^e$. Therefore, the unhedged firm underinvests relative to the socially efficient investment. The stock market value of the unhedged firm may also be written as follows:

$$S^u = V^u + V_A - V_L - B(b) + \left(P^b + P^u\right)$$

$$= V^u - \left(B(b) - P^b\right) + \Delta + P^u$$

$$= V^u - I + \Delta + P^u$$

$$= npv^u + \Delta + P^u$$

The Buy-In

The liability claims in the pension plan are one source of the insolvency risk for the firm. This risk becomes more of a problem for the firm as longevity increases. The firm can absorb the risk or chose to manage it in some way. Buy-in and buy-out instruments represent a growing segment of the markets for longevity risk transfer. The buy-in is considered first.

A buy-in is a transaction with an insurer in which the firm purchases an annuity that pays the pension claims as they are realized. Without loss of generality, we will suppose that the firm uses debt to finance the annuity purchase from an insurer. Then the constraint in the constrained maximization problem becomes

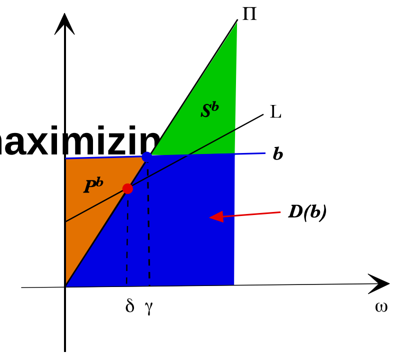
$$D(b) = I + \int_0^{\zeta} L \; dP$$

Hence, a debt instrument replaces the pension liability, and the stock value for the buy-in becomes

$$S^i = \int_{\gamma}^{\zeta} (\Pi + A - L - b + L) dP = \int_{\gamma}^{\zeta} (\Pi + A - b) dP$$

where S^i denotes the hedged corporate equity value. Similarly, let I^i denote the stock value maximizin investment decision.

Figure 2: The Hedged Firm



Case 2: Insolvency Risk and A Buy-In

The Lagrange function is $L(I, b, \lambda) = S^i + \lambda\left(D(b) - I - \int_0^{\zeta} L \; dP\right)$ and the first order conditions are as follows:

$$D_1 L = \int_{\gamma}^{\zeta} D_1 \Pi \, dP - \lambda\left(\int_0^{\gamma} D_1 \Pi \, dP - 1\right) = 0 \qquad (19)$$

$$D_2 L = -\int_{\gamma}^{\zeta} dP + \lambda \int_{\gamma}^{\zeta} dP = 0 \qquad (20)$$

$$D_3 L = D(b) - I - V_L = 0 \qquad (21)$$

From (20) and (19) it follows that $I^i = I^e > I^u$ and so the buy-in resolves the under-investment problem and maximizes the risk adjusted net present value

$$S^i = npv^i + \Delta \qquad (22)$$

A similar analysis shows that a buy-out may also be used to resolve the under-investment problem.

Case 3: Insolvency Risk and a Longevity Bond

Consider a longevity bond. Let B(ω) be the longevity bond payoff *then* in state ω. Suppose there is no basis risk so that B(ω) = L(ω). Then if the corporation hedges with a longevity bond, the corporate payoff becomes

$$\Pi + A - L + B - b = \Pi + A - b$$

Suppose the corporation raises the money for the investment and the longevity bond with a debt issue. Then the financing constraint is

$$D(b) = I + \int_0^\zeta B(\omega)\,\mathrm{d}P(\omega)$$

where the debt value is

$$D(b) = \int_0^\delta (\Pi + A)\,\mathrm{d}P + \int_\delta^\zeta b\,\mathrm{d}P$$

The Longevity Bond

The stock market value of the corporation with the longevity bond is

$$S^b = \int_{\delta}^{\zeta} (\Pi + A - b) \, \mathrm{d}P$$

The Lagrange function for this constrained maximization problem is

$$L(I, b, \lambda) = S^b(I, b) + \lambda \left(D(b) - I - \int_{0}^{\zeta} B(\omega) \, \mathrm{d}P \right)$$

and the first order conditions are:

$$D_1 L = \int_{\delta}^{\zeta} D_1 \Pi \, \mathrm{d}P + \lambda \left(\int_{0}^{\delta} D_1 \Pi \, \mathrm{d}P - 1 \right) = 0$$

$$D_2 L = - \int_{\delta}^{\zeta} \mathrm{d}P + \lambda \int_{\delta}^{\zeta} \mathrm{d}P = 0$$

The Longevity Bond

It follows that the optimal investment $I^b = I^e > I^u$ since the first order conditions yield

$$\int_0^{\zeta} D_1 \Pi \, dP = 1$$

It may also be noted that the stock value of the corporation hedged with a longevity bond may be equivalently expressed as

$$S^b = \int_0^{\zeta} max\{0, \Pi + A - b\} dP$$

$$= \int_0^{\zeta} (\Pi + A - b) \, dP + \int_0^{\zeta} max\{0, b - (\Pi + A)\} dP$$

$$= V + V_A - B(b) + P^b$$

$$= V + V_A - D(b)$$

$$= V - I + V_A - B_L$$

$$= npv + \Delta$$

The Longevity Bond

It follows that the longevity bond also solves the under-investment problem that exists for the unhedged firm.

If basis risk is introduced, then the payoff on the longevity bond is not perfectly correlated with the longevity risk of the pension fund then a longevity bond may still eliminate or reduce the under-investment.

Concluding Remarks

In the absence of positive net present value projects, the publicly held and traded corporation does not have an incentive to hedge longevity risk with any of the insurance or capital market instruments designed for hedging that risk.

Buy-ins and buy-outs have become the most popular instruments in hedging longevity risk and in the presence of insolvency risk these instruments can reduce or eliminate the agency costs associated with the under-investment problem.

Buy-ins and buy-outs transfer the risk from corporations to insurers, but the risk is stored by insurers and at some point, will become a risk that will threaten insolvency for the insurers.

The longevity bond is a capital market instrument that in theory can achieve the same risk reduction for the corporation and solve the under-investment problem. Preliminary analysis suggests that this is true even if there is basis risk.

Thank you!

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